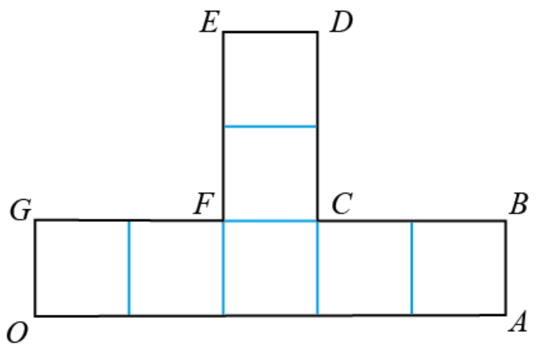
Exercise 3F

1 a



Let O be the origin and OA be the positive x-axis.

$$7\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=5\begin{pmatrix}2.5\\0.5\end{pmatrix}+2\begin{pmatrix}2.5\\2\end{pmatrix}$$

$$=\frac{1}{7}\begin{pmatrix}17.5\\6.5\end{pmatrix}$$

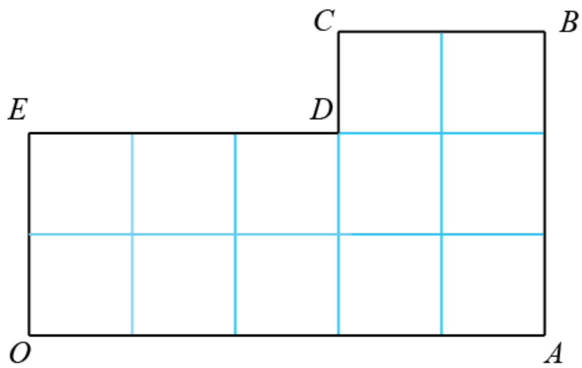
$$=\begin{pmatrix}\frac{5}{2}\\\frac{13}{14}\end{pmatrix}$$

When the lamina is suspended from O let the angle between OA and the downward vertical be θ.

$$\tan\theta=\frac{\frac{13}{14}}{\frac{5}{2}}$$

$\theta = 20.376\ldots$
$= 20.4°$ (3 s.f.)

1 b



Let O be the origin and OA be the positive x-axis.

$$12\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 10\begin{pmatrix}2.5\\ 1\end{pmatrix} + 2\begin{pmatrix}4\\ 2.5\end{pmatrix}$$

$$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \frac{1}{12}\begin{pmatrix}33\\ 15\end{pmatrix}$$

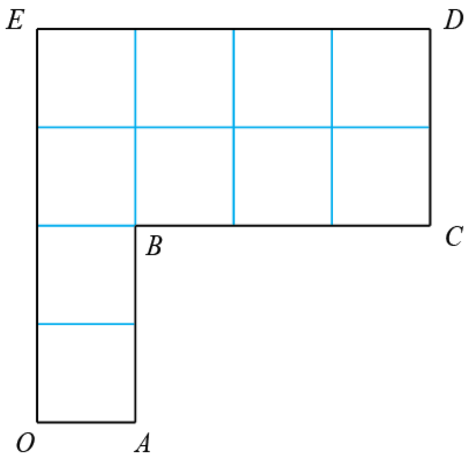
$$= \begin{pmatrix}\frac{33}{12}\\ \frac{5}{4}\end{pmatrix}$$

When the lamina is suspended from O let the angle between OA and the downward vertical be θ.

$$\tan\theta = \frac{\frac{5}{4}}{\frac{33}{12}}$$

$\theta = 24.443\ldots$
$= 24.4°$ (3 s.f.)

