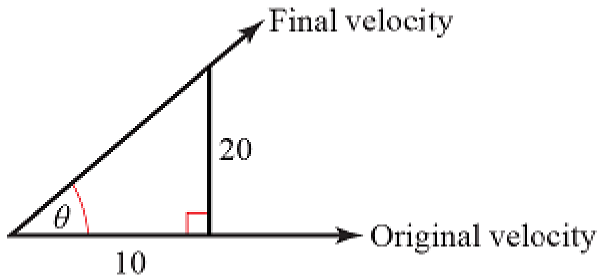
Review Exercise 2

1 a $-4\mathbf{i}+4\mathbf{j}=0.2\mathbf{v}-0.2\times 30\mathbf{i}$

$\mathbf{v}=10\mathbf{i}+20\mathbf{j}\,\mathrm{m\,s^{-1}}$ ← Impulse = change of momentum

b



$\tan\theta=\frac{20}{10}$

$\theta=63.4^\circ$

c K.E. lost

$=\frac{1}{2}\times 0.2\times 30^2-\frac{1}{2}\times 0.2\left(10^2+20^2\right)$

$=40\,\mathrm{J}$

2 a $\mathbf{v}=\left(t^2+2\right)\mathbf{i}-6t\mathbf{j}$

$\mathbf{a}=2t\mathbf{i}-6\mathbf{j}$ ← Differentiate to find **a.**

$\mathbf{F}=0.75(2t\mathbf{i}-6\mathbf{j})$ ← Use $\mathbf{F}=m\mathbf{a}$ to find **F**.

$t=4$ ← Make $t=4$

$\mathbf{F}=0.75(8\mathbf{i}-6\mathbf{j})$

$=6\mathbf{i}-4.5\mathbf{j}$

$|\mathbf{F}|=\sqrt{\left(6^2+4.5^2\right)}=7.5\,\mathrm{N}$ ← The magnitude of **F** is the modulus of the vector.

b $\mathbf{I}=p\mathbf{i}-p\mathbf{j}$ ← Impulse is parallel to $\mathbf{i}-\mathbf{j}$

$|\mathbf{I}|=\sqrt{\left(p^2+p^2\right)}=p\sqrt{2}$

But $|\mathbf{I}|=9\sqrt{2}\ \therefore p=9$ ← Impulse = change of momentum

$9\mathbf{i}-9\mathbf{j}=0.75(\mathbf{v}-(27\mathbf{i}-30\mathbf{j}))$

$36\mathbf{i}-36\mathbf{j}=3\mathbf{v}-81\mathbf{i}+90\mathbf{j}$ ← For the initial velocity, $t=5$

$3\mathbf{v}=117\mathbf{i}-126\mathbf{j}$

$\mathbf{v}=(39\mathbf{i}-42\mathbf{j})\,\mathrm{m\,s^{-1}}$

3 a $\mathbf{I} = 0.2\left((15\mathbf{i}+15\mathbf{j})-(-10\mathbf{i})\right)$

$= 5\mathbf{i}+3\mathbf{j}$

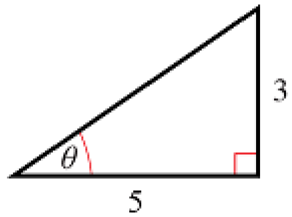
$|\mathbf{I}| = \sqrt{(5^2+3^2)} = \sqrt{34}$

$= 5.83\text{ N s}$

Use Impulse = change of momentum

The magnitude of the impulse is the modulus of the vector.

b



$\tan\theta = \frac{3}{5}$

$\theta = 31^\circ$ (nearest degree)

c K.E. gained

$= \frac{1}{2}\times 0.2\times(15^2+15^2)-\frac{1}{2}\times 0.2\times 10^2$

$= 35\text{ J}$

4 a $\mathbf{v} = \int(2\mathbf{i}+6t\mathbf{j})\,dt$

$= 2t\mathbf{i}+3t^2\mathbf{j}+\mathbf{c}$

When $t=0$, $\mathbf{v} = 2\mathbf{i}-4\mathbf{j}$

$\therefore \mathbf{c} = 2\mathbf{i}-4\mathbf{j}$

$\mathbf{v} = (2t+2)\mathbf{i}+(3t^2-4)\mathbf{j}$

Don't forget the (vector) constant of integration.

b $t=2 \Rightarrow \mathbf{v} = 6\mathbf{i}+8\mathbf{j}$

$3\mathbf{i}-1.5\mathbf{j} = 0.5(\mathbf{V}-(6\mathbf{i}+8\mathbf{j}))$

$6\mathbf{i}-3\mathbf{j} = \mathbf{V}-6\mathbf{i}-8\mathbf{j}$

$\mathbf{V} = 12\mathbf{i}+5\mathbf{j}$

$|\mathbf{V}| = \sqrt{(12^2+5^2)} = 13$

Speed $= 13\text{ m s}^{-1}$

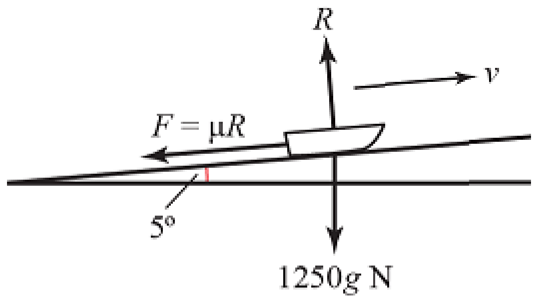
Find the velocity immediately before the impact.

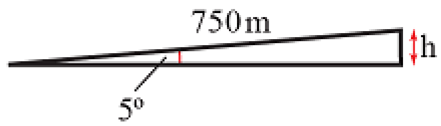
Impulse = change of momentum

Speed is the modulus of the velocity.

5 $m = 1250$ kg, $\mu = 0.05$, $s = 750$ m

a





Resolving perpendicular to the slope:

$R = mg\cos 5$

Friction is limiting, so F = μR

$F = \mu mg\cos 5$
$F = 0.05 \times 1250 \times 9.8\cos 5 = 610.16...$

The frictional force between the sled and the slope is 610 N (3 s.f.)

b The frictional force acts along the slope, so work done against friction, W_F:
$W_F = Fs$
$W_F = 610 \times 750 = 457500$
The work done against friction is 458 kJ (3 s.f.)

c Work done against gravity, $W_G = mgh$
$h = 750\sin 5$
$W_G = 1250 \times 9.8 \times 750\sin 5 = 800\,743$
The work done against gravity is 801 kJ (3 s.f.)

6 $m = 4$ kg, $h = 40$ m

a From the conservation of energy:
K.E. gained = P.E. lost

Final K.E. $= mgh$
Final K.E. $= 4 \times 9.8 \times 40 = 1568$

When the rock hits the sea, its kinetic energy is 1568 J

b Work will be done against the opposing air resistance means that the final kinetic energy will therefore be reduced, as not all the P.E lost will be converted to K.E.

7 Work done = mgh

$19\,600 = 1000 \times 9.8 \times 25 \sin\theta$ (where θ is the angle of the slope)

$$\sin\theta = \frac{19\,600}{1000 \times 9.8 \times 25}$$

$$= \frac{2}{25}$$

$\theta = \sin^{-1}\left(\frac{2}{25}\right)$ as required

8 $m = 200$ kg, $u = 2$ m s^{-1}, $v = 1.5$ m s^{-1}, $s = 200$ m

a Loss of kinetic energy = initial K.E. – final K.E.

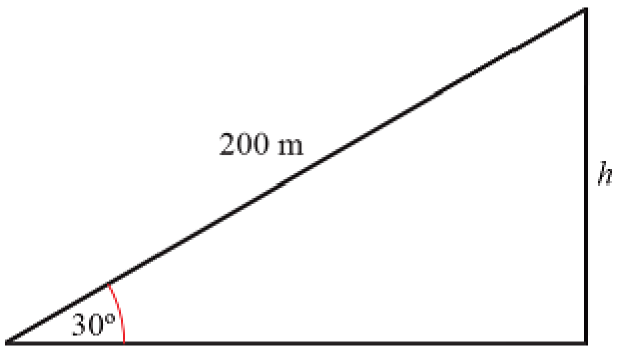
K.E. lost $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

K.E. lost $= \frac{1}{2}m\left(u^2 - v^2\right)$

K.E. lost $= \frac{1}{2} \times 200\left(2^2 - 1.5^2\right) = 175$

The cable car loses 175 J of kinetic energy.

b Potential energy gained, P.E. = mgh

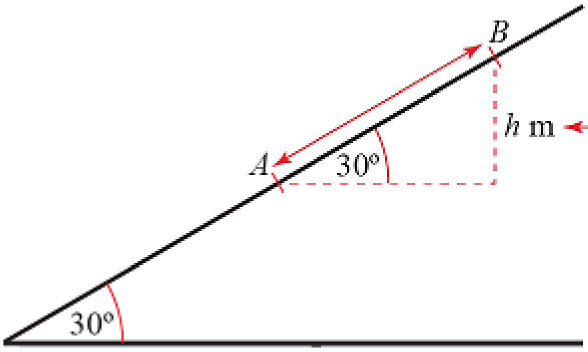


$h = 200\sin 30$

P.E. $= 200 \times 9.8 \times 200\sin 30 = 196\,000$

The potential energy gained is 196 kJ (3 s.f.)

9



The change in the potential energy of P depends on the vertical distance it has moved. You find this using trigonometry.

a Let the vertical distance moved by P be h m.

$$\frac{h}{3} = \sin 30^\circ \Rightarrow h = 3\sin 30^\circ = 1.5$$

The potential energy gained by P is given by
$$\text{P.E.} = mgh = 2 \times 9.8 \times 1.5 = 29.4$$

Let the speed of P at B be v m s^{-1}

The kinetic energy lost by P is given by

$$\text{K.E.} = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$$
$$= \frac{1}{2}2 \times 10^2 - \frac{1}{2}2v^2 = 100 - v^2$$

Using the principle of conservation of energy

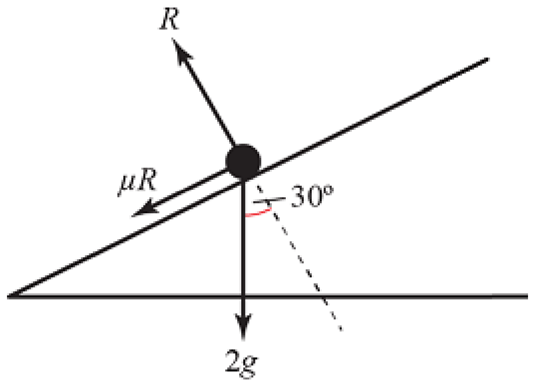
$$100 - v^2 = 29.4$$
$$v^2 = 100 - 29.4 = 70.6$$
$$v = \sqrt{70.6} = 8.402\ldots$$

If no forces other than gravity are acting on the particle, as mechanical energy is conserved, the loss of kinetic energy must equal the gain in potential energy.

