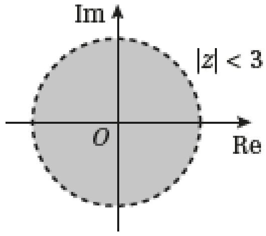
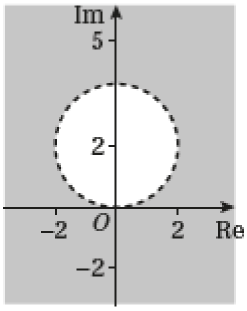
Exercise 4C

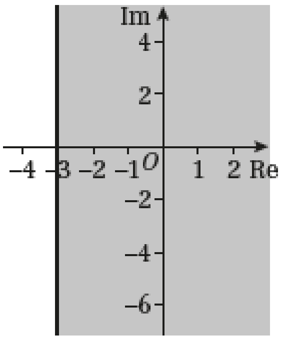
1 a $|z| < 3$



b $|z - 2\text{i}| > 2$



c $|z + 7| \geq |z - 1|$



1 d $|z+6| > |z+2+8\text{i}|$

$|x+y\text{i}+6| > |x+y\text{i}+2+8\text{i}|$

$|(x+6)+y\text{i}| > |(x+2)+\text{i}(y+8)|$

$|(x+6)+y\text{i}|^2 > |(x+2)+\text{i}(y+8)|^2$

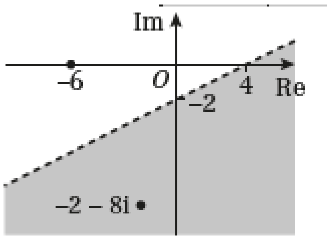
$(x+6)^2+y^2 > (x+2)^2+(y+8)^2$

$x^2+12x+36+y^2 > x^2+4x+4+y^2+16y+64$

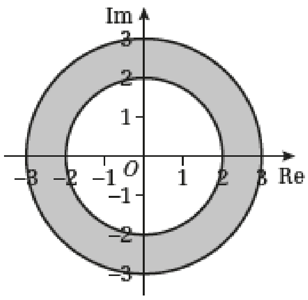
$8x+36 > 16y+68$

$16y < 8x-32$

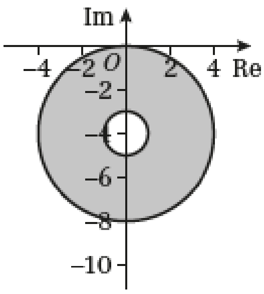
$y < \frac{1}{2}x-2$



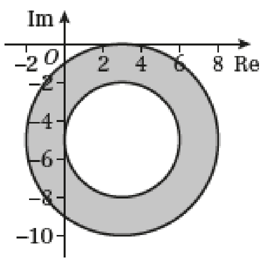
e $2 \leq |z| \leq 3$



f $1 \leq |z+4\text{i}| \leq 4$



g $1 \leq |z+4\text{i}| \leq 4$



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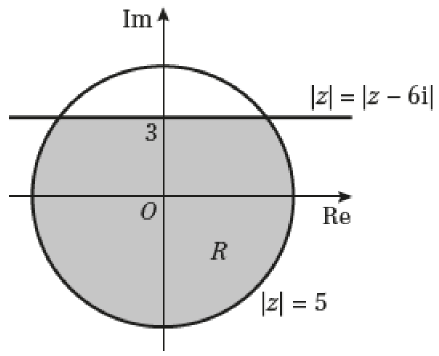


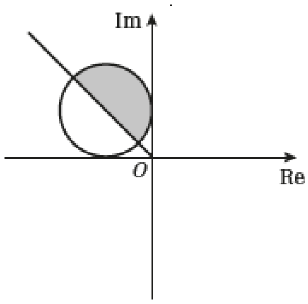
$|z| \leqslant 5$

$|z| \leqslant |z-6\mathrm{i}|$

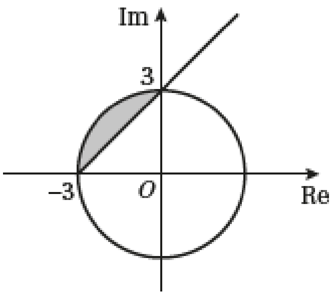
$|z| = 5$ represents a circle centre $(0, 0)$, radius 5

$|z| = |z-6\mathrm{i}|$ represents a perpendicular bisector of the line joining $(0, 0)$, to $(0, 6)$ and has the equation $y = 3$.

