Review exercise 2

1 a $k = -1$

At point A, $x = 0$

$f(x) = 3e^0 - 1$

$= 2$

$A(0, 2)$ The y-coordinate of A is 2.

b At point B, $y = 0$

$3e^{-x} - 1 = 0$

$3e^{-x} = 1$

$e^{-x} = \frac{1}{3}$

$\ln(e^{-x}) = \ln\frac{1}{3}$

$-x = \ln\frac{1}{3}$

$x = -\ln\frac{1}{3}$

$= \ln\left(\frac{1}{3}\right)^{-1}$

$= \ln 3$ (which is the x-coordinate of B)

2 $T = 400e^{-0.05t} + 25,\ t \geq 0$

a let $t = 0$

$T = 400 \times e^0 + 25 = 425$ °C

b let $T = 300$

$300 = 400e^{-0.05t} + 25$

$300 - 25 = 400e^{-0.05t}$

$275 = 400e^{-0.05t}$

$\frac{275}{400} = e^{-0.05t}$

Take ln of both sides:

$\ln\left(\frac{275}{400}\right) = -0.05t$

$\frac{-1}{0.05}\ln\left(\frac{275}{400}\right) = t$

$t = 7.49$ minutes

c $T = 400e^{-0.05t} + 25$

$\frac{dT}{dt} = 400e^{-0.05t} \times -0.05$

$= -20e^{-0.05t}$

let $t = 50$

$\frac{dT}{dt} = -20e^{-0.05t\times 50}$

$= -20e^{-2.5}$

$= 1.64$

The rate the temperature is decreasing is 1.64 °C/min

d $T = 400e^{-0.05t} + 25,\ t \geqslant 0$

$e^{-0.05t}$ tends to 0, so effectively the minimum value of T is 25 °C. Therefore, 20 °C is not possible,

3 a $\ln x + \ln 3 = \ln 6$
$\ln 3x = \ln 6$
$3x = 6$
$x = 2$

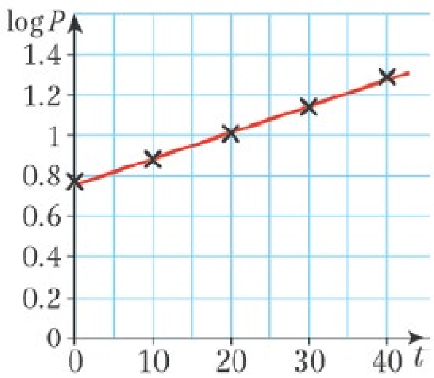
b $e^x + 3e^{-x} = 4$
$e^x + \dfrac{3}{e^x} = 4$
$e^{2x} + 3 = 4e^x$
$e^{2x} - 4e^x + 3 = 0$

let $y = e^x$
$y^2 - 4y + 3 = 0$
$(y - 3)(y - 3) = 0$
$y = 3$ or 1
$y = e^x$
$e^x = 3$ or $e^x = 1$
$x = 0$
$x = \ln 3$ or $x = 0$

4 a

Time in years since 1970, t	$\log P$
0	0.77
10	**0.88**
20	**1.01**
30	**1.14**
40	**1.29**

b



c As $P = ab^t$
$\log P = \log (ab^t)$
$\log P = \log a + \log b^t$
$\log P = \log a + t \log b$

This is a linear relationship where the gradient is log b and the intercept is log a.

4 d Intercept = 0.77

$\log a = 0.77$

$a = 10^{0.77}$

$= 5.888\ldots$

≈ 5.9 (2 s.f.)

