

Practice Exam Paper

$$1 \text{ a } mv \frac{dv}{dx} = -4x$$

$$m = \frac{3}{2}$$

$$\frac{3}{2}v \frac{dv}{dx} = -4x$$

$$v \frac{dv}{dx} = -\frac{8}{3}x$$

$$\frac{1}{2}v^2 = -\frac{4}{3}x^2 + c$$

$$v^2 = -\frac{8}{3}x^2 + c$$

When $v = 8$, $x = 0$

$$(8)^2 = -\frac{8}{3}(0)^2 + c$$

$$c = 64$$

Therefore

$$v^2 = -\frac{8}{3}x^2 + 64$$

b When the particle stops $v = 0$, therefore:

$$(0)^2 = -\frac{8}{3}x^2 + 64$$

$$\frac{8}{3}x^2 = 64$$

$$x^2 = 24$$

$$x = 2\sqrt{6} \text{ m}$$

$$2 \text{ a } S = \pi \int_0^{\frac{\pi}{4}} y^2 dx$$

$$= \pi \int_0^{\frac{\pi}{4}} \cos^2 x dx$$

$$= \frac{\pi}{2} \int_0^{\frac{\pi}{4}} (1 + \cos 2x) dx$$

$$= \frac{\pi}{2} \left[x + \frac{1}{2} \sin 2x \right]_0^{\frac{\pi}{4}}$$

$$= \frac{\pi}{2} \left[\frac{\pi}{4} + \frac{1}{2}(1) \right]$$

$$= \frac{\pi^2}{8} + \frac{\pi}{4}$$

$$= \frac{\pi}{8}(\pi + 2) \text{ as required.}$$

$$\mathbf{b} \quad \bar{x} = \frac{\int_0^{\frac{\pi}{4}} xy^2 dx}{\int_0^{\frac{\pi}{4}} y^2 dx}$$

From part a:

$$\int_0^{\frac{\pi}{4}} y^2 dx = \frac{\pi}{8}(\pi + 2)$$

$$\int_0^{\frac{\pi}{4}} xy^2 dx = \int_0^{\frac{\pi}{4}} x \cos^2 x dx$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{4}} x(1 + \cos 2x) dx$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{4}} x dx + \frac{1}{2} \int_0^{\frac{\pi}{4}} x \cos 2x dx$$

$$= \frac{1}{4} \left[x^2 \right]_0^{\frac{\pi}{4}} + \frac{1}{2} \left[\frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x \right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{4} \left[x^2 \right]_0^{\frac{\pi}{4}} + \frac{1}{4} \left[x \sin 2x \right]_0^{\frac{\pi}{4}} + \frac{1}{8} \left[\cos 2x \right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{4} \left[\frac{\pi^2}{16} \right] + \frac{1}{4} \left[\frac{\pi}{4} (1) \right] - \frac{1}{8} [1]$$

$$= \frac{\pi^2}{64} + \frac{\pi}{16} - \frac{1}{8}$$

$$= \frac{\pi^2 + 4\pi - 8}{64}$$

$$\bar{x} = \frac{\frac{\pi^2 + 4\pi - 8}{64}}{\frac{1}{8}(\pi + 2)}$$

$$= \frac{\pi^2 + 4\pi - 8}{8(\pi + 2)}$$

$$3 \text{ a } F = \frac{GmM}{x^2}$$

$$G = \frac{gR^2}{M}$$

$$F = \frac{gR^2}{M} \times \frac{mM}{x^2} \\ = \frac{mgR^2}{x^2}$$

$$3 \text{ b } mv \frac{dv}{dx} = -\frac{mgR^2}{x^2}$$

$$v \frac{dv}{dx} = -\frac{gR^2}{x^2}$$

$$\int_{2U}^0 v \, dv = -gR^2 \int_{\frac{3R}{2}}^x \frac{1}{x^2} \, dx$$

$$\int_{2U}^0 v \, dv = -gR^2 \int_{\frac{3R}{2}}^x \frac{1}{x^2} \, dx$$

$$\frac{1}{2} [v^2]_{2U}^0 = -gR^2 \left[-\frac{1}{x} \right]_{\frac{3R}{2}}^x$$

$$-\frac{1}{2} (2U)^2 = gR^2 \left(\frac{1}{x} - \frac{2}{3R} \right)$$

$$-2U^2 = gR^2 \left(\frac{3R - 2x}{3xR} \right)$$

$$-6U^2 xR = 3gR^3 - 2gR^2 x$$

$$-6U^2 xR + 2gR^2 x = 3gR^3$$

$$x = \frac{3gR^2}{2(gR - 3U^2)}$$

This is from the centre of the Earth. We need from the surface, so using $h = x - R$ gives:

