Exercise 4B

1 a Let X be the random discrete variable $X \sim \text{Po}(3)$ and let T denote the sum of the 10 sample observations, so $T \sim \text{Po}(10\times 3)$, i.e. $T \sim \text{Po}(30)$

If the sample mean = 2.5, then $T = 10\times 2.5 = 25$. So the probability that the sample mean is less than 25 is $\text{P}(T \leqslant 25)$

By calculation $\text{P}(T \leqslant 25) = 0.2084$ (4 d.p.)

b By the central limit theorem, $\bar{X} \approx\sim \text{N}\left(3, \frac{3}{10}\right)$, i.e. $X \approx\sim \text{N}(3, 0.3)$

Using a calculator, $\text{P}(\bar{X} \leqslant 2.5) = 0.1807$ (4 d.p.)

The two answers are not very close. This is because the estimate found in part **b** is not very accurate as the sample size is too small.

2 $X \sim \text{B}(10, 0.2)$

$\text{E}(X) = np = 10\times 0.2 = 2$

$\text{Var}(X) = np(1-p) = 2\times 0.8 = 1.6$

By the central limit theorem $\bar{X} \approx\sim \text{N}\left(2, \frac{1.6}{20}\right)$, i.e. $\bar{X} \approx\sim \text{N}(2, 0.8)$

$\text{P}(\bar{X} \leqslant 2.4) \approx 0.9214$ (4 d.p.)

4 a Let X be the number of thunderstorms hitting the town each month, then $X \sim \text{Po}(3)$

$\text{P}(X = 4) = \frac{\text{e}^{-3}3^4}{4!} = 0.1680$ (4 d.p.)

b $\text{E}(X) = \text{Var}(X) = 3$

By the central limit theorem $\bar{X} \approx\sim \text{N}\left(3, \frac{3}{12}\right)$, i.e. $\bar{X} \approx\sim \text{N}(3, 0.25)$

$\text{P}(\bar{X} \leqslant 2.5) \approx 0.1587$ (4 d.p.)

5 $$\mathrm{E}(X)=\frac{a+b}{2}$$

$$=\frac{(a-3)+(3a+5)}{2}$$

$$=2a+1$$

$$\mathrm{Var}(X)=\frac{1}{12}(b-a)^2$$

$$=\frac{1}{12}\left[(3a+5)-(a-3)\right]^2$$

$$=\frac{1}{12}(2a+8)^2$$

$$=\frac{4}{12}(a+4)^2$$

$$=\frac{(a+4)^2}{3}$$

Therefore:

$$X\sim N\left(2a+1,\frac{\frac{(a+4)^2}{3}}{40}\right)$$

$$X\sim N\left(2a+1,\frac{(a+4)^2}{120}\right)$$

6 **a** Let the discrete random variable C be the number of calls received by the telephonist in the five-minute period before her break, then $C\sim\mathrm{Po}(10)$. Let T be the total number of calls received in this period for the 30 days the telephonist records the calls, then $T=30\bar{C}$

By the central limit theorem $\bar{C}\approx\sim\mathrm{N}\left(10,\frac{10}{30}\right)$

$$\mathrm{P}(T>350)=\mathrm{P}\left(\bar{C}>\frac{350}{30}\right)=1-\mathrm{P}\left(\bar{C}<\frac{350}{30}\right)\approx 1-0.9981=0.0019\text{ (4 d.p.)}$$

b $\mathrm{P}(C<9)\approx 0.0416$ (4 d.p.)