Gradient $= \dfrac{1.29 - 0.77}{40 - 0} = \dfrac{0.52}{40} = 0.013$

$\log b = 0.013$

$b = 10^{0.013}$

$= 1.03\ldots$

≈ 1.0

$a = 5.9$, $b = 1.0$

5 a $\mathrm{f}: x \to \ln(5x - 2),\ x \in \mathbb{R},\ x > \dfrac{2}{5}$

Let $y = \ln(5x - 2)$

Swapping x and y gives:

$x = \ln(5y - 2)$

Solving for y gives:

$x = \ln(5y - 2)$

$\mathrm{e}^x = 5y - 2$

$y = \dfrac{\mathrm{e}^x + 2}{5}$

Therefore,

$\mathrm{f}^{-1}(x) = \dfrac{\mathrm{e}^x + 2}{5}$

b $x \in \mathbb{R}$

c $\ln(5x - 2) = 2$

$5x - 2 = \mathrm{e}^2$

$x = \dfrac{\mathrm{e}^2 + 2}{5}$

$= 1.877\ldots$

$= 1.878$ (3 d.p.)

6 a $\mathrm{f}: x \to \mathrm{e}^x + k,\ x \in \mathbb{R},\ k > 0$

Since $\mathrm{e}^x > 0$, $\mathrm{f}(x)$ has the range $\mathrm{f}(x) > k$

b $\mathrm{f}(\ln k) = \mathrm{e}^{\ln k} + k$

$= k + k$

$= 2k$

6 c Let $y = e^x + k$

Swapping x and y gives:

$x = e^y + k$

Solving for y gives:

$x = e^y + k$

$e^y = x - k$

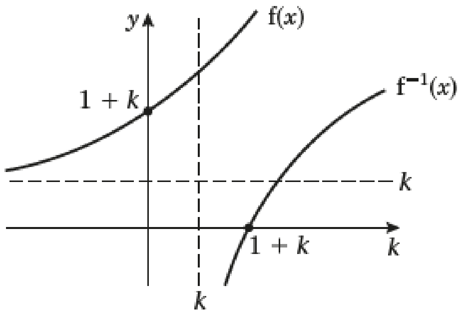
$y = \ln(x - k)$

$f^{-1}(x) = \ln(x - k)$

The domain of $f^{-1}(x)$ is equal to the range of $f(x)$

Therefore, the domain of $f^{-1}(x)$ is $x > k$

d



7 a $f : x \to \ln(4 - 2x),\ x \in \mathbb{R},\ x < 2$

Let $y = \ln(4 - 2x)$

Swapping x and y gives:

$x = \ln(4 - 2y)$

Solving for y gives:

