Chapter review

1 $H \sim \mathrm{N}(178, 4^2)$

a Using the normal CD function, $\mathrm{P}(H > 185) = 0.04059... = 0.0406$ (4 d.p.)

b Using the normal CD function, $\mathrm{P}(H < 180) = 0.69146...$
The probability that three men, selected at random, all satisfy this criterion is
$\mathrm{P}(H < 180)^3 = 0.33060... = 0.3306$ (4 d.p.) .

c Using the inverse normal function, $\mathrm{P}(H > h) = 0.005 \Rightarrow h = 188.03...$
To the nearest centimetre, the height of a door frame needs to be at least 188 cm.

2 $W \sim \mathrm{N}(32.5,\ 2.2^2)$

a Using the normal CD function, $\mathrm{P}(W < 30) = 0.12790...$
The percentage of sheets weighing less than 30kg is 12.8% (3 s.f.).

b Using the normal CD function, $\mathrm{P}(31.6 < W < 34.8) = 0.51085...$
So 51.1% of sheets satisfy Bob's requirements (1 d.p.).

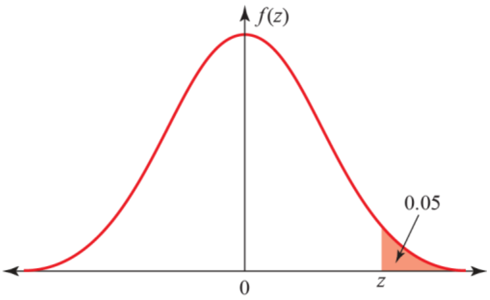
3 $T \sim \mathrm{N}(48, 8^2)$

a Using the normal CD function, $\mathrm{P}(T > 60) = 0.06680...$
The probability that a battery will last for more than 60 hours is 0.0668 (4 d.p.).

b Using the normal CD function, $\mathrm{P}(T < 35) = 0.05208...$
The probability that a battery will last for less than 35 hours is 0.0521 (4 d.p.).

4 $X \sim \mathrm{N}(24, \sigma^2)$

a $\mathrm{P}(X > 30) = 0.05 \Rightarrow \mathrm{P}\left(Z > \frac{30-\mu}{\sigma}\right) = 0.05$



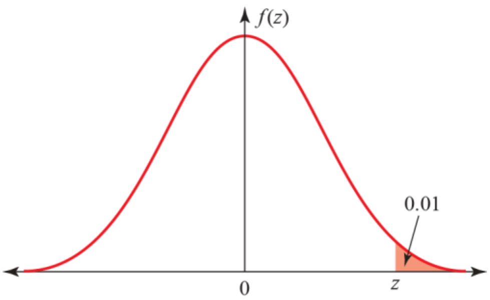
Using the inverse normal function, $z = -1.64485...$

so $1.64485... = \frac{30-24}{\sigma}$

$$\sigma = \frac{6}{1.64485...} = 3.647... = 3.65 \text{ (3 s.f.)}$$

b Using the normal CD function, $\mathrm{P}(X < 20) = 0.13636... = 0.136$ (3 d.p.)

c $\mathrm{P}(X > d) = 0.01 \Rightarrow \mathrm{P}\left(Z > \frac{d-\mu}{\sigma}\right) = 0.01$



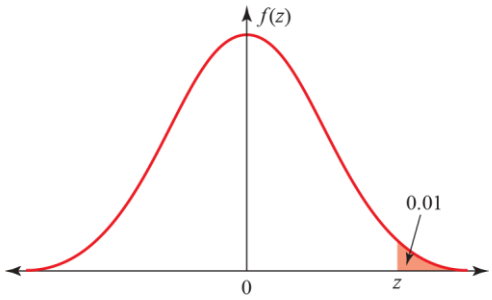
Using the inverse normal function, $z = 2.32634...$

so $2.32634... = \frac{d-24}{3.647...}$

$$d = 32.485... = 32.5 \text{ (3 s.f.)}$$

5 $L \sim \mathrm{N}(120,\ \sigma^2)$

a $\mathrm{P}(L > 140) = 0.01 \Rightarrow \mathrm{P}\left(Z > \dfrac{140-\mu}{\sigma}\right) = 0.01$



Using the inverse normal function, $z = 2.32634...$

so $2.32634... = \dfrac{140-120}{\sigma}$

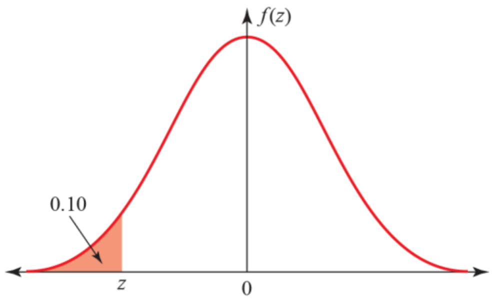
$$\sigma = \frac{20}{2.32634...} = 8.59716...$$