$$h = \frac{3gR^2}{2(gR - 3U^2)} - R$$

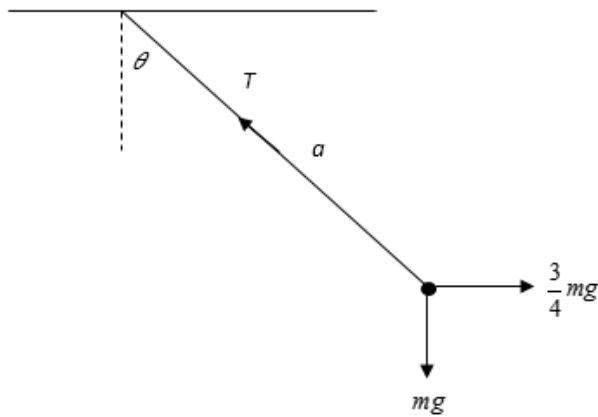
$$h = \frac{3gR^2 - 2R(gR - 3U^2)}{2(gR - 3U^2)}$$

$$= \frac{3gR^2 - 2gR^2 + 6RU^2}{2(gR - 3U^2)}$$

$$= \frac{gR^2 + 6RU^2}{2(gR - 3U^2)}$$

$$= \frac{R(gR + 6U^2)}{2(gR - 3U^2)} \text{ as required.}$$

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$$\text{Res}(\leftarrow) \quad T \cos \theta = \frac{3}{4}mg$$

$$\text{Res}(\uparrow) \quad T \sin \theta = mg$$

$$\frac{T \sin \theta}{T \cos \theta} = \frac{mg}{\frac{3}{4}mg}$$

$$\tan \theta = \frac{4}{3} \Rightarrow \sin \theta = \frac{4}{5} \text{ and } \cos \theta = \frac{3}{5}$$

$$\frac{4}{5}T = mg$$

$$T = \frac{5}{4}mg$$

By Hooke's law:

$$T = \frac{\lambda x}{l}$$

$$x = \frac{Tl}{\lambda}$$

Substituting gives:

$$x = \frac{\frac{5}{4}mga}{2mg}$$

$$= \frac{5}{8}a$$

5 (to come)

- 6 a Let the P have zero potential energy.

$$\text{Initial KE} = \frac{1}{2} \times 2m \times u^2 = mu^2$$

$$\text{Initial PE} = -2mg \times \frac{3a}{2} = -3mga$$

$$\text{Generally KE} = \frac{1}{2} \times 2m \times v^2 = mv^2$$

$$\text{and PE} = -2mg \times \frac{3a}{2} \cos \theta = -3mga \cos \theta$$

Using conservation of energy:

$$mu^2 - 3mga = mv^2 - 3mga \cos \theta$$

$$v^2 = u^2 - 3ga + 3ga \cos \theta$$

The force acting towards the centre is:

$$T - 2mg \cos \theta = \frac{2mv^2}{\frac{3}{2}a}$$

$$T = \frac{4m(u^2 - 3ga + 3ga \cos \theta)}{3a} + 2mg \cos \theta$$

$$= \frac{4mu^2}{3a} - 4mg + 6mg \cos \theta$$

At the top, $T \geq 0$ and $\theta = 180^\circ$

$$\frac{4mu^2}{3a} - 4mg - 6mg \geq 0$$

$$\frac{4mu^2}{3a} \geq 10mg$$

$$u^2 \geq \frac{15}{2}ga$$

$$u \geq \sqrt{\frac{15}{2}ga}$$

- b i When $u = \sqrt{5ga}$ and $v = \sqrt{ga}$

Substituting into $v^2 = u^2 - 3ga + 3ga \cos \theta$ gives:

$$ga = 5ga - 3ga + 3ga \cos \theta$$

$$3 \cos \theta = -1$$

$$\cos \theta = -\frac{1}{3}$$

$$\theta = 109.5^\circ \text{ (4 s.f.)}$$

ii height = $\frac{3}{2}a(1 - \cos \theta)$

$$= \frac{3}{2}a \left(1 - \left(-\frac{1}{3} \right) \right)$$

$$= 2a$$

7 a Res(\uparrow) $T = 2g = 19.6$

By Hooke's law:

$$T = \frac{\lambda x}{l} = \frac{49x}{1.2}$$

$$\frac{49x}{1.2} = 19.6$$

$$x = 0.48 \text{ m}$$

b Using $F = ma$

$$2g - T = 2\ddot{x}$$

$$2g - \frac{49}{1.2}(0.48 + x) = 2\ddot{x}$$

$$2\ddot{x} = -\frac{245}{6}x$$

$$\ddot{x} = -\frac{245}{12}x \text{ therefore SHM}$$

$$\omega = \sqrt{\frac{245}{12}} \text{ and } T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\sqrt{\frac{245}{12}}}$$

$$T^2 = \frac{4\pi^2}{\frac{245}{12}}$$

$$= \frac{48\pi^2}{245}$$

$$T = \frac{4}{7}\pi\sqrt{\frac{3}{5}} \text{ s}$$

c $v^2 = \omega^2(a^2 - x^2)$

At E, $v = \frac{2}{35}$ and $x = 0$

Substituting gives:

$$\frac{4}{1225} = \frac{245}{12}a^2$$

$$a = 0.0126 \text{ m}$$

Therefore the particle moves 0.0126 m before coming to rest.