$x = \ln(4 - 2y)$

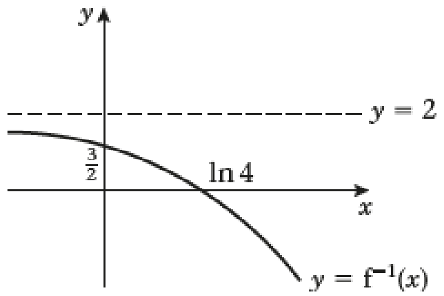
$e^x = 4 - 2y$

$y = \dfrac{4 - e^x}{2}$

Therefore,

$f^{-1}(x) = \dfrac{4 - e^x}{2}$

7 b



c $f^{-1}(x) = \frac{4 - e^x}{2}$

$f^{-1}(x) < 2$

d $g : x \to e^x,\ x \in \mathbb{R}$

$gf(x) = g(\ln(4 - 2x))$

$= e^{\ln(4-2x)}$

$= 4 - 2x$

$gf(0.5) = 4 - 2(0.5)$

$= 3$

8 a $f : x \to 2x + \ln 2,\ x \in \mathbb{R}$

$g : x \to e^{2x},\ x \in \mathbb{R}$

$gf(x) = g(2x + \ln 2)$

$= e^{2(2x+\ln 2)}$

$= e^{4x} \times e^{2\ln 2}$

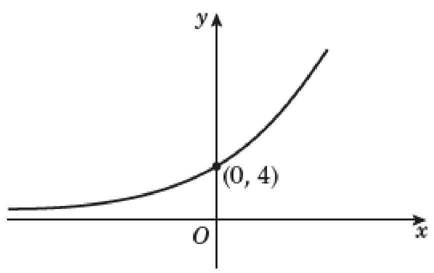
$= e^{4x} \times (e^{\ln 2})^2$

$= e^{4x} \times 2^2$

$= 4e^{4x}$

$gf : x \to 4e^{4x},\ x \in \mathbb{R}$ as required

b



c $gf(x) > 0$

8 d $\text{gf}(x) = 4e^{4x}$

$\frac{d}{dx}\left[\text{gf}(x)\right] = 16e^{4x}$

When

$\frac{d}{dx}\left[\text{gf}(x)\right] = 3$

$16e^{4x} = 3$

$e^{4x} = \frac{3}{16}$

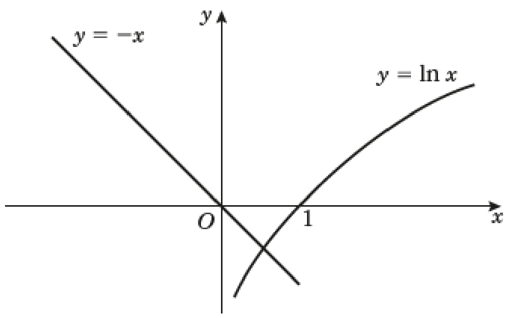
$4x \ln e = \ln\left(\frac{3}{16}\right)$

$x = \frac{1}{4}\ln\left(\frac{3}{16}\right)$

$= -0.418...$

$= -0.418 \quad (3 \text{ s.f.})$

9 a



b $x + \ln x = 0 \quad \Rightarrow \quad x = -\ln x$

$x = \frac{2x + x}{3} = \frac{2x - \ln x}{3}$

So, x can be written in the form $\frac{2x - \ln x}{3}$.

9 c $x_{n+1} = \dfrac{2x_n - \ln x_n}{3}$, $x_0 = 1$

$$x_1 = \frac{2(1) - \ln(1)}{3}$$
$$= 0.6666667$$
$$x_2 = \frac{2(0.6666667) - \ln(0.6666667)}{3}$$
$$= 0.5795995$$
$$x_3 = \frac{2(0.5795995) - \ln(0.5795995)}{3}$$
$$= 0.5682056$$
$$x_4 = \frac{2(0.5682056) - \ln(0.5682056)}{3}$$
$$= 0.5672277$$
$$x_5 = \frac{2(0.5672277) - \ln(0.5672277)}{3}$$
$$= 0.5671500$$
$$x_6 = \frac{2(0.5671500) - \ln(0.5671500)}{3}$$
$$= 0.5671438$$
$$x_7 = \frac{2(0.5671438) - \ln(0.5671438)}{3}$$
$$= 0.5671433$$

Therefore,
$x = 0.56714$ (5 d.p.)

10 $y = \frac{1}{2}x^2 + 4\cos x$

$$\frac{dy}{dx} = x - 4\sin x$$

When $x = \dfrac{\pi}{2}$:

$$y = \frac{\pi^2}{8} \text{ and } \frac{dy}{dx} = \frac{\pi}{2} - 4 = \frac{\pi - 8}{2}$$

So gradient of normal is $-\dfrac{2}{\pi - 8}$

Equation of normal is

$$y - \frac{\pi^2}{8} = -\frac{2}{\pi - 8}\left(x - \frac{\pi}{2}\right)$$
$$y(8 - \pi) - \frac{\pi^2}{8}(8 - \pi) = 2\left(x - \frac{\pi}{2}\right)$$
$$8y(8 - \pi) - \pi^2(8 - \pi) = 16x - 8\pi$$
$$8y(8 - \pi) - 16x - \pi^2(8 - \pi) + 8\pi = 0$$
$$8y(8 - \pi) - 16x + \pi(\pi^2 - 8\pi + 8) = 0$$

11 $y = e^{3x} - \ln(x^2)$
$= e^{3x} - 2\ln x$

$\frac{dy}{dx} = 3e^{3x} - \frac{2}{x}$

When $x = 2$:

