Practice exam paper

1 a $u_y = 20\sin 30 = 10 \text{ m s}^{-1}$, $v_y = 0$, $a_y = -9.8 \text{ m s}^{-2}$

Using $v^2 = u^2 = 2as$ gives:

$(0)^2 = (10)^2 + 2(-9.8)s$

$19.6s = 100$

$s = \frac{250}{49}$ m

Therefore the greatest height of the projectile, h, is:

$h = \frac{250}{49} + 10$

$= \frac{740}{49}$ m

$= 15.1$ m (3 s.f.)

b At the highest point in the flight of the projectile:

$u_y = 0$, $s_y = -\frac{740}{49}$ m, $a_y = -9.8 \text{ m s}^{-2}$

Using $v^2 = u^2 + 2as$ gives:

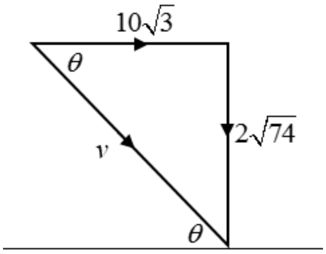
$v_y^2 = (0)^2 + 2(-9.8)\left(-\frac{740}{49}\right)$

$= 296$

$v_y = \pm 2\sqrt{74}$

$v_x = 20\cos 30$

$= 10\sqrt{3}$



$v = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{(2\sqrt{74})^2 + (10\sqrt{3})^2}$

$= 2\sqrt{149} \text{ m s}^{-1}$

$= 24.4 \text{ ms}^{-1}$ (3 s.f.)

c $\tan\theta = \frac{2\sqrt{74}}{10\sqrt{3}}$

$\theta = 44.807...$

$= 44.8°$ (3 s.f.)

2 a $v = \dfrac{2t^2 - 3t - 2}{\sqrt{t}}$ for $t \geq 1$

To find the time when the particle changes direction:

$2t^2 - 3t - 2 = 0$

$(2t+1)(t-2) = 0$

$t = -\dfrac{1}{2}$ or $t = 2$

Since $t \geq 1$, $t = 2$

So the particle changes direction at $t = 2$ s

$$v = \frac{2t^2 - 3t - 2}{\sqrt{t}}$$

$$= 2t^{\frac{3}{2}} - 3t^{\frac{1}{2}} - 2t^{-\frac{1}{2}}$$

$$s = \int_1^2 \left(2t^{\frac{3}{2}} - 3t^{\frac{1}{2}} - 2t^{-\frac{1}{2}}\right) dt + \int_2^5 \left(2t^{\frac{3}{2}} - 3t^{\frac{1}{2}} - 2t^{-\frac{1}{2}}\right) dt$$

Total distance travelled, d, is:

$$d = \int_1^2 \left|\left(2t^{\frac{3}{2}} - 3t^{\frac{1}{2}} - 2t^{-\frac{1}{2}}\right)\right| dt + \int_2^5 \left|\left(2t^{\frac{3}{2}} - 3t^{\frac{1}{2}} - 2t^{-\frac{1}{2}}\right)\right| dt$$

$$= \left|\left[\frac{4}{5}t^{\frac{5}{2}} - 2t^{\frac{3}{2}} - 4t^{\frac{1}{2}}\right]_1^2\right| + \left|\left[\frac{4}{5}t^{\frac{5}{2}} - 2t^{\frac{3}{2}} - 4t^{\frac{1}{2}}\right]_2^5\right|$$

$$= \left|\left[\left(\frac{4}{5}(2)^{\frac{5}{2}} - 2(2)^{\frac{3}{2}} - 4(2)^{\frac{1}{2}}\right) - \left(\frac{4}{5}(1)^{\frac{5}{2}} - 2(1)^{\frac{3}{2}} - 4(1)^{\frac{1}{2}}\right)\right]\right|$$

$$+ \left|\left[\left(\frac{4}{5}(5)^{\frac{5}{2}} - 2(5)^{\frac{3}{2}} - 4(5)^{\frac{1}{2}}\right) - \left(\frac{4}{5}(2)^{\frac{5}{2}} - 2(2)^{\frac{3}{2}} - 4(2)^{\frac{1}{2}}\right)\right]\right|$$