The speed of P at B is 8.4 m s^{-1} (2 s.f.)

As a numerical value of g has been used, you should round your final answer to 2 significant figures. Three significant figures are also acceptable.

9 b



Let the normal reaction between the particle and the plane have magnitude R N.

$R(\nwarrow): R = 2g\cos 30^\circ$

The frictional force is given by

$F = \mu R = \mu 2g\cos 30^\circ$

The kinetic energy lost by P is given by

$$\text{K.E.} = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$$
$$= \frac{1}{2}2\times 10^2 - \frac{1}{2}2\times 7^2 = 51$$

The potential energy gained by P is the same as in **a**.

The vertical height moved is the same as in **a.**

The total loss of mechanical energy, in J, is
$51 - 29.4 = 21.6$

The work done by friction is given by:

$$\text{work done} = \text{force}\times\text{distance moved}$$
$$W = \mu R\times 3 = \mu 2g\cos 30^\circ \times 3 = \mu\times 50.922\ldots$$

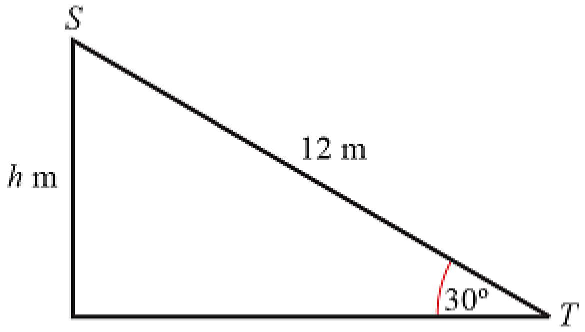
Using the work–energy principle:

$\mu\times 50.922\ldots = 21.6 \Rightarrow \mu = 0.424\ldots$

The work done by the friction is equal to the total loss of energy of the particle.

The coefficient of friction is 0.42 (2 s.f.)

10 a



In moving from S to T, P descends a vertical distance of h m, where

$$\frac{h}{12} = \sin 30^\circ \Rightarrow h = 12\sin 30^\circ = 6$$

The change in the potential energy of P depends on the vertical distance it has moved. You find this using trigonometry.

The potential energy, in J, lost by P is given by
$mgh = 0.6 \times 9.8 \times 6 = 35.28$

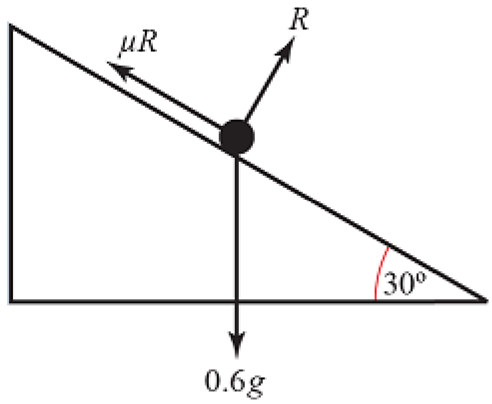
The kinetic energy, in J, lost by P is given by

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = \frac{1}{2}m\left(u^2 - v^2\right)$$
$$= \frac{1}{2} \times 0.6 \times \left(10^2 - 9^2\right) = 5.7$$

The total loss of energy of P is
$(35.28 + 5.7)$ J $= 40.98$ J $= 41$ J (2 s.f.)

As P moves from S to T both kinetic and potential energy are lost.

10 b



Let the normal reaction between the particle and the plane have magnitude R N.

$\text{R}(\nearrow): R = 0.6g\cos 30^\circ$

The frictional force is given by

$F = \mu R = \mu 0.6g\cos 30^\circ = \mu \times 5.09229\ldots$

The work done by friction is given by:

$\text{work done} = \text{force} \times \text{distance moved}$

$W = F \times 12 = \mu \times 61.106\ldots$

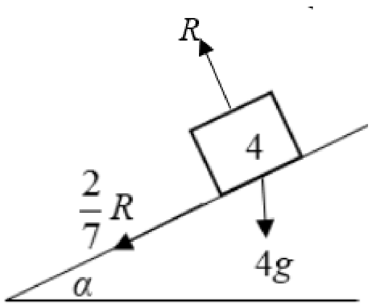
Using the work–energy principle

$\mu \times 61.106\ldots = 40.98 \Rightarrow \mu = 0.6706$

The coefficient of friction is 0.67 (2 s.f.)

Friction opposes motion and acts up the plane. The work done by friction against the motion of the particle equals the total loss of energy of the particle. You should use the unrounded answer from **a** for the total energy loss.

11 a



Res(↖) $R = 4g\cos\alpha$

$= 3.2g$

Therefore:

$F = \frac{2}{7}(3.2g)$

$= \frac{32}{35}g$

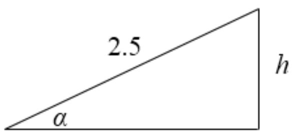
Work done = Fs

$= \frac{32}{35}g(2.5)$

$= \frac{80}{35}g$

$= 22.4$ J

b



$\sin\alpha = \frac{h}{2.5} \Rightarrow h = 2.5\sin\alpha$

$\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{3}{5}$

$h = 2.5\left(\frac{3}{5}\right)$

$= 1.5$

Work done = change in energy

work $= \frac{1}{2}mv^2 - mgh$

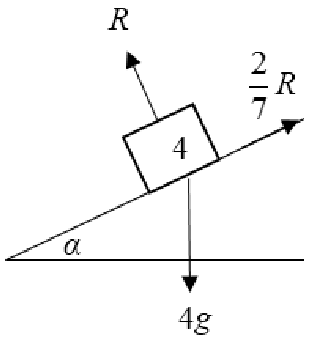
$22.4 = \frac{1}{2}(4)v^2 - 4g(1.5)$

$2v^2 = 81.2$

$v = 6.371\ldots$

$= 6.37$ m s^{-1} (3 s.f.)

11 c



Res(↖) $R = 4g\cos\alpha$

$= 3.2g$

Res(↙) $4g\sin\alpha - \frac{2}{7}R = 4a$

$\frac{12}{5}g - \frac{32}{35}g = 4a$

$4a = \frac{52}{35}g$

$a = \frac{13}{35}g$ m s^{-2}

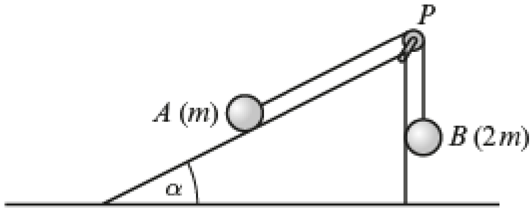
Using $v^2 = u^2 + 2as$ with $u = 0$, $s = 2.5$ and $a = \frac{13}{35}g$ gives:

$v^2 = (0)^2 + 2\left(\frac{13}{35}g\right)(2.5)$

$v = 4.266...$

$= 4.27$ m s^{-1}

12 a



$\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{3}{5}$

When A has moved a distance of h:

$\sin\alpha = \frac{x}{h}$ (where x represents the vertical height moved by A)

$x = \frac{3}{5}h$

Potential energy gained by A is:

$E_A = mgh$

$= \frac{3}{5}mgh$

Potential energy lost by B is:

$E_B = mgh$

$= 2mgh$

Therefore the potential energy lost by the system is:

$2mgh - \frac{3}{5}mgh = \frac{7}{5}mgh$

b Res(↖) $R = mg\cos\alpha$

$R = \frac{4}{5}mg$

Therefore:

$F = \frac{5}{8} \times \frac{4}{5}mg$

$= \frac{1}{2}mg$

Therefore, work done by friction is:

$Fs = \frac{1}{2}mgh$

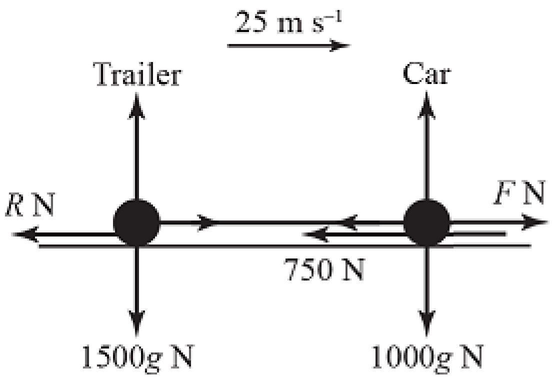
Work done = change in energy

$\frac{1}{2}mgh = \frac{7}{5}mgh - \frac{3}{2}mv^2$

$\frac{3}{2}mv^2 = \frac{9}{10}mgh$

$v^2 = \frac{3}{5}gh$

13 a



Let F N be the magnitude of the driving force produced by the engine of the car.

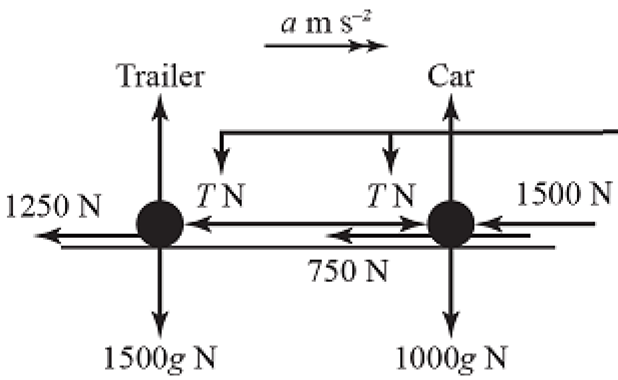
$$50\text{ kW} = 50\,000\text{ W}$$
$$P = Fv$$
$$50\,000 = F \times 25 \Rightarrow F = 2000$$

For the car and trailer combined:

$$\text{R}(\rightarrow): F - 750 - R = 0$$
$$R = F - 750 = 2000 - 750 = 1250, \text{ as required}$$

When you consider the car and trailer combined, the tensions at the ends of the tow–bar cancel one another out and can be ignored.

b



As the car brakes, the forces in the tow-bar are thrusts and act in the directions shown in this diagram. The forces in the tow-bar in **a** are tensions and act in the opposite directions to thrusts.