$y = e^6 - \ln 4$ and $\frac{dy}{dx} = 3e^6 - 1$

Equation of tangent is

$y - (e^6 - \ln 4) = (3e^6 - 1)(x - 2)$

$y - e^6 + \ln 4 = (3e^6 - 1)x - 6e^6 + 2$

$y - (3e^6 - 1)x - 2 + \ln 4 + 5e^6 = 0$

12 a $y = (2x-3)^2 e^{2x}$

Let $u = (2x-3)^2 \Rightarrow \frac{du}{dx} = 4(2x-3)$

and $v = e^{2x} \Rightarrow \frac{dv}{dx} = 2e^{2x}$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= 2(2x-3)^2 e^{2x} + 4(2x-3)e^{2x}$

$= 2e^{2x}(2x-3)(2x-3+2)$

$= 2e^{2x}(2x-3)(2x-1)$

b $\frac{dy}{dx} = 0 \Rightarrow 2x - 3 = 0$ or $2x - 1 = 0$

So $x = \frac{3}{2}$ or $\frac{1}{2}$

When $x = \frac{3}{2}$, $y = 0$

When $x = \frac{1}{2}$, $y = 4e$

So coordinates of stationary points are $\left(\frac{3}{2}, 0\right)$ and $\left(\frac{1}{2}, 4e\right)$.

13 a $y=\dfrac{(x-1)^2}{\sin x}$

Let $u=(x-1)^2 \Rightarrow \dfrac{du}{dx}=2(x-1)$

and $v=\sin x \Rightarrow \dfrac{dv}{dx}=\cos x$

$$\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$$
$$=\frac{2(x-1)\sin x-(x-1)^2\cos x}{\sin^2 x}$$
$$=\frac{(x-1)(2\sin x-x\cos x+\cos x)}{\sin^2 x}$$

b When $x=\dfrac{\pi}{2}$:

$y=\left(\dfrac{\pi}{2}-1\right)^2$ and $\dfrac{dy}{dx}=2\left(\dfrac{\pi}{2}-1\right)$

Equation of tangent is

$$y-\left(\frac{\pi}{2}-1\right)^2=2\left(\frac{\pi}{2}-1\right)\left(x-\frac{\pi}{2}\right)$$
$$=(\pi-2)\left(x-\frac{\pi}{2}\right)$$
$$y=(\pi-2)x-\frac{\pi}{2}(\pi-2)+\left(\frac{\pi}{2}-1\right)^2$$
$$=(\pi-2)x-\frac{\pi^2}{2}+\pi+\frac{\pi^2}{4}-\pi+1$$
$$=(\pi-2)x+\left(1-\frac{\pi^2}{4}\right)$$

14 a $y = \text{cosec}\, x = \dfrac{1}{\sin x}$

Let $u = \sin x \Rightarrow \dfrac{du}{dx} = \cos x$

and $y = \dfrac{1}{u} \Rightarrow \dfrac{dy}{du} = -\dfrac{1}{u^2}$

Using the chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$
$$= -\frac{1}{\sin^2 x} \times \cos x$$
$$= -\frac{1}{\sin x} \times \frac{1}{\tan x}$$
$$= -\text{cosec}\, x \cot x$$

b $x = \text{cosec}\, 6y$

$$\frac{dx}{dy} = -6\,\text{cosec}\, 6y \cot 6y$$

$$\text{cosec}^2 6y = 1 + \cot^2 6y$$
$$\Rightarrow \cot 6y = \sqrt{x^2 - 1}$$

$$\frac{dx}{dy} = -6x\sqrt{x^2 - 1}$$
$$\frac{dy}{dx} = -\frac{1}{6x\sqrt{x^2 - 1}}$$

15 $y = \arcsin x$

So $x = \sin y$

$\Rightarrow \dfrac{dx}{dy} = \cos y$ and $\dfrac{dy}{dx} = \dfrac{1}{\cos y}$

$$\sin^2 y + \cos^2 y = 1$$
$$\cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$$