So the standard deviation of the volume dispensed is 8.60 ml (3 s.f.).

b Using the normal CD function, $\mathrm{P}(L < 110) = 0.12237...$

The probability that the machine dispenses less than 110ml is 0.122 (3 s.f.).

c $\mathrm{P}(L < c) = 0.10 \Rightarrow \mathrm{P}\left(Z < \dfrac{c-\mu}{\sigma}\right) = 0.10$



Using the inverse normal function, $z = -1.28155...$

so $-1.2816 = \dfrac{c-120}{8.59716...}$

$$c = 108.982...$$

To the nearest millilitre, the largest volume leading to a complaint is 109 ml.

6 a $\mathrm{P}(X < 20) = 0.25$ and $\mathrm{P}(X < 40) = 0.75$

Using the inverse normal function (or the percentage points table),

$$\mathrm{P}(X < 20) = 0.25 \Rightarrow \mathrm{P}\left(Z < \frac{20-\mu}{\sigma}\right) = 0.25 \Rightarrow z_1 = -0.67448...$$

$$\mathrm{P}(X < 40) = 0.75 \Rightarrow \mathrm{P}\left(Z < \frac{40-\mu}{\sigma}\right) = 0.75 \Rightarrow z_2 = 0.67448...$$

So $-0.6745\sigma = 20 - \mu$ **(1)**

and $0.6745\sigma = 40 - \mu$ **(2)**

(2) − **(1)**: $1.3489\sigma = 20$

$\sigma = 14.826...$

Substituting into **(2)**:

$\mu = 40 - 0.6745 \times 14.826... = 29.99...$

So $\mu = 30$ and $\sigma = 14.8$ (3 s.f.)

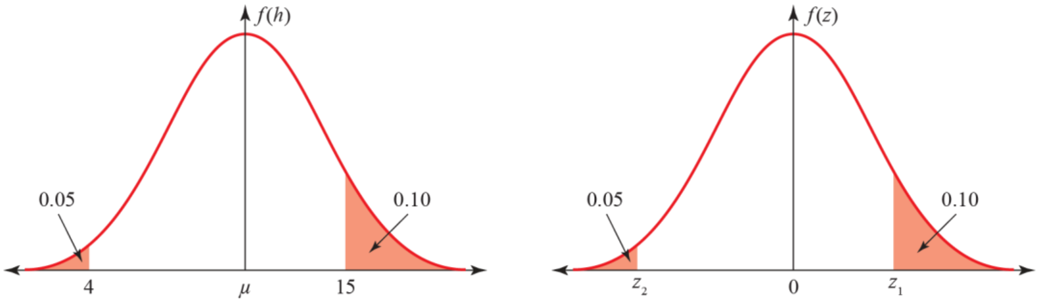
b Using the inverse normal CD function with $\mu = 30$ and $\sigma = 14.826...$,

$\mathrm{P}(X < a) = 0.1 \Rightarrow a = 10.999...$ and $\mathrm{P}(X < b) = 0.9 \Rightarrow b = 49.000...$

So the 10% to 90% interpercentile range is $49.0 - 11.0 = 38.0$

7 $$\mathrm{P}(H > 15) = 0.10 \Rightarrow \mathrm{P}\left(Z > \frac{15-\mu}{\sigma}\right) = 0.10 \Rightarrow z_1 = 1.28155...$$

$$\mathrm{P}(H < 4) = 0.05 \Rightarrow \mathrm{P}\left(Z > \frac{4-\mu}{\sigma}\right) = 0.05 \Rightarrow z_2 = -1.64485...$$



So $-1.6449\sigma = 4 - \mu$

$1.2816\sigma = 15 - \mu$

Subtract $2.9265\sigma = 11$

$\sigma = 3.7587... = 3.76\,\text{cm}$ (3 s.f.)

$\mu = 15 - 1.2816\sigma = 10.2\,\text{cm}$ (3 s.f.)

8 a $T \sim \mathrm{N}(80, 10^2)$

Using the normal CD function, $\mathrm{P}(T > 85) = 0.30853... = 0.3085$ (4 d.p.)