Let the acceleration of the car while braking be a m s^{-2}

For the car and trailer combined:

$$\text{R}(\rightarrow): F = ma$$
$$-1500 - 750 - 1250 = 2500a$$
$$2500a = -3500 \Rightarrow a = -\frac{3500}{2500} = -1.4$$

The deceleration of the car is therefore 1.4 m s^{-2}

13 c Let the magnitude of the thrust in the tow-bar while braking be T N.

For the trailer alone

$$R(\rightarrow):\ F = ma$$
$$-1250 - T = 1500a = 1500 \times (-1.4)$$
$$T = 1500 \times 1.4 - 1250 = 850$$

The magnitude of the thrust in the tow-bar while braking is 850 N.

d To find the distance travelled in coming to rest

$$v^2 = u^2 + 2as$$
$$0^2 = 25^2 + 2 \times (-1.4)s$$
$$s = \frac{25^2}{2.8}$$

The work done, in J, by the braking force of 1500 N is given by

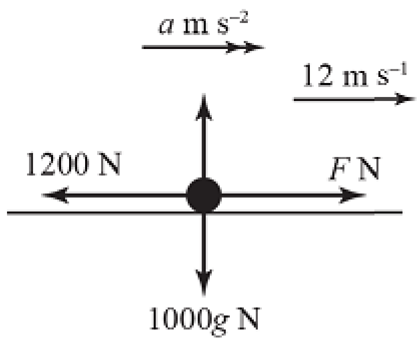
$$W = 1500 \times s = 1500 \times \frac{25^2}{2.8} = 334\,821$$

Work done = force × distance moved

The work done by the braking force in bringing the car and the trailer to rest is 335 kJ (3 s.f.)

e The resistance could be modelled as varying with speed.

14



a Let F N be the magnitude of the driving force produced by the engine of the car.

$$24\text{ kW} = 24\,000\text{ W}$$
$$P = Fv$$
$$24\,000 = F \times 12 \Rightarrow F = 2000$$

$$R(\rightarrow): F = ma$$
$$F - 1200 = 1000a$$
$$2000 - 1200 = 1000a \Rightarrow a = \frac{800}{1000} = 0.8\text{ m s}^{-2}$$

14 b The kinetic energy, in J, lost as the car is brought to rest is

$$\frac{1}{2}mu^2 = \frac{1}{2}1000\times14^2 = 98\,000$$

The final kinetic energy is zero.

$$\text{Work done by resistance} = \text{Energy lost}$$
$$\text{Resistance}\times\text{ distance} = \text{Energy lost}$$
$$1200d = 98\,000$$
$$d = \frac{98\,000}{1200} = 81\frac{2}{3}\text{ m}$$

You use the work-energy principle. The work done by the resistance (1200 N) in bringing the car to rest is equal to the kinetic energy lost.

c Resistance usually varies with speed.

As the speed slows down, the resistance to motion usually decreases. In this case, this might mean that the car would travel further.

15 a From A to B, the cyclist descends

$$(20-12)\text{m} = 8\text{ m}$$

Whatever the path taken, the potential energy lost in travelling from A to B depends solely on the difference in levels between A and B.

The potential energy, in J, lost in travelling from A to B is given by

$$mgh = 80\times9.8\times8 = 6272$$

The kinetic energy, in J, lost in travelling from A to B is given by

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = \frac{1}{2}m\left(u^2 - v^2\right)$$
$$= 40\left(8^2 - 5^2\right) = 1560$$

The total mechanical energy lost is
$(6272+1560)\text{ J} = 7832\text{ J}$

The work done by resistance due to non-gravitational forces is given by

$$W = \text{force}\times\text{distance moved}$$
$$= 20\times500 = 10\,000$$

$$(10\,000-7832)\text{ J} = 2168\text{ J}$$

The non-gravitational resistances to motion have worked 10 000 J against the motion. However, the mechanical energy lost is only 7832 J. The difference between these values is the work that has been done by the cyclist.

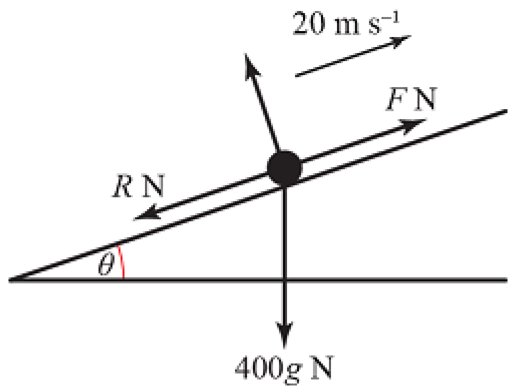
The work done by the cyclist in moving from A to B is 2200 J (2 s.f.)

15 b At B, let the force generated by the cyclist be FN.

$$\text{R}(\rightarrow): F = ma$$
$$F - 20 = 80 \times 0.5$$
$$\text{So } F = 60N$$
$$P = Fv$$
$$= 60 \times 5 = 300$$

The power generated by the cyclist is 300 W

16



Let F N be the magnitude of the driving force produced by the engine.

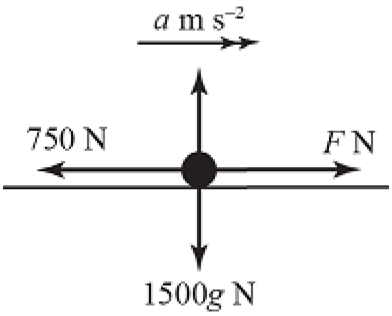
$$10\,\text{kW} = 10\,000\,\text{W}$$
$$P = Fv$$
$$10\,000 = F \times 20 \Rightarrow F = 500$$

Before you use the formula $P = Fv$, you have to convert kilowatts to watts.

$$R(\nearrow): F = ma$$
$$F - R - 400g\sin\theta = 0$$
$$R = F - 400g\sin\theta$$
$$= 500 - 400 \times 9.8 \times \frac{1}{14} = 220$$

As the car is travelling at a constant speed, its acceleration is zero.

17 a



Let the acceleration of the lorry be a m s^{-2} and the driving force of the engine have magnitude F N.

$36\text{ kW} = 36\,000\text{ W}$ ← kW must be converted to W.

$P = Fv$

$36\,000 = F \times 20 \Rightarrow F = 1800$

$\text{R}(\rightarrow): F = ma$

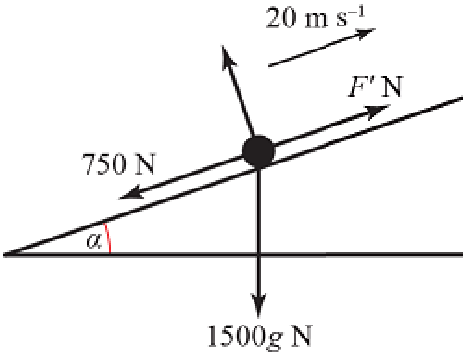
$F - 750 = 1500a$

$1800 - 750 = 1500a$

$$a = \frac{1800 - 750}{1500} = 0.7$$

The acceleration of the lorry when the speed is 20 m s^{-1} is 0.7 m s^{-2} ← This result is only true at one instant in time. The speed would now increase and the driving force and acceleration decrease.

17 b



Let the driving force of the engine have magnitude F' N.

The driving forces in **a** and **b** are different and it is a good idea to avoid confusion by using different symbols for the forces.

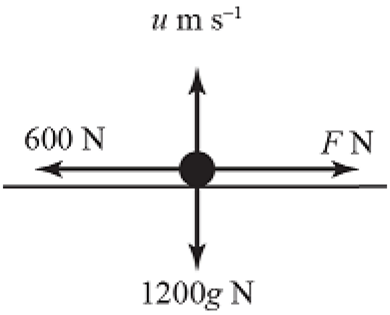
$$\text{R}(\nearrow): F' = ma$$
$$F' - 750 - 1500g\sin\alpha = 0$$
$$F' = 750 + 1500 \times 9.8 \times \frac{1}{10} = 2220$$
$$P = Fv$$
$$= 2220 \times 20 = 44\,400$$

In this part of the question the lorry is moving at a constant speed and the acceleration is zero.

The rate at which the lorry is now working is 44.4 kW.

This question does not ask for a particular form of the answer, so you could give your answer in either W or kW. Two or three significant figures are acceptable.

18 a



Let the speed of the car be u m s^{-1} and the driving force of the engine have magnitude F N.

$21\text{ kW} = 21000\text{ W}$

$R(\rightarrow): F - 600 = 0 \Rightarrow F = 600$

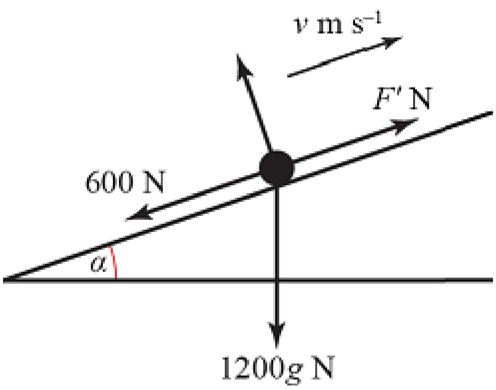
$P = Fv$

$21000 = 600v \Rightarrow v = \dfrac{21000}{600} = 35$

The speed of the car is 35 m s^{-1}

In both parts of this question, as the car is moving with constant speed, the acceleration is zero. So the vector sum of the forces acting on the car is zero.

b



Let the speed of the car be u ms^{-1} and the driving force of the engine have magnitude F'N.

The driving forces in **a** and **b** are different and it is a good idea to avoid confusion by using different symbols for the forces.

$R(\nearrow): F' - 1200g\sin\alpha - 600 = 0$

$F' = 1200 \times 9.8 \times \dfrac{1}{14} + 600 = 1440$

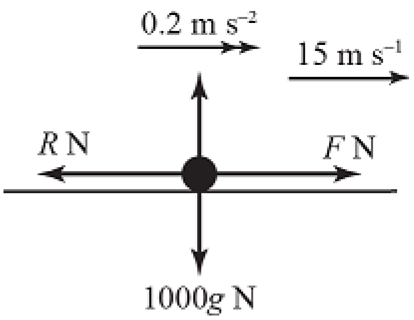
$P = Fv$

$21\,000 = 1440v \Rightarrow v = 14.58\dot{3}$

The constant speed of the car as it moves up the hill is 15 m s^{-1} (2 s.f.)

Although there is an exact answer, $14\frac{7}{12}$, a numerical value for g has been used in the question and the answer should be rounded to 2 significant figures. Three significant figures (14.6) is also acceptable.