So $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1 - x^2}}$

16 $\int_a^3 (12-3x)^2\,dx = 78$

$\left[-\frac{1}{9}(12-3x)^3\right]_a^3 = -\frac{27}{9}+\frac{1}{9}(12-3a)^3$

$-3+\frac{1}{9}(12-3a)^3 = 78$

$\frac{1}{9}(12-3a)^3 = 81$

$(12-3a)^3 = 729$

$12-3a = 9$

$a = 1$

17 a $\cos(5x+2x) = \cos 5x\cos 2x - \sin 5x\sin 2x$

$\cos(5x-2x) = \cos 5x\cos 2x + \sin 5x\sin 2x$

Adding:

$\cos 7x + \cos 3x = 2\cos 5x\cos 2x$

b $\int 6\cos 5x\cos 2x\,dx$

$= 3\int(\cos 7x+\cos 3x)\,dx$

$= \frac{3}{7}\sin 7x + \sin 3x + c$

18 Consider $y = e^{x^4} \Rightarrow \frac{dy}{dx} = 4x^3 e^{x^4}$

So $\int_0^m mx^3 e^{x^4}\,dx = \left[\frac{m}{4}e^{x^4}\right]_0^m$

$= \frac{m}{4}e^{m^4} - \frac{m}{4}$

So $\frac{m}{4}e^{m^4} - \frac{m}{4} = \frac{3}{4}(e^{81}-1)$

$\frac{m}{4}(e^{m^4}-1) = \frac{3}{4}(e^{81}-1)$

$m = 3$

19 a $\text{f}(x)=\dfrac{5x^2-8x+1}{2x(x-1)^2}$

$$\frac{5x^2-8x+1}{2x(x-1)^2}=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$$

$$\begin{aligned}5x^2-8x+1&=2A(x-1)^2+2Bx(x-1)+2Cx\\&=2A(x^2-2x+1)+2Bx(x-1)+2Cx\\&=2Ax^2-4Ax+2A+2Bx^2-2Bx+2Cx\\&=(2A+2B)x^2+(-4A-2B+2C)x+2A\end{aligned}$$

Comparing coefficients

For constant:

$2A=1\Rightarrow A=\dfrac{1}{2}$

For x^2:

$2A+2B=5$

$2\left(\dfrac{1}{2}\right)+2B=5\Rightarrow B=2$

For x:

$-4A-2B+2C=-8$

$-4\left(\dfrac{1}{2}\right)-2(2)+2C=-8\Rightarrow C=-1$

$$\frac{5x^2-8x+1}{2x(x-1)^2}=\frac{1}{2x}+\frac{2}{x-1}-\frac{1}{(x-1)^2}$$

b $$\int\left(\frac{5x^2-8x+1}{2x(x-1)^2}\right)\text{d}x=\int\left(\frac{1}{2x}+\frac{2}{x-1}-\frac{1}{(x-1)^2}\right)\text{d}x$$

$$=\frac{1}{2}\ln|x|+2\ln|x-1|+\frac{1}{(x-1)}+c$$

19 c $\int_4^9 \left(\frac{5x^2-8x+1}{2x(x-1)^2}\right) dx = \int_4^9 \left(\frac{1}{2x}+\frac{2}{x-1}-\frac{1}{(x-1)^2}\right) dx$

$$= \left[\frac{1}{2}\ln|x| + 2\ln|x-1| + \frac{1}{(x-1)}\right]_4^9$$

$$= \left(\frac{1}{2}\ln 9 + 2\ln 8 + \frac{1}{8}\right) - \left(\frac{1}{2}\ln 4 + 2\ln 3 + \frac{1}{3}\right)$$

$$= \frac{1}{2}\ln 9 + 2\ln 8 - \frac{1}{2}\ln 4 - 2\ln 3 + \frac{1}{8} - \frac{1}{3}$$