$$= \left|\left[(-6.788...) - (-5.2)\right]\right| + \left|\left[(13.416...) - (-6.788...)\right]\right|$$

$$= |-1.588...| + |20.204...|$$

$$= 21.792...$$

$$= 21.8 \text{ m (3 s.f.)}$$

b $v = 2t^{\frac{3}{2}} - 3t^{\frac{1}{2}} - 2t^{-\frac{1}{2}}$

$$a = \frac{dv}{dt} = 3t^{\frac{1}{2}} - \frac{3}{2}t^{-\frac{1}{2}} + t^{-\frac{3}{2}}$$

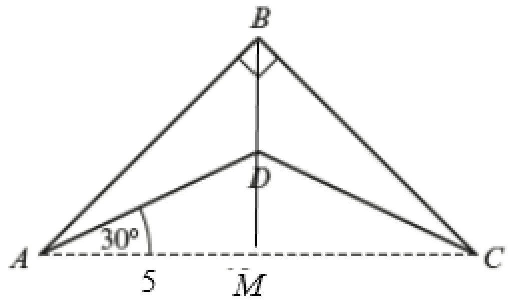
When $t = 2$:

$$\frac{dv}{dt} = 3(2)^{\frac{1}{2}} - \frac{3}{2}(2)^{-\frac{1}{2}} + (2)^{-\frac{3}{2}}$$

$$= 3.535...$$

$$= 3.54 \text{ ms}^{-2}$$

3



$\tan 45 = \frac{MB}{5}$

$MB = 5$

Area of $ABC = \frac{1}{2}\times 10\times 5$

$= 25\text{ cm}^2$

The centre of mass of ABC lies:

$\frac{1}{3}\times 5 = \frac{5}{3}$ cm above M

$\tan 30 = \frac{MD}{5}$

$MD = \frac{5\sqrt{3}}{3}$

Area of $ADC = \frac{1}{2}\times 10\times \frac{5\sqrt{3}}{3}$

$= \frac{25\sqrt{3}}{3}\text{ cm}^2$

The centre of mass of ADC lies:

$\frac{1}{3}\times\frac{5\sqrt{3}}{3} = \frac{5\sqrt{3}}{9}$ cm above M

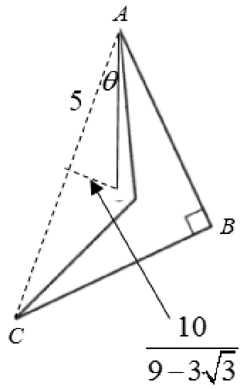
$$\left(25 - \frac{25\sqrt{3}}{3}\right)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 25\begin{pmatrix}5\\ \frac{5}{3}\end{pmatrix} - \frac{25\sqrt{3}}{3}\begin{pmatrix}5\\ \frac{5\sqrt{3}}{9}\end{pmatrix}$$

$$\left(\frac{75-25\sqrt{3}}{3}\right)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}125\\ \frac{125}{3}\end{pmatrix} - \begin{pmatrix}\frac{125\sqrt{3}}{3}\\ \frac{125}{9}\end{pmatrix}$$

$$= \begin{pmatrix}\frac{375-125\sqrt{3}}{3}\\ \frac{250}{9}\end{pmatrix}$$

3 (continued)

$$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\begin{pmatrix}5\\ \dfrac{10}{9-3\sqrt{3}}\end{pmatrix}$$



$$\tan\theta=\frac{\dfrac{10}{9-3\sqrt{3}}}{5}$$

$\theta=27.734...$

$\angle BAC=45°$

Therefore AB makes an angle of:

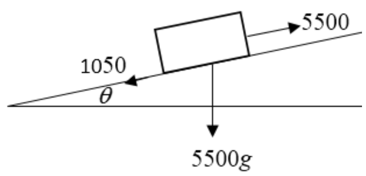
$45 - 27.734\ldots = 17.265\ldots$

$= 17.3°$ with the vertical

4 a power $= Fv$

$110\,000=20F$

$F=5500$ N



Res(↗) $5500-1050-5500g\sin\theta=0$

$$\sin\theta=\frac{5500-1050}{5500g}$$

$$=\frac{89}{1078}$$

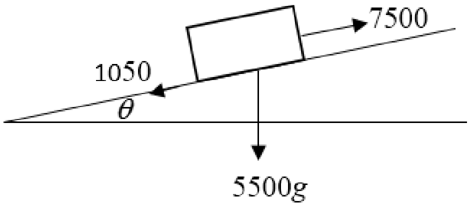
$$\theta=\sin^{-1}\left(\frac{89}{1078}\right)$$

$= 4.74°$ (3 s.f.)