19

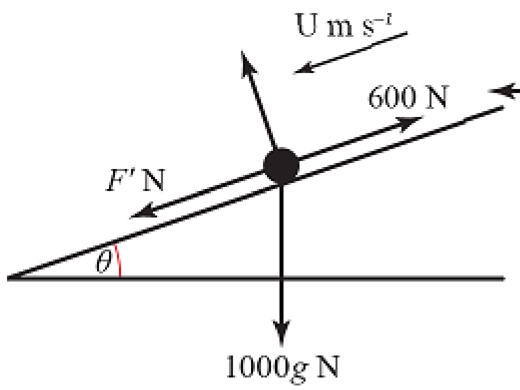


a Let F N be the magnitude of the driving force produced by the engine of the car.

$$12\text{ kW} = 12\,000\text{ W}$$
$$P = Fv$$
$$12\,000 = F \times 15$$
$$F = \frac{12\,000}{15} = 800$$
$$R(\rightarrow): F = ma$$
$$F - R = 1000 \times 0.2$$
$$R = F - 1000 \times 0.2$$
$$= 800 - 200 = 600, \text{ as required}$$

Using Newton's second law, the vector sum of the forces on the car equals the mass times acceleration.

b



Resistance acts against motion. As the car is travelling down the hill, the resistance of 600 N acts up the hill.

Let the driving force of the engine have magnitude F' N.

$$R(\swarrow): F' + 1000g\sin\theta - 600 = 0$$
$$F' = 600 - 1000 \times 9.8 \times \frac{1}{40} = 355$$

As the car is travelling at a constant speed, there is no acceleration.

$$7\text{ kW} = 7000\text{ W}$$
$$P = Fv$$
$$7000 = 355U$$
$$U = \frac{7000}{355} = 19.718\ldots = 20\ (2\text{ s.f.})$$

20 $m = 600$ kg, $R = \left(500 + 2v^2\right)$ N, $v = 15$ m s^{-1}, $P = ?$

a The engine must create a force F where $F = R$

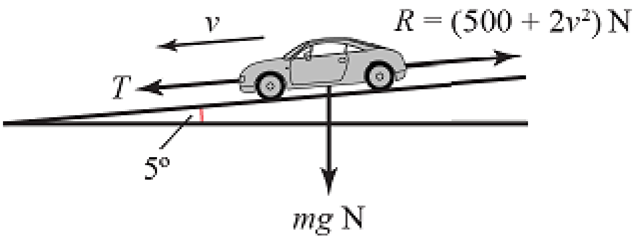
$P = Fv$

$P = \left(500 + 2v^2\right)v$

$P = \left(500 + \left(2 \times 15^2\right)\right) \times 15 = 14\,250$

For the motorcycle to maintain a constant speed of 15 m s^{-1} on a horizontal road, the engine must deliver 14.3 kW (3 s.f.)

b



Resolving parallel to the slope:

$T = \left(500 + 2v^2\right) - mg \sin 5$

So the power required is:

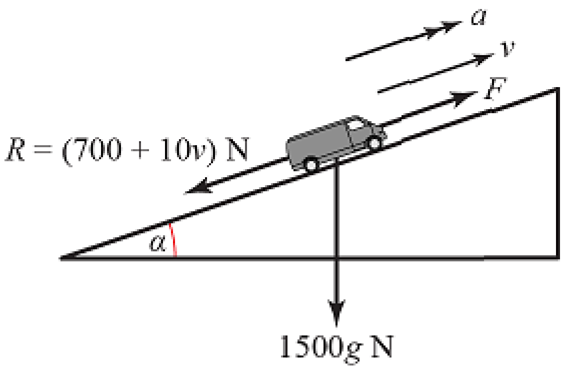
$P = \left(500 + 2v^2 - mg \sin 5\right)v$

$P = \left(500 + \left(2 \times 15^2\right) - (600 \times 9.8 \sin 5)\right) \times 15 = 6562.8...$

For the motorcycle to maintain a constant speed of 15 m s^{-1} when travelling down a road inclined at 5° to the horizontal, the engine must deliver 6.6 kW (2 s.f.)

21 $m = 1500\text{ kg}$, $R = (700 + 10v)\text{ N}$, $v = 30\text{ m s}^{-1}$

a



$P = 60\,000\text{ W}$
The force provided by the engine, F, is given by:

$$P = Fv$$
$$60\,000 = 30F$$
$$F = 2000$$

Using Newton's second law of motion up the hill:

$$2000 - (700 + 10v) - mg\sin\alpha = ma$$
$$2000 - (700 + 300) - \left(1500 \times 9.8 \times \tfrac{1}{12}\right) = 1500a$$
$$a = -\frac{225}{1500} = -0.15$$

The initial deceleration of the van is 0.15 m s^{-2}

b $P = 80\,000\text{ W}$
The force provided by the engine is now given by:

$$P = F'v$$
$$80\,000 = F'v$$
$$F' = \frac{80\,000}{v}$$

When the van reaches its maximum speed, the acceleration will be zero.
Therefore, by Newton's second law,
the resultant force on the van (in the direction of the acceleration)
will be zero. So

$$\frac{80\,000}{v} - (700 + 10v) - \left(1500 \times 9.8 \times \tfrac{1}{12}\right) = 0$$
$$10v^2 + 1925v - 80\,000 = 0$$
$$v = 35.14\ldots \text{ or } v = -227.64\ldots$$

Only the positive root is relevant.

When the engine operates at 80 kW, the van maintains a constant uphill speed of 35.1 m s^{-1} (3 s.f.)

22 a The vertical distance fallen by P in moving from A to C is (45 – 30) m = 15 m

Using the principle of conservation of energy, kinetic energy gained = potential energy lost

$$\frac{1}{2}\cancel{m}v^2 - \frac{1}{2}\cancel{m}u^2 = \cancel{m}gh$$

$$\frac{1}{2}\times 24.5^2 - \frac{1}{2}u^2 = 9.8\times 15$$

$$u^2 = 24.5^2 - 2\times 9.8\times 15 = 306.25$$

$$u = \sqrt{306.25} = 17.5, \text{as required}$$

The mass of the particle cancels throughout this equation. The calculations in this question are independent of the mass of P.

This equation has a similar form to $v^2 = u^2 + 2as$. However, it would be an error to use this formula, which is a formula for motion in a straight line, as P is not moving in a straight line.

b $\text{R}(\rightarrow): u_x = u\cos\theta = 17.5\times\frac{4}{5} = 14$

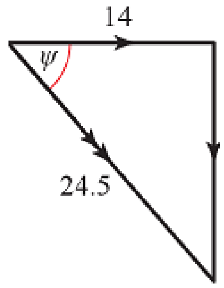
The horizontal component of the velocity is constant throughout the motion.

Let the required angle be ψ

$$\cos\psi = \frac{14}{24.5} = \frac{4}{7}$$

$$\psi = 55.15\ldots^\circ = 55^\circ \text{ (nearest degree)}$$

At C, the velocity of P and its components are illustrated in this diagram.



ψ can now be found using trigonometry. There is no need to find the vertical component of the velocity at C.

c $R(\uparrow): u_y = u\sin\theta = 17.5\times\frac{3}{5} = 10.5$

To find the time taken for P to move from A to D

$$R(\uparrow): s = ut + \frac{1}{2}at^2$$

$$-45 = 10.5t - 4.9t^2$$

$$4.9t^2 - 10.5t - 45 = 0$$

$$49t^2 - 105t - 450 = 0$$

$$(7t - 30)(7t + 15) = 0$$

$$t = \frac{30}{7}, \text{as } t > 0$$

These factors are difficult to spot and you can use the formula for a quadratic. You should, however, obtain an exact answer.

$$\text{R}(\rightarrow): \text{distance} = \text{speed}\times\text{time}$$

$$= 14\times\frac{30}{7} = 60$$

$$BD = 60\text{ m}$$

23 a The kinetic energy, in J, gained in moving from A to B is

$$\frac{1}{2}mv^2 = \frac{1}{2}80 \times 20^2 = 16\,000$$

The potential energy, in J, lost in moving from A to B is

$$mgh = 80 \times 9.8 \times (32.5 - 8.1) = 19\ 129.6$$

The net loss of mechanical energy is

$$(19129.6 - 16\,000)\text{J} = 3129.6\text{J}$$

The net loss in mechanical energy is the work done by the resistance to motion.

The work done by the resisting force of R newtons, in J, is given by

$$\text{Work} = \text{force} \times \text{distance}$$
$$= R \times 60$$

By the work–energy principle

$$60R = 3129.6$$
$$R = \frac{3129.6}{60} = 52.16 = 52\text{ N}\,(2\text{ s.f.})$$

b $\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{3}{5}, \cos\alpha = \frac{4}{5}$

You can sketch a 3, 4, 5, triangle to check these relations.

$$\text{R}(\rightarrow): u_x = 20\cos\alpha = 20 \times \frac{4}{5} = 16$$

$$R(\uparrow): u_y = 20\sin\alpha = 20 \times \frac{3}{5} = 12$$

To find the time taken to move from B to C

$$\text{R}(\uparrow): s = ut + \frac{1}{2}at^2$$
$$-8.1 = 12t - 4.9t^2$$
$$4.9t^2 - 12t - 8.1 = 0$$
$$49t^2 - 120t - 81 = 0$$
$$(t-3)(49t+27) = 0$$
$$t = 3, \text{as } t > 0$$

Rearranging the quadratic and multiplying by 10.

The time taken to move from B to C is 3 s.

c $\text{distance} = \text{speed} \times \text{time}$

$$= 16 \times 3 = 48$$

The horizontal distance from B to C is 48 m.

23 d Let the speed of the skier immediately before reaching C be w m s^{-1}

Using the conservation of energy

$$\frac{1}{2}\not{m}w^2 - \frac{1}{2}\not{m}v^2 = \not{m}gh$$

$$w^2 = v^2 + 2gh$$

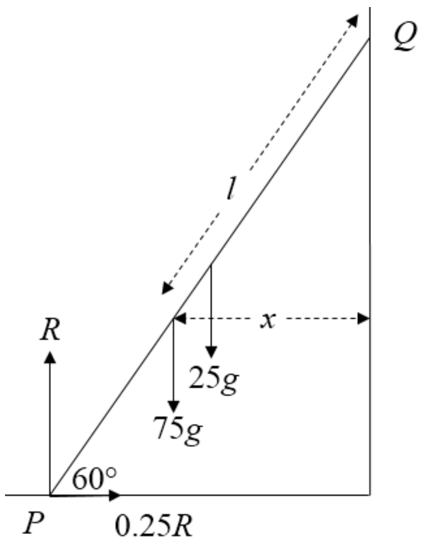
$$= 20^2 + 2\times 9.8\times 8.1 = 558.76$$

$$w = \sqrt{558.76} = 23.638\ldots$$

Cancelling the m and rearranging the formula. This result is similar to $v^2 = u^2 + 2as$. However, it would be an error to use this formula, which is a formula for motion in a straight line, as the skier is not moving in a straight line. You must establish the result using the principle of conservation of energy.