$$= \ln 3 + \ln 64 - \ln 2 - \ln 9 - \frac{5}{24}$$

$$= \ln 192 - \ln 18 - \frac{5}{24}$$

$$= \ln\left(\frac{32}{3}\right) - \frac{5}{24} \text{ as required}$$

20 a $\frac{5x+3}{(2x-3)(x+2)} \equiv \frac{A}{2x-3} + \frac{B}{x+2}$

$$\equiv \frac{A(x+2)+B(2x-3)}{(2x-3)(x+2)}$$

$$5x+3 \equiv A(x+2)+B(2x-3)$$

Let $x=-2$: $-7 = B(-7)$ so $B=1$

Let $x=\frac{3}{2}$: $\frac{21}{2} = A\left(\frac{7}{2}\right)$ so $A=3$

So $\frac{5x+3}{(2x-3)(x+2)} \equiv \frac{3}{2x-3} + \frac{1}{x+2}$

b $\int_2^6 \frac{5x+3}{(2x-3)(x+2)} dx$

$$= \int_2^6 \frac{3}{2x-3} dx + \int_2^6 \frac{1}{x+2} dx$$

$$= \left[\tfrac{3}{2}\ln(2x-3) + \ln(x+2)\right]_2^6$$

$$= \left(\tfrac{3}{2}\ln 9 + \ln 8\right) - \left(\tfrac{3}{2}\ln 1 + \ln 4\right)$$

$$= \ln 9^{\frac{3}{2}} + \ln 8 - 0 - \ln 4$$

$$= \ln 9^{\frac{3}{2}} + \ln\frac{8}{4}$$

$$= \ln 27 + \ln 2$$

$$= \ln 54$$

21 Let $I = \int_1^e (x^2+1)\ln x \, dx$

Let $u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x}$

and $\frac{dv}{dx} = x^2 + 1 \Rightarrow v = \frac{x^3}{3} + x$

Using the integration by parts formula:

$$I = \left[\left(\frac{x^3}{3}+x\right)\ln x\right]_1^e - \int_1^e \frac{1}{x}\left(\frac{x^3}{3}+x\right)dx$$

$$= \left(\frac{e^3}{3}+e\right)\times 1 - \left(\frac{1}{3}+1\right)\times 0 - \int_1^e \left(\frac{x^2}{3}+1\right)dx$$

$$= \frac{e^3}{3}+e-0-\left[\frac{x^3}{9}+x\right]_1^e$$

$$= \frac{e^3}{3}+e-\left(\left(\frac{e^3}{9}+e\right)-\left(\frac{1}{9}+1\right)\right)$$

$$= \frac{2e^3}{9}+\frac{10}{9}$$

$$= \tfrac{1}{9}(2e^3+10)$$

22 a $g(x) = x^3 - x^2 - 1$

$g(1.4) = 1.4^3 - 1.4^2 - 1 = -0.216 < 0$
$g(1.5) = 1.5^3 - 1.5^2 - 1 = 0.125 > 0$

The change of sign implies that the root α is in $[1.4, 1.5]$.

b $g(1.4655) = -0.00025... < 0$
$g(1.4665) = 0.00326... > 0$

The change of sign implies that the root α satisfies $1.4655 < \alpha < 1.4665$, and so $\alpha = 1.466$ correct to 3 decimal places.

23 a $p(x) = \cos x + e^{-x}$

$p(1.7) = \cos 1.7 + e^{-1.7} = 0.054... > 0$
$p(1.8) = \cos 1.8 + e^{-1.8} = -0.062... < 0$

The change of sign implies that the root α is in $[1.7, 1.8]$.

23 b $p(1.7455) = \cos 1.7455 + e^{-1.7455}$
$= 0.00074... > 0$
$p(1.7465) = \cos 1.7465 + e^{-1.7465}$
$= -0.00042... < 0$

The change of sign implies that the root α satisfies $1.7455 < \alpha < 1.7465$, and so $\alpha = 1.746$ correct to 3 decimal places.

24 a $f(x) = e^{x-2} - 3x + 5 = 0$
$e^{x-2} = 3x - 5$
$x - 2 = \ln(3x - 5)$
$x = \ln(3x-5) + 2$, for $3x - 5 > 0 \Rightarrow x > \frac{5}{3}$

b Using $x_0 = 4$:
$x_1 = \ln 7 + 2 = 3.9459$
$x_2 = \ln(3 \times 3.9459 - 5) + 2 = 3.9225$
$x_3 = \ln(3 \times 3.9225 - 5) + 2 = 3.9121$
All correct to 4 decimal places.