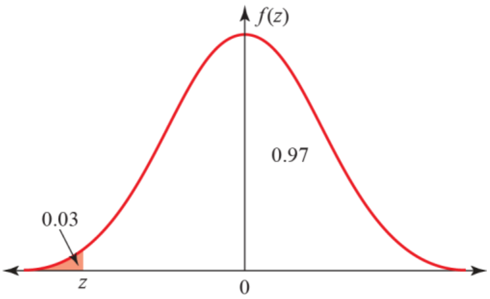
b $S \sim \mathrm{N}(100, 15^2)$

Using the normal CD function, $\mathrm{P}(S > 105) = 0.36944... = 0.3694$ (4 d.p.)

c The student's score on the first test was better, since fewer of the students got this score or higher.

9 $J \sim \mathrm{N}(108, \sigma^2)$

a $\mathrm{P}(J < 100) = 0.03 \Rightarrow \mathrm{P}\left(Z < \dfrac{100-\mu}{\sigma}\right) = 0.03$



Using the inverse normal function, $z = -1.88079...$

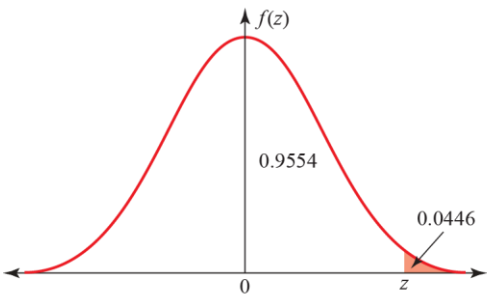
so $-1.88079... = \dfrac{100-108}{\sigma}$

$$\sigma = 4.2535... = 4.25\,\mathrm{g}\ (3\text{ s.f.}).$$

The standard deviation is 4.25 g (3 s.f.).

b Using the normal CD function, $\mathrm{P}(J > 115) = 0.0499... = 0.050$ (3 d.p.)

10 $T \sim \mathrm{N}(\mu, 3.8^2)$ and $\mathrm{P}(T > 15) = 0.0446$



a $\mathrm{P}(T > 15) = 0.0446 \Rightarrow \mathrm{P}\left(Z > \dfrac{X-\mu}{\sigma}\right) = 0.0446 \Rightarrow z = 1.70$

so $1.70 = \dfrac{15-\mu}{3.8}$

$$\mu = 15 - 3.8 \times 1.70$$
$$= 8.54 \text{ minutes (3 s.f.)}$$

b $\mathrm{P}(T < 5) = \mathrm{P}\left(Z < \dfrac{5-8.54}{3.8}\right)$

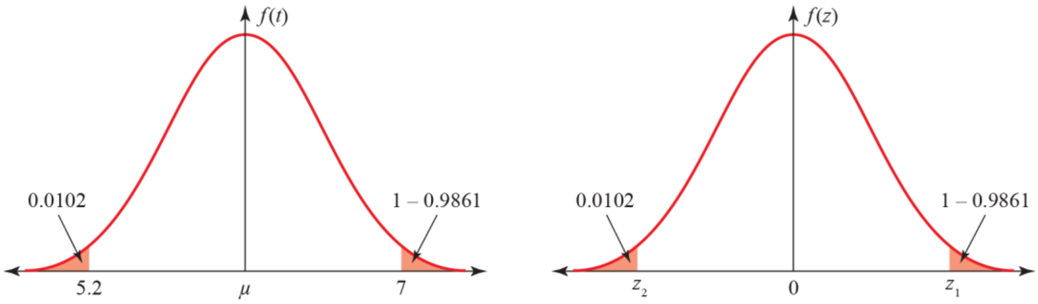
$$= \mathrm{P}(Z < -0.93...)$$
$$= 0.17577... = 0.1758 \text{ (4 d.p.)}$$

11 $T \sim \mathrm{N}(\mu, \sigma^2)$

Using the inverse normal function,

$$\mathrm{P}(T < 7) = 0.9861 \Rightarrow \mathrm{P}\left(Z < \frac{7-\mu}{\sigma}\right) = 0.9861 \Rightarrow z_1 = 2.20009...$$

$$\mathrm{P}(T < 5.2) = 0.0102 \Rightarrow \mathrm{P}\left(Z < \frac{5.2-\mu}{\sigma}\right) = 0.0102 \Rightarrow z_2 = -2.31890...$$



So $\quad 2.2001\sigma = 7 - \mu \quad$ (**1**)

and $\quad -2.3189\sigma = 5.2 - \mu \quad$ (**2**)

(**1**) – (**2**): $\quad 4.5190\sigma = 1.8$

$\sigma = 0.3983...$

Substituting into (**1**):

$\mu = 7 - 2.2001 \times 0.3983... = 6.123...$

So the mean thickness of the shelving is 6.12 mm and the standard deviation is 0.398 mm (3 s.f.).