The speed of the skier immediately before reaching C is 24 m s^{-1} (2 s.f.)

24



Res(↑) $R = 100g$

$\mu R = 25g$

Taking moments about Q gives:

$3R = 75gx + 25g\times 3\cos 60 + 25g\times 6\sin 60$

$75gx = 300g - 37.5g - 75g\sqrt{3}$

$$x = \frac{300 - 37.5 - 75\sqrt{3}}{75}$$

$= 1.767...$

$$\cos 60 = \frac{1.767...}{l}$$

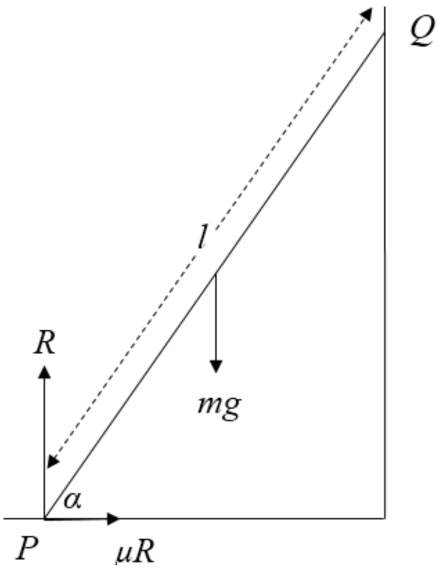
$l = 3.535...$

Since the ladder is 6 m long, the builder can walk:

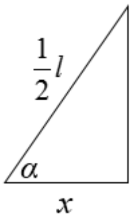
$6 - 3.535\ldots = 2.464\ldots$

$= 2.46$ m up the ladder

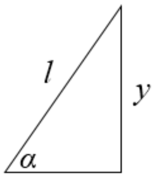
25



Res(↑) $R = mg \Rightarrow \mu R = \mu mg$



$x = \frac{1}{2} l \cos\alpha$



$y = l \sin\alpha$

Taking moments about Q gives:

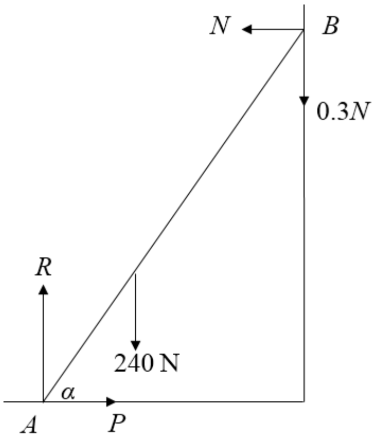
$$mg \times \frac{1}{2} l \cos\alpha + \mu mg \times l \sin\alpha = R \times l \cos\alpha$$

$$\mu mgl \sin\alpha = mgl \cos\alpha - \frac{1}{2} mgl \cos\alpha$$

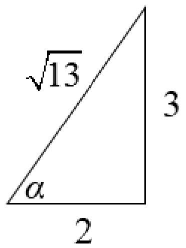
$$\mu mgl \sin\alpha = \frac{1}{2} mgl \cos\alpha$$

$$\mu = \frac{1}{2\tan\alpha}$$

26



Res(↑) $R + 0.3N = 240$
Res(→) $P = N$
$R = 240 - 0.3P$ **(1)**



$\tan\alpha = \frac{3}{2} \Rightarrow \sin\alpha = \frac{3}{\sqrt{13}}$ and $\cos\alpha = \frac{2}{\sqrt{13}}$

Taking moments about B gives:

$240 \times 4\cos\alpha + 6\sin\alpha P = 6\cos\alpha R$

$240 \times \frac{8}{\sqrt{13}} + \frac{18}{\sqrt{13}}P = \frac{12}{\sqrt{13}}R$

$1920 + 18P = 12R$

$R = \frac{1920 + 18P}{12}$ **(2)**

Equating **(1)** and **(2)** gives:

$\frac{1920 + 18P}{12} = 240 - 0.3P$

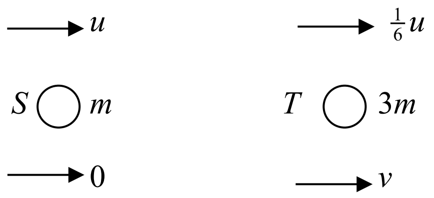
$1920 + 18P = 2880 - 3.6P$

$21.6P = 960$

$P = 44.444....$

$= 44.4$ N (3 s.f.)

27



$$mu + 3m \times \frac{1}{6}u = 3mv$$

Conservation of momentum.

$$\frac{3}{2}u = 3v$$

$$u = 2v$$

$$e\left(u - \frac{1}{6}u\right) = v$$

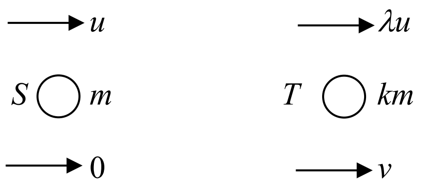
Newton's Law of Restitution.

$$\therefore \quad \frac{5}{6}eu = \frac{1}{2}u$$

$$e = \frac{6}{5} \times \frac{1}{2}$$

$$e = \frac{3}{5}$$

28 a



$mu + km\lambda u = kmv$ — Conservation of momentum.

$u(1 + k\lambda) = kv$ **(1)**

$v = e(u - \lambda u) = eu(1 - \lambda)$ **(2)** — Newton's Law of Restitution.

Eliminate v from **(1)** and **(2)**

$u(1 + k\lambda) = keu(1 - \lambda)$

$$e = \frac{1 + k\lambda}{k(1 - \lambda)}$$

b

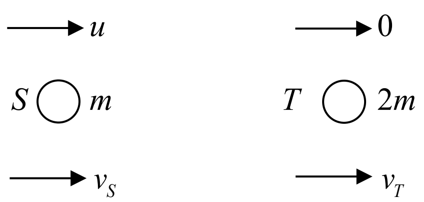
$e \leqslant 1$

$\Rightarrow 1 + k\lambda \leqslant k - k\lambda$ — Any coefficient of restitution satisfies $0 \leqslant e \leqslant 1$

$$\frac{1}{1 - 2\lambda} \leqslant k$$

but $0 < \lambda < \frac{1}{2} \Rightarrow 0 < 1 - 2\lambda < 1$ and $k > 1$

29 a



$mu = mv_S + 2mv_T$

$u = v_S + 2v_T$ **(1)** — Conservation of momentum.

$eu = v_T - v_S$ **(2)** — Newton's Law of Restitution.

(1) + (2): $u + eu = 3v_T$

$$v_T = \frac{1}{3}u(1 + e)$$

29 b i from **(2)**

$$eu = \frac{1}{3}u(1+e) - v_S$$

$$v_S = \frac{1}{3}u(1+e) - eu$$

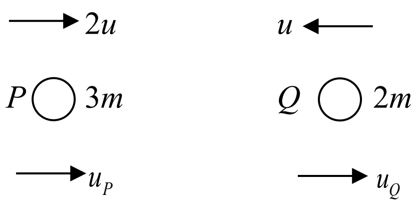
$$v_S = \frac{1}{3}u(1-2e)$$

but $e > \frac{1}{2} \Rightarrow 1 - 2e < 0$ ← Speed must be positive.

$\therefore$ Speed of S is $\frac{1}{3}u(2e-1)$

ii The arrow in the diagram was the wrong way round, as shown in **b i**, so the direction of motion was reversed.

30 a



← Speed must be positive.

$$3m \times 2u - 2mu = 3mu_P + 2mu_Q$$ ← Conservation of momentum.

$$\therefore \quad 4u = 3u_P + 2u_Q \quad \textbf{(1)}$$

$$e(2u+u) = u_Q - u_P$$ ← Newton's Law of Restitution.

$$3eu = u_Q - u_P \quad \textbf{(2)}$$

Eliminating u_P between **(1)** and **(2)**:

$$4u = 3(u_Q - 3eu) + 2u_Q$$

$$4u = 5u_Q - 9eu$$

$$u_Q = \frac{1}{5}u(9e+4)$$

30 b Using **(2)**

$$u_P = u_Q - 3eu$$
$$= \frac{1}{5}u(9e+4) - 3eu$$
$$= \frac{2}{5}u(2-3e)$$

But

$$u_P < 0$$

Direction of motion of P is reversed.

$$\therefore 2 - 3e < 0$$
$$e > \frac{2}{3}$$
$$\therefore \quad \frac{2}{3} < e \leqslant 1$$

Use the general condition $0 \le e \le 1$

c For Q

$$\frac{32}{5}mu = 2m \times \frac{1}{5}u(9e+4) + 2mu$$
$$32 = 2(9e+4) + 10$$
$$18e = 14$$
$$e = \frac{7}{9}$$

Impulse = change of momentum.

31 For the fall, down positive:

$u = 0\text{ m s}^{-1}$, $a = g$, $t = 2\text{ s}$, $v = ?$

$$v = u + at$$
$$v = 0 + 2g = 2g$$

Speed after the bounce, v', is given by Newton's law of restitution:

$$v' = ev = \frac{6}{7} \times 2g$$

For the return, up positive: $u = \frac{12}{7}g\text{ m s}^{-1}$, $a = -g = -9.8\text{ m s}^{-2}$, $v = 0\text{ m s}^{-1}$, $s = ?$

$$v^2 = u^2 + 2as$$
$$0 = \left(\frac{12}{7}g\right)^2 - 2gs$$
$$2s = \left(\frac{12}{7}\right)^2 g$$
$$s = \frac{1}{2} \times \frac{144}{49} \times 9.8 = 14.4$$

The ball rises to a height of 14.4 m on the first bounce.

32 a Distance travelled = s, $t_{in} = 2$ s, $t_{out} = 3$ s

When travelling towards the wall, average speed, $u = \frac{s}{t_{in}} = \frac{s}{2}$

When travelling away from the wall, average speed, $v = \frac{s}{t_{out}} = \frac{s}{3}$

Using Newton's law of restitution:
$v = eu$

$\frac{s}{3} = e\frac{s}{2}$

$e = \frac{\frac{s}{3}}{\frac{s}{2}} = \frac{2}{3}$

The coefficient of restitution is $\frac{2}{3}$.

b If the plane is rough, then the sphere will experience a frictional force and decelerate as it travels to and from the wall.

