Integration 11L

1 a $\lim_{\delta x \to 0} \sum_{x=3}^{12} x^2 \ \delta x = \int_3^{12} x^2 \mathrm{d}x$

This is the limit as the width of the strip tends towards 0.

$\int_3^{12} x^2 \mathrm{d}x = \left[\frac{x^3}{3}\right]_3^{12}$

$= 576 - 9$

$= 567$

b $\lim_{\delta x \to 0} \sum_{x=9}^{25} \sqrt{x} \ \delta x = \int_9^{25} \sqrt{x} \, \mathrm{d}x$

This is the limit as the width of the strip tends towards 0.

$\int_9^{25} \sqrt{x} \, \mathrm{d}x = \int_9^{25} x^{\frac{1}{2}} \, \mathrm{d}x$

$= \left[\frac{2x^{\frac{3}{2}}}{3}\right]_9^{25}$

$= \frac{250}{3} - \frac{54}{3}$

$= \frac{196}{3}$

c $\lim_{\delta x \to 0} \sum_{x=5}^{10} \left(x\sqrt{x-1}\right) \delta x = \int_5^{10} \left(x\sqrt{x-1}\right) \mathrm{d}x$

This is the limit as the width of the strip tends towards 0.

Let $I = \int_5^{10} \left(x\sqrt{x-1}\right) \mathrm{d}x$

Let $u = x - 1$

$\frac{\mathrm{d}u}{\mathrm{d}x} = 1$

So replace dx with du
and replace $\sqrt{x-1}$ with u
and replace x with $u + 1$

Change the limits:

x	u
10	9
5	4

c $I = \int_4^9 \left((u+1)u^{\frac{1}{2}}\right) \mathrm{d}u$

$= \int_4^9 \left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right) \mathrm{d}u$

$= \left[\frac{2u^{\frac{5}{2}}}{5} + \frac{2u^{\frac{3}{2}}}{3}\right]_4^9$

$= \frac{2 \times 9^{\frac{5}{2}}}{5} + \frac{2 \times 9^{\frac{3}{2}}}{3} - \frac{2 \times 4^{\frac{5}{2}}}{5} - \frac{2 \times 4^{\frac{3}{2}}}{3}$

$= \frac{486}{5} + \frac{54}{3} - \frac{64}{5} - \frac{16}{3}$

$= \frac{1456}{15}$

2 $\lim_{\delta x \to 0} \sum_{x=2}^{3} \ln x \ \delta x = \int_2^3 \ln x \, \mathrm{d}x$

This is the limit as the width of the strip tends towards 0.

Let $I = \int_2^3 \ln x \, \mathrm{d}x$

Let $u = \ln x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}$

$\frac{\mathrm{d}v}{\mathrm{d}x} = 1 \quad \Rightarrow v = x$

$I = [x \ln x]_2^3 - \int_2^3 x \times \frac{1}{x} \mathrm{d}x$

$= 3\ln 3 - 2\ln 2 - \int_2^3 1 \mathrm{d}x$

$= \ln 3^3 - \ln 2^2 - [x]_2^3$

$= \ln\left(\frac{27}{4}\right) - 3 + 2$

$= -1 + \ln\left(\frac{27}{4}\right)$

3 $\lim_{\delta x \to 0} \sum_{x=2}^{5} \sqrt[3]{x} \ \delta x = \int_2^5 \sqrt[3]{x} \ dx$

This is the limit as the width of the strip tends towards 0.

$$\int_2^5 \sqrt[3]{x} \ dx = \int_2^5 x^{\frac{1}{3}} \ dx$$

$$= \left[\frac{3x^{\frac{4}{3}}}{4} \right]_2^5$$

$$= 6.41240... - 1.88988...$$

$$= 4.523 \text{ (4 s.f.)}$$