4 b power $= Fv$

$150\,000 = 20F$

$F = 7500$ N



Res(↗) $7500 - 1050 - 5500g\sin\theta = 5500a$

$5500a = 7500 - 1050 - 4450$

$a = \frac{4}{11}$ m s^{-2}

$= 0.364\text{ms}^{-2}$ (3 s.f.)

5 a By the conservation of momentum

Momentum before = momentum after

$2m \times 4 + m \times 2 = 2mv_P + mv_Q$

$2v_P + v_Q = 10 \quad (1)$

By Newtons's law of restitution

$e = \frac{\text{speed of separation}}{\text{speed of approach}}$

$0.8 = \frac{v_Q - v_P}{2}$

$v_Q = 1.6 + v_P \quad (2)$

Substituting (2) into (1) gives:

$2v_P + (1.6 + v_P) = 10$

$3v_P = 8.4$

$v_P = 2.8$ m s^{-1}

Since $v_Q = 1.6 + v_P$

$v_Q = 4.4$ m s^{-1}

b Before the collision:

$E_{\text{before}} = \frac{1}{2} \times 2m \times 4^2 + \frac{1}{2} \times m \times 2^2$

$= 18m$

After the collision:

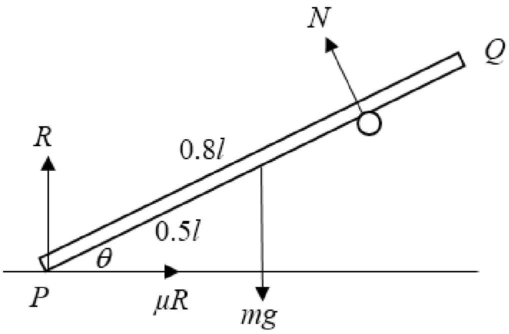
$E_{\text{after}} = \frac{1}{2} \times 2m \times 2.8^2 + \frac{1}{2} \times m \times 4.4^2$

$= 17.52m$

Percentage KE lost $= \frac{18 - 17.52}{18} \times 100$

$= \frac{8}{3}\%$

6 a



Taking moments about P gives:

$mg \times 0.5l\cos\theta = 0.8lN$

$N = \frac{5}{8}mg\cos\theta$

b When $\theta = 30°$

$N = \frac{5\sqrt{3}}{16}mg \qquad (1)$

Res (↑) $R + N\cos\theta = mg$

$R = mg - \frac{5\sqrt{3}}{16}mg\cos\theta$

$= mg - \frac{5\sqrt{3}}{16}mg\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{17}{32}mg \qquad$ **(2)**

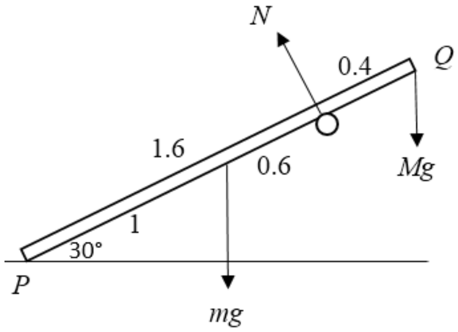
Res(→) $N\sin\theta = \mu R \qquad (3)$

Substituting (1) and (2) into (3) gives:

$\frac{5\sqrt{3}}{16}mg\left(\frac{1}{2}\right) = \frac{17}{32}\mu mg$

$\mu = \frac{5\sqrt{3}}{17}$ as required

6 c



Taking moments about the pivot gives:

$0.4Mg = 0.6mg$

$M = 1.5m$