If the times it takes to travel between the wall and P are the same as in part **a**, then, although the average speed in each direction remains the same, the sphere hits the wall at a lower speed (u is smaller) and leaves it at a greater speed (v is greater) than the values calculated.

Since the coefficient of restitution is given by $e = \frac{v}{u}$, it would therefore have a bigger value than that calculated in part **a**.

33 a For the fall, down positive: $u = 0$ m s^{-1}, $a = g$, $s = 50$ m, $v = ?$
$v^2 = u^2 + 2as$

$v^2 = 2g \times 50 = 100g$

Speed after the bounce, v', is given by Newton's law of restitution:
$v' = ev$

$v'^2 = e^2v^2 = 100ge^2$

For the return, up positive: $v = 0$ m s^{-1}, $u = v'$, $a = -$g, $s = 35$ m

$v^2 = u^2 + 2as$

$0 = 100ge^2 - (2g \times 35)$

$100e^2 = 70$

$e = \frac{\sqrt{70}}{10}$

The coefficient of restitution is $\frac{\sqrt{70}}{10}$

33 b For the first fall, down positive: $u = 0$ m s^{-1}, $a = g$, $s = 50$ m, $t = t_1$

$s = ut + \frac{1}{2}at^2$

$50 = \frac{1}{2}gt_1^2$

$t_1^2 = \frac{100}{g}$

For the second fall, down positive: $u = 0$ m s^{-1}, $a = g$, $s = 35$ m, $t = t_2$

$35 = \frac{1}{2}gt_2^2$

$t_2^2 = \frac{70}{g}$

The ball takes the same time to rise to 35 m after the first bounce so total time, t, is given by:

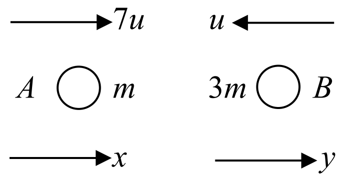
$t = t_1 + 2t_2$

$t = \frac{10}{\sqrt{g}} + 2\frac{\sqrt{70}}{\sqrt{g}}$

If $g = 9.8 \Rightarrow t = \frac{10}{\sqrt{9.8}} + 2\frac{\sqrt{70}}{\sqrt{9.8}}$

$t = 8.54$ s

34 a



$e = 0.25$

Draw a diagram.

$$7mu - 3mu = mx + 3my$$

$$4u = x + 3y \quad \textbf{(1)}$$

Conservation of momentum.

$$0.25 \times (7u + u) = y - x$$

$$2u = y - x \quad \textbf{(2)}$$

Newton's Law of Restitution.

$$\textbf{(1)} + \textbf{(2)}: \; 6u = 4y$$

$$y = \frac{3u}{2}$$

Solve **(1)** and **(2)** simultaneously.

$$\text{In } \textbf{(2)}: \; 2u = \frac{3u}{2} - x$$

$$x = -\frac{u}{2}$$

The minus sign shows the arrow in the diagram is pointing in the wrong direction.

A has speed $\frac{u}{2}$

B has speed $\frac{3u}{2}$

Speed is always positive.

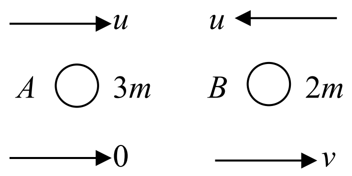
b K.E. lost

$$= \frac{1}{2} \times m \times (7u)^2 + \frac{1}{2} \times 3m \times u^2 - \left(\frac{1}{2} m \times \left(\frac{u}{2} \right)^2 + \frac{1}{2} \times 3m \left(\frac{3u}{2} \right)^2 \right)$$

$$= \frac{1}{2} m \times 49u^2 + \frac{3}{2} mu^2 - \left(\frac{mu^2}{8} + \frac{27mu^2}{8} \right)$$

$$= \frac{45}{2} mu^2$$

35



a $3mu - 2mu = 2mv$

$mu = 2mv$ ← Conservation of momentum.

$e(u+u) = v$

$2ue = v$ ← Newton's Law of Restitution.

Eliminating v:

$mu = 2m(2ue)$

$e = \frac{1}{4}$

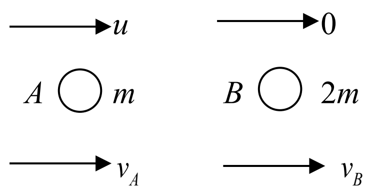
b K.E. lost

$= \frac{1}{2} \times 3mu^2 + \frac{1}{2} \times 2mu^2 - \left(0 + \frac{1}{2} \times 2m\left(\frac{1}{2}u\right)^2\right)$ ← $v = 2ue = 2u \times \frac{1}{4}$ $= \frac{1}{2}u$

$= \frac{5}{2}mu^2 - \frac{1}{4}mu^2$

$= \frac{9}{4}mu^2$

36 a



$mu = mv_A + 3mv_B$

$y = v_A + 3v_B$ **(1)** ← Conservation of momentum.

$eu = v_B - v_A$ **(2)** ← Newton's Law of Restitution.

(1) + (2): $u + eu = 4v_B$

$v_B = \frac{1}{4}(1+e)u$

b Using **(2)**:

$v_A = v_B - eu$

$= \frac{1}{4}(1+e)u - eu$

$= \frac{1}{4}(1-3e)u$

36 c K.E. after impact

$$= \frac{1}{2}mv_A^2 + \frac{1}{2}\times 3mv_B^2$$

$$= \frac{1}{2}m\left(\frac{1}{4}(1-3e)u\right)^2 + \frac{3}{2}m\left(\frac{1}{4}(1+e)u\right)^2$$

$$= \frac{1}{2}m\frac{u^2}{16}(1-6e+9e^2) + \frac{3}{2}m\frac{u^2}{16}(1+2e+e^2)$$

$$= \frac{mu^2}{32}(1-6e+9e^2+3+6e+3e^2)$$

$$= \frac{mu^2}{32}(4+12e^2)$$

$$= \frac{mu^2}{8}(1+3e^2)$$

K.E. after impact $= \frac{1}{6}mu^2$

$$\therefore \quad \frac{1}{8}(1+3e^2) = \frac{1}{6}$$

$$6+18e^2 = 8$$

$$18e^2 = 2$$

$$e^2 = \frac{1}{9}$$

$$e = \frac{1}{3} \quad (e > 0)$$

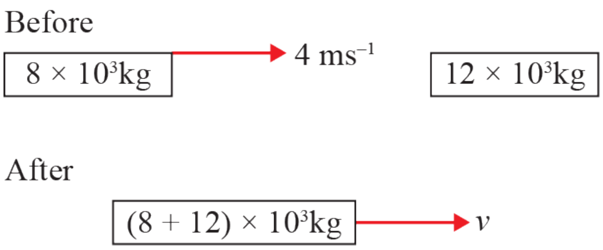
d $v_A = \frac{u}{4}(1-3e)$

$$= \frac{u}{4}(1-3\times\frac{1}{3})$$

$$= 0$$

$\therefore$ A is at rest.

37 Initially: $m = 8 \times 10^3$ kg, $v = 4$ m s^{-1}, kinetic energy = E_{ki}



$$E_k = \frac{1}{2}mv^2$$

$$E_{ki} = \frac{1}{2} \times 8 \times 10^3 \times 4^2 = 64 \times 10^3$$

Finally: $m = (8 \times 10^3 + 12 \times 10^3) = 20 \times 10^3$ kg, $v = 1.5$ m s^{-1}, kinetic energy = E_{kf}

$$E_{kf} = \frac{1}{2} \times 20 \times 10^3 \times 1.5^2 = 22.5 \times 10^3$$

Change in kinetic energy:

$$\Delta E_k = E_{ki} - E_{kf}$$

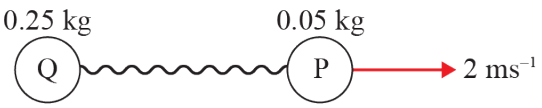
$$\Delta E_k = 64 \times 10^3 - 22.5 \times 10^3$$

$$\Delta E_k = 41.5 \times 10^3$$

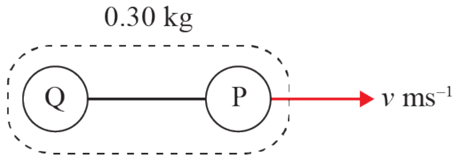
The loss in kinetic energy is 41.5 kJ.

38 Initially: $m = 0.05$ kg, $v = 2$ m s^{-1}, kinetic energy = E_{ki}

Before



After



$$E_k = \frac{1}{2}mv^2$$

$$E_{ki} = \frac{1}{2} \times \frac{1}{20} \times 2^2 = 0.1$$

Once string is taut, speed of the particles, v, is found using conservation of momentum:

$$0.05 \times 2 = (0.05 + 0.25)v$$
$$0.1 = 0.3v$$
$$v = \frac{1}{3}$$

and the total final kinetic energy E_{kf} is

$$E_{kf} = \frac{1}{2} \times \frac{3}{10} \times \frac{1}{3}^2 = \frac{1}{60}$$

Change in kinetic energy:

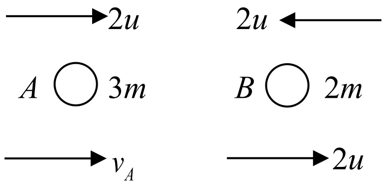
$$\Delta E_k = E_{ki} - E_{kf}$$

$$\Delta E_k = \frac{1}{10} - \frac{1}{60}$$

$$\Delta E_k = \frac{1}{12}$$

The loss in kinetic energy is $\frac{1}{12}$ J.

39 a



Direction of motion of B is reversed but speed is unchanged.

$$3m \times 2u - 2m \times 2u = 3mv_A + 2m \times 2u$$

$$2u = 3v_A + 4u$$

Conservation of momentum.

$$v_A = -\frac{2}{3}u$$

$$e(2u + 2u) = 2u - v_A$$

Newton's Law of Restitution.