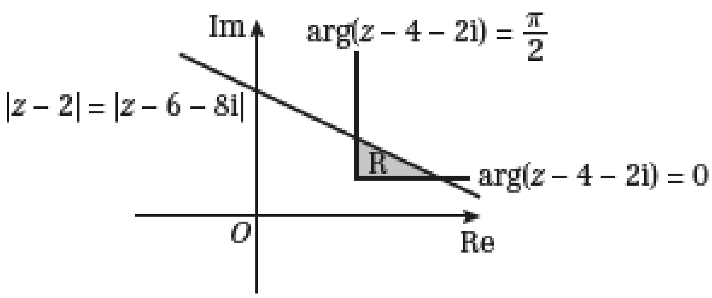
3 $|z+1-\mathrm{i}| \leq 1$ and $0 \leq \arg z \leq \frac{3\pi}{4}$



4 $\{z \in \mathbb{C} : |z| \leq 3\} \cap \left\{z \in \mathbb{C} : \frac{\pi}{4} \leq \arg(z+3) \leq \pi\right\}$



5 a (i-iii), b



$|z-2| = |z-6-8\mathrm{i}|$ represents a perpendicular bisector of the line joining (2, 0) to (6, 8).

6 a i $|z+10| = |z-6-4\sqrt{2}\text{i}|$

$|x+y\text{i}+10| = |x+y\text{i}-6-4\sqrt{2}\text{i}|$

$|(x+10)+y\text{i}| = |(x-6)+\text{i}(y-4\sqrt{2})|$

$|(x+10)+y\text{i}|^2 = |(x-6)+\text{i}(y-4\sqrt{2})|^2$

$(x+10)^2+y^2 = (x-6)^2+(y-4\sqrt{2})^2$

$x^2+20x+100+y^2 = x^2-12x+36+y^2-8\sqrt{2}y+32$

$32x+32 = -8\sqrt{2}y$

$y = -2\sqrt{2}x-2\sqrt{2}$ **(1)**

ii $|z+1| = 3$ is the circle (−1, 0) and radius 3

Therefore:

$(x+1)^2+y^2 = 9$ **(2)**

b $(x+1)^2+(-2\sqrt{2}x-2\sqrt{2})^2 = 9$

$x^2+2x+1+8x^2+16x+8 = 9$

$9x^2+18x = 0$

$9x(x+2) = 0$

$x = 0$ or $x = -2$

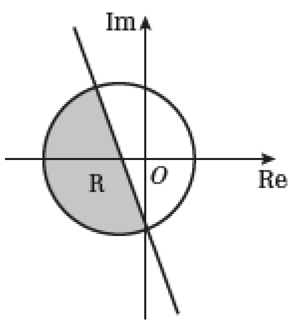
when $x = 0$, $y = -2\sqrt{2}$

when $x = -2$, $y = 2\sqrt{2}$

Therefore:

$z = -2\sqrt{2}\text{i}$ **or** $z = -2+2\sqrt{2}\text{i}$

c



Challenge

$|z+8+4\mathrm{i}|=|z+2+12\mathrm{i}|$

$|x+y\mathrm{i}+8+4\mathrm{i}|=|x+y\mathrm{i}+2+12\mathrm{i}|$

$|(x+8)+\mathrm{i}(y+4)|=|(x+2)+\mathrm{i}(y+12)|$

$|(x+8)+\mathrm{i}(y+4)|^2=|(x+2)+\mathrm{i}(y+12)|^2$

$(x+8)^2+(y+4)^2=(x+2)^2+(y+12)^2$

$x^2+16x+64+y^2+8y+16=x^2+4x+4+y^2+24y+144$

$12x+80=16y+148$

$y=\frac{3}{4}x-\frac{17}{4}$