25 a $f(x) = \frac{1}{(x-2)^3} + 4x^2$

$f(0.2) = \frac{1}{(0.2-2)^3} + 4 \times 0.2^2$
$= -0.011... < 0$

$f(0.3) = \frac{1}{(0.3-2)^3} + 4 \times 0.3^2$
$= 0.156... > 0$

The change of sign implies that the root α is in $[0.2, 0.3]$.

b $f(x) = \frac{1}{(x-2)^3} + 4x^2 = 0$

$\frac{1}{(x-2)^3} = -4x^2$

$(x-2)^3 = -\frac{1}{4x^2}$

$x - 2 = \sqrt[3]{\frac{-1}{4x^2}}$

$x = \sqrt[3]{\frac{-1}{4x^2}} + 2$

25 c Using $x_0 = 1$:

$$x_1 = \sqrt[3]{\frac{-1}{4}} + 2 = 1.3700$$

$$x_2 = \sqrt[3]{\frac{-1}{4 \times 1.3700^2}} + 2 = 1.4893$$

$$x_3 = \sqrt[3]{\frac{-1}{4 \times 1.4893^2}} + 2 = 1.5170$$

$$x_4 = \sqrt[3]{\frac{-1}{4 \times 1.5170^2}} + 2 = 1.5228$$

All correct to 4 decimal places.

d $$\text{f}(1.5235) = \frac{1}{(1.5235 - 2)^3} + 4 \times 1.5235^2$$

$$= 0.0412... > 0$$

$$\text{f}(1.5245) = \frac{1}{(1.5245 - 2)^3} + 4 \times 1.5245^2$$

$$= -0.0050... < 0$$

The change of sign implies that the root α satisfies $1.5235 < \alpha < 1.5245$, and so $\alpha = 1.524$ correct to 3 decimal places.

Challenge

1 $y=-\frac{3}{(4-6x)^2}\quad x\neq\frac{2}{3}$

When $x = 1$, $y=-\frac{3}{4}$

Let $u = (4 - 6x)$ so $y = -3u^{-2}$

$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=-3(-2)u^{-3}\times(-6)$

$=-\frac{36}{(4-6x)^3}$

When $x = 1$:

$\frac{dy}{dx}=-\frac{36}{(4-6(1))^3}$

$=\frac{9}{2}$

Since the gradient of the normal is the negative reciprocal of the gradient of the tangent, the normal has a gradient of $-\frac{2}{9}$

Using $y-y_1=m(x-x_1)$ with $m=-\frac{2}{9}$ at $\left(1,-\frac{3}{4}\right)$

$y+\frac{3}{4}=-\frac{2}{9}(x-1)$

$4y+3=-\frac{8}{9}(x-1)$

$36y+27=-8x+8$

$8x+36y+19=0$

2 **a** $f(0) = 0^3 - k(0) + 1 = 1$
$g(0) = e^2(0) = e^0 = 1$
Therefore, $f(0) = g(0) = 1$
$P(0, 1)$

b $f'(x) = 3x^2 - k$
Gradient at $x = 0$
$f'(0) = 3(0)^2 - k = -k$

Gradient of $g(x)$ at $x = 0$ is $\frac{1}{k}$

$g'(x) = 2e^{2x}$
$g'(0) = 2e^{2(0)} = 2e^0 = 2$

$\frac{1}{k}=2$

$k=\frac{1}{2}$

3 $V = \frac{2}{3}\pi r^3 \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}r} = 2\pi r^2$

$S = 3\pi r^2 \Rightarrow \frac{\mathrm{d}S}{\mathrm{d}r} = 6\pi r$

We are told that $\frac{\mathrm{d}V}{\mathrm{d}t} = 6$

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{\mathrm{d}S}{\mathrm{d}r} \times \frac{\mathrm{d}r}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t}$$

$$= 6\pi r \times \frac{1}{2\pi r^2} \times 6$$

$$= \frac{18}{r}$$