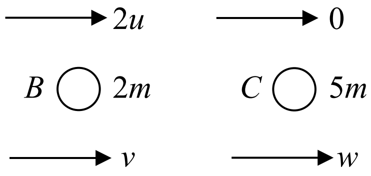
$$4eu = 2u - v_A$$

$$\therefore \quad 4eu = 2u + \frac{2}{3}u$$

$$4eu = \frac{8}{3}u$$

$$e = \frac{2}{3}$$

39 b



$2m \times 2u = 2mv \times 5mw$

$4u = 2v + 5w$ **(3)**

Conservation of momentum.

$\frac{3}{5} \times 2u = w - v$ **(4)**

Newton's Law of Restitution.

Eliminate w from **(3)** and **(4)**

$4u = 2v + 5\left(\frac{6u}{5} + v\right)$

$4u = 2v + 6u + 5v$

$7v = -2u$

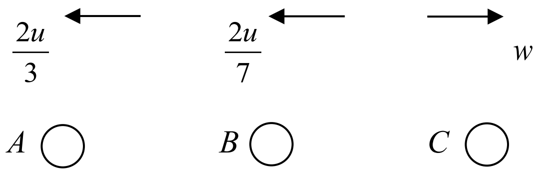
$v = -\frac{2}{7}u$

B will not hit *C* again as they move in opposite directions. You must investigate the possibility of *B* hitting *A* again.

From **a**

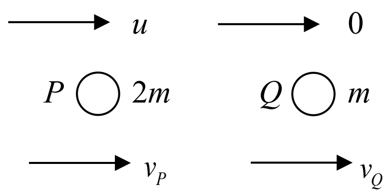
$v_A = -\frac{2}{3}u$

After the collision between *B* and *C*:



As speed *A* > speed *B* there will be no further collisions.

40 a



$$2mu = 2mv_P + mv_Q$$ Conservation of momentum.

$$2u = 2v_P + v_Q \quad \textbf{(1)}$$

$$\frac{1}{3}u = v_Q - v_P \quad \textbf{(2)}$$ Newton's Law of Restitution.

$$\textbf{(1)} + 2 \times \textbf{(2)}: \frac{8u}{3} = v_Q + 2v_Q$$

$$3v_Q = \frac{8u}{3}$$

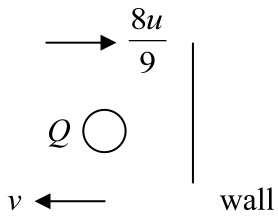
$$v_Q = \frac{8u}{9}$$

Using **(2)**

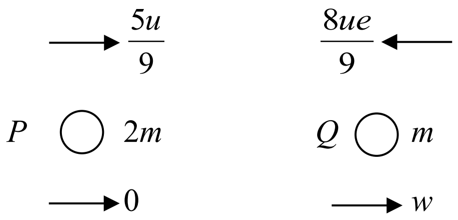
$$v_P = v_Q - \frac{1}{3}u$$

$$v_P = \frac{8u}{9} - \frac{1}{3}u$$

$$v_P = \frac{5u}{9}$$

40 b 

$v = \frac{8ue}{9}$ ← Newton's Law of Restitution.



Here, $e = \frac{1}{3}$ again.

$2m \times \frac{5u}{9} - m \times \frac{8ue}{9} = mw$ ← Conservation of momentum.

$10u - 8ue = 9w$ **(3)** ← Newton's Law of Restitution.

$\frac{1}{3}\left(\frac{5u}{9} + \frac{8ue}{9}\right) = w$

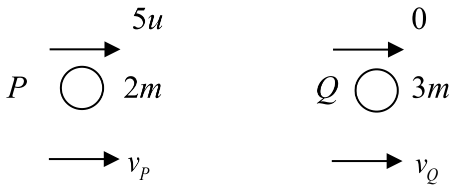
$5u + 8ue = 27w$ **(4)**

Eliminating w between **(3)** and **(4)**:

$$3(10u - 8ue) = 5u + 8ue$$
$$30u - 24ue = 5u + 8ue$$
$$32ue = 25u$$
$$e = \frac{25}{32}$$

c Q is now moving towards the wall once more. After Q hits the wall, it will return to collide with P once more.

41 a



$2m \times 5u = 2mv_P + 3mv_Q$ — Conservation of momentum.

$10u = 2v_P + 3v_Q$ **(1)**

$e \times 5u = v_Q - v_P$ **(2)** — Newton's Law of Restitution.

(1) + 2 × **(2)**:

$10u + 10eu = 3v_Q + 2v_Q$

$10u + 10eu = 5v_Q$

$v_Q = 2u + 2eu = 2(1+e)u$

b From **(2)**

Find direction of motion for P, as if P is moving towards the wall there must be a second collision between P and Q.

$v_P = v_Q - 5eu$

$= 2(1+e)u - 5eu$

$v_P = 2 \times 1.4u - 5 \times 0.4u$ — $e = 0.4$ in **b**.

$= 0.8u$

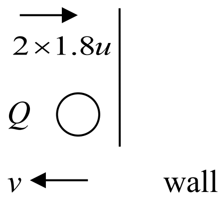
$v_P > 0 \therefore$ P moves towards the wall and will collide with Q after Q rebounds from the wall.

c $e = 0.8$ — e = 0.8

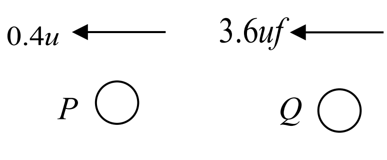
$v_P = 2 \times 1.8u - 5 \times 0.8u$

$= -0.4u$

Q hits the wall:



$v = 3.6uf$ — Newton's Law of Restitution.



For a second collision

$3.6uf > 0.4u$

$f > \frac{0.4}{3.6} = \frac{1}{9}$

Range for f is

$\frac{1}{9} < f \leqslant 1$ — All coefficients of restitution are less than or equal to 1.

42 a By the conservation of momentum
momentum before = momentum after

$2m \times 2u + 3m \times u = 2mv_A + 3mv_B$

$2v_A + 3v_B = 7u$ **(1)**

By Newtons's law of restitution

$e = \frac{\text{speed of separation}}{\text{speed of approach}}$

$e = \frac{v_B - v_A}{2u - u}$

$v_B - v_A = ue$

$v_A = v_B - ue$ **(2)**

Substituting **(2)** into **(1)** gives:

$2(v_B - ue) + 3v_B = 7u$

$5v_B - 2ue = 7u$

$v_B = \frac{7u + 2ue}{5}$

$v_B = \frac{u(7 + 2e)}{5}$

b $v_A = v_B - ue$, therefore:

$v_A = \frac{u(7 + 2e)}{5} - ue$

$= \frac{7u + 2ue - 5ue}{5}$

$= \frac{u(7 - 3e)}{5}$

c $\frac{7u - 3ue}{5} = \frac{11}{10}u$

$70u - 30ue = 55u$

$30ue = 15u$

$e = \frac{1}{2}$

42 d After the collision B has velocity:

$$v_B = \frac{u\left(7+2\left(\frac{1}{2}\right)\right)}{5}$$
$$= \frac{8}{5}u$$

Using $s = ut + \frac{1}{2}at^2$ gives:

$$d = \frac{8}{5}ut \Rightarrow t = \frac{5d}{8u}$$

After the collision A has velocity:

$$v_A = \frac{u\left(7-3\left(\frac{1}{2}\right)\right)}{5}$$
$$= \frac{11u}{10}$$

Using $s = ut + \frac{1}{2}at^2$ gives:

$$s = \frac{11u}{10}\left(\frac{5d}{8u}\right)$$
$$= \frac{11d}{16}$$

Since A has travelled $\frac{11d}{16}$ since the collision, it is a distance of $\frac{5d}{16}$ at the instant B hits the barrier.

e By Newtons's law of restitution

$$e = \frac{\text{speed of rebound}}{\text{speed of approach}}$$
$$\frac{11}{16} = \frac{v_B}{\frac{8}{5}u}$$
$$v_B = \frac{11u}{10}$$

Since A and B are travelling with equal speeds in opposite directions they will collide at the midpoint of the distance from A to the barrier at the instant B hits the barrier, therefore they collide at distance $\frac{5d}{32}$ from the barrier.

43 a Using $v^2 = u^2 + 2as$ gives:

$$v^2 = (0)^2 + 2(9.8)(2)$$
$$= 39.2$$
$$v = \frac{14\sqrt{5}}{5}$$

By Newtons's law of restitution

$$e = \frac{\text{speed of rebound}}{\text{speed of approach}}$$
$$\frac{4}{5} = \frac{\text{speed of rebound}}{\frac{14\sqrt{5}}{5}}$$
$$\text{speed of rebound} = \frac{56\sqrt{5}}{25}$$

Using $v^2 = u^2 + 2as$ gives:

$$(0)^2 = \left(\frac{56\sqrt{5}}{25}\right)^2 + 2(-9.8)s$$
$$19.6s = \frac{3136}{125}$$
$$s = \frac{32}{25}$$
$$= 1.28 \text{ m}$$

So the ball bounces to 0.64 of its original height.

This continues with each successive bounce so the total distance travelled, d, is

$d = 2 + 1.28 + 1.28 + 0.8192 + 0.8192 + 0.524288 + 0.524288 + \ldots$

$= 2 + 2(1.28 + 1.28 \times 0.64 + 1.28 \times 0.64^2 + \ldots$

The expression inside the brackets is a geometric sequence with $a = 1.28$ and $r = 0.64$

Using $S_\infty = \frac{a}{1-r}$ gives:

$$d = 2 + 2\left(\frac{1.28}{1-0.64}\right)$$
$$= \frac{82}{9} \text{ m}$$

b The model predicts an infinite number of bounces which is not realistic.

44 For the first collision

By the conservation of momentum

momentum before = momentum after

$m \times 4 + 2m \times 0 = mv_A + 2mv_B$

$v_A + 2v_B = 4$ **(1)**

By Newtons's law of restitution

$e = \dfrac{\text{speed of separation}}{\text{speed of approach}}$

$0.7 = \dfrac{v_B - v_A}{4}$

$v_B - v_A = 2.8$

$v_A = v_B - 2.8$ **(2)**

Substituting **(2)** into **(1)** gives:

$(v_B - 2.8) + 2v_B = 4$

$3v_B = 6.8$

$v_B = \dfrac{34}{15}$

Since $v_A = v_B - 2.8$

$v_A = \dfrac{34}{15} - 2.8$

$= -\dfrac{8}{15}$

So after the first collision $v_A = -\dfrac{8}{15}$ and $v_B = \dfrac{34}{15}$

For the second collision

By the conservation of momentum

momentum before = momentum after

$2m \times \dfrac{34}{15} + 3m \times 0 = 2mv_B + 3mv_C$

$2v_B + 3v_C = \dfrac{68}{15}$ **(3)**

By Newtons's law of restitution

$e = \dfrac{\text{speed of separation}}{\text{speed of approach}}$

$0.4 = \dfrac{v_C - v_B}{\frac{34}{15}}$

$v_C - v_B = \dfrac{68}{75}$

$v_B = v_C - \dfrac{68}{75}$ **(4)**

Substituting **(4)** into **(3)** gives:

44 (continued)

$$2\left(v_C - \frac{68}{75}\right) + 3v_C = \frac{68}{15}$$

$$5v_C = \frac{476}{75}$$

$$v_C = \frac{476}{375}$$

Since $v_B = v_C - \frac{68}{75}$

$$v_B = \frac{476}{75} - \frac{68}{75}$$

$$= \frac{136}{375}$$

So after the second collision the velocities of the spheres are:

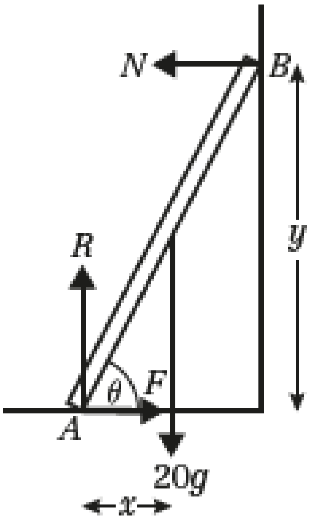
$$v_A = -\frac{8}{15}$$

$$v_B = \frac{136}{375}$$

$$v_C = \frac{476}{375}$$

Therefore there are exactly two collisions.

45 a



Res(↑) $R = 20g \Rightarrow \mu R = 5g$ N

b Res(→) $N = 5g$

Taking moments about A

$20gx = 5gy$

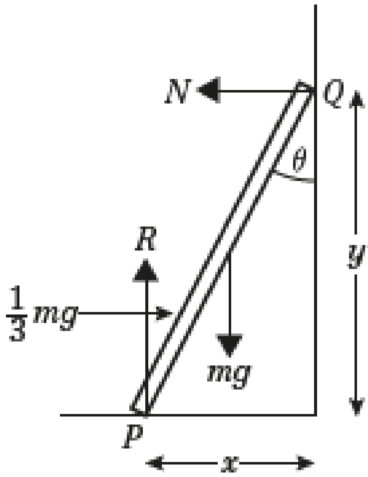
$20g \times 5\cos\theta = 5g \times 10\sin\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{100g}{50g}$

$\tan\theta = 2$

$\theta = \tan^{-1}(2)$ as required

46



Res(↑) $R = mg$

Res(→) $N = \frac{1}{3}mg$

Taking moments about Q

$Rx = \frac{1}{2}mgx + \frac{1}{3}mg \times \frac{3}{4}y$

$mgx = \frac{1}{2}mgx + \frac{1}{4}mgy$

$x = \frac{1}{2}y$

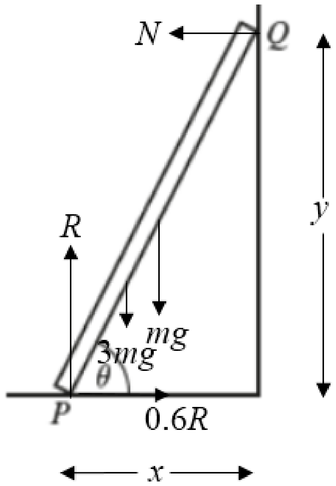
$l\sin\theta = \frac{1}{2}l\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{1}{2}$

$\tan\theta = \frac{1}{2}$

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$ as required

47



Res(↑) $R = 4mg$

Res(→) $N = 0.6R$

$= 2.4mg$

Taking moments about P

$$Ny = \frac{1}{5}x \times 3mg + \frac{1}{2}x \times mg$$

$$2.4y = \frac{3}{5}x + \frac{1}{2}x$$

$$y = \frac{11}{24}x$$

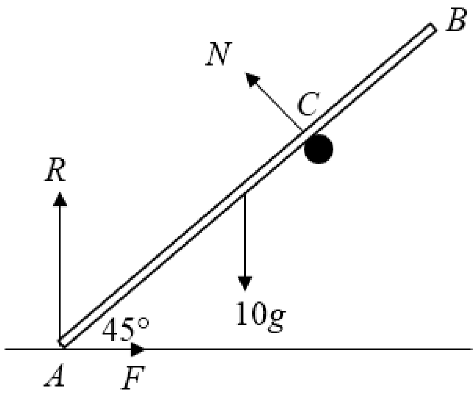
$$\tan\theta = \frac{y}{x}$$

$$= \frac{11}{24}$$

$$\theta = \tan^{-1}\left(\frac{11}{24}\right)$$

$$\theta = 24.6^\circ \text{ (3 s.f.)}$$

48 a



Res(↑) $R + N\cos 45 = 10g$

$R = 10g - \frac{\sqrt{2}}{2}N$ **(1)**

Taking moments about A

$4N = 10g \times 3\cos 45$

$N = \frac{15\sqrt{2}}{4}g$ **(2)**

Substituting **(2)** into **(1)** gives:

$R = 10g - \frac{\sqrt{2}}{2}\left(\frac{15\sqrt{2}}{4}g\right)$

$= 10g - \frac{15}{4}g$

$= \frac{25}{4}g$ N

b From part **a**

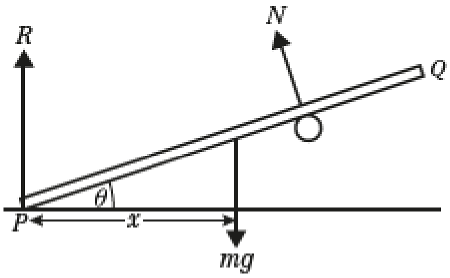
$N = \frac{15\sqrt{2}}{4}g$ N

c Res(→) $F = N\cos 45$

$F = \frac{15\sqrt{2}}{4}g \times \frac{\sqrt{2}}{2}$

$F = \frac{15}{4}g$ N

49



Res(↑) $R + N\cos\theta = mg$

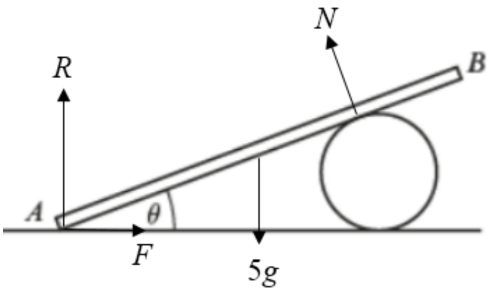
Taking moments about P gives:

$mgx = 0.75lN$

$mg \times \frac{1}{2} l\cos\theta = \frac{3}{4} lN$

$N = \frac{2}{3} mg\cos\theta$ as required

50 a



Res(↑) $R + N\cos\theta = 5g$ **(1)**

Taking moments about A gives:

$4N = 5g \times 2.5\cos\theta$

$N = \frac{25}{8} g\cos\theta$ **(2)**

Substituting **(2)** into **(1)** gives:

$R + \left(\frac{25}{8} g\cos\theta\right)\cos\theta = 5g$

$R = 5g - \frac{25}{8} g\cos^2\theta$

$\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \cos^2\theta = \frac{3}{4}$

Therefore:

$R = 5g - \frac{25}{8} g\left(\frac{3}{4}\right)$

$= \frac{85}{32} g$ N

50 b $\mu R = N\sin\theta$ **(1)**

$R = \frac{85}{32}g$ N **(2)**

$N = \frac{25\sqrt{3}}{16}g$ **(3)**

Substituting **(2)** and **(3)** into **(1)** gives:

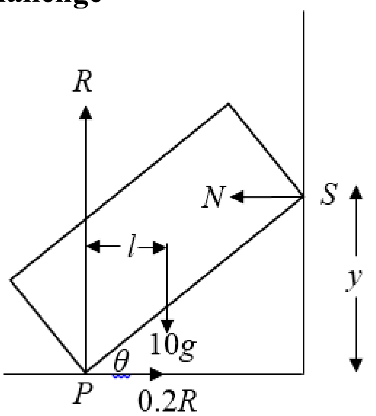
$\mu \times \frac{85}{32}g = \frac{25\sqrt{3}}{16}g \times \frac{1}{2}$

$\frac{85}{32}\mu g = \frac{25\sqrt{3}}{32}g$

$\mu = \frac{25\sqrt{3}}{85}$

$= \frac{5\sqrt{3}}{17}$ as required

Challenge



Res(↑) $R = 10g \Rightarrow \mu R = 2g$

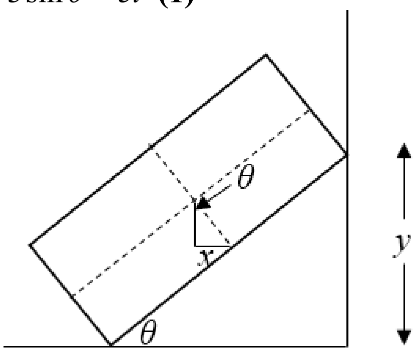
Res(→) $N = 0.2R \Rightarrow N = 2g$

Taking moments about P gives:

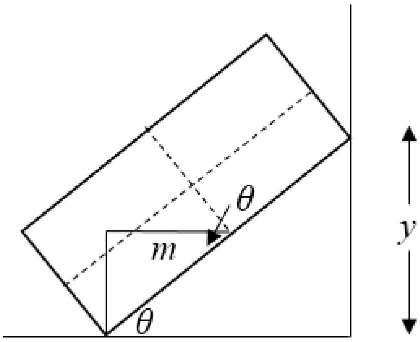
$Ny = 10gl$

$y = 5l$

$3\sin\theta = 5l$ **(1)**



$\sin\theta = \frac{x}{0.5} \Rightarrow x = 0.5\sin\theta$



$\cos\theta = \frac{m}{1.5} \Rightarrow x = 1.5\cos\theta$

$l = m - x$

$l = 1.5\cos\theta - 0.5\sin\theta$ **(2)**

Substituting **(2)** into **(1)** gives:

$3\sin\theta = 5(1.5\cos\theta - 0.5\sin\theta)$

$0.6\sin\theta = 1.5\cos\theta - 0.5\sin\theta$

$1.1\sin\theta = 1.5\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{1.5}{1.1}$

$\tan\theta = \frac{15}{11}$

$\theta = \tan^{-1}\left(\frac{15}{11}\right)$

$\theta = 53.7^\circ$ (3 s.f.)