1 c



Let O be the origin and OA be the positive x-axis.

$$10\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 4\begin{pmatrix}0.5\\ 2\end{pmatrix} + 6\begin{pmatrix}2.5\\ 3\end{pmatrix}$$

$$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \frac{1}{10}\begin{pmatrix}17\\ 26\end{pmatrix}$$

$$= \begin{pmatrix}\frac{17}{10}\\ \frac{13}{5}\end{pmatrix}$$

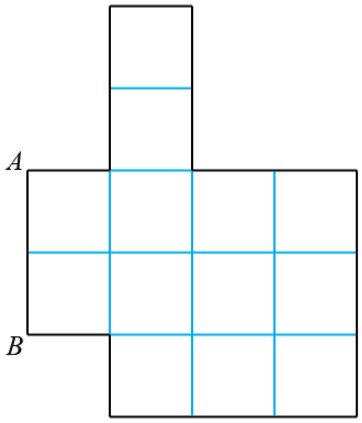
When the lamina is suspended from O let the angle between OA and the downward vertical be θ.

$$\tan\theta = \frac{\frac{13}{5}}{\frac{17}{10}}$$

$\theta = 56.821\ldots$

$= 56.8°$ (3 s.f.)

2



Let A be the origin

$$13\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 2\begin{pmatrix}1.5\\ 1\end{pmatrix} + 8\begin{pmatrix}2\\ -1\end{pmatrix} + 3\begin{pmatrix}2.5\\ -2.5\end{pmatrix}$$

$$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \frac{1}{13}\begin{pmatrix}26.5\\ -13.5\end{pmatrix}$$

$$= \begin{pmatrix}\frac{53}{26}\\ -\frac{27}{26}\end{pmatrix}$$

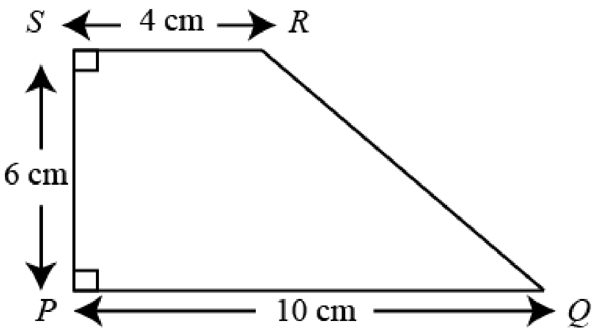
When the lamina is suspended from A let the angle between AB and the downward vertical be θ.

$$\tan\theta = \frac{\frac{53}{26}}{\frac{27}{26}}$$

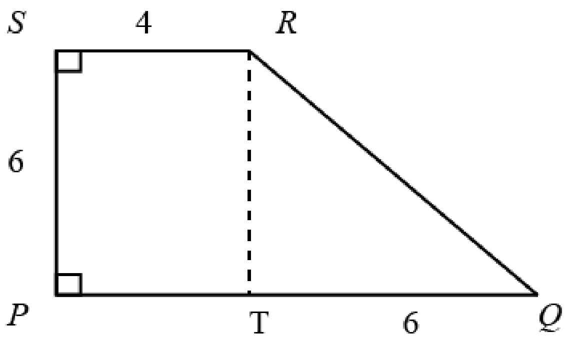
$\theta = 63.004\ldots$
$= 63.0°$ (3 s.f.)

3 Angle between AB and downward vertical = 63.004…° (from Q2)
Therefore, angle between AB and horizontal = 90 – 63.004…. = 27.0° (3 s.f.)

4



Let P be the origin and PQ be the positive x-axis.
Let the point directly below R that lies on PQ be T.



The total area of the shape is:

$$(4\times6)+\frac{1}{2}(6\times6)=42$$

Let PQ be the origin.

$$42\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=24\begin{pmatrix}2\\3\end{pmatrix}+18\begin{pmatrix}6\\2\end{pmatrix}$$

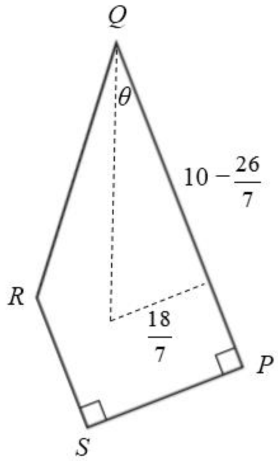
$$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\frac{1}{42}\begin{pmatrix}156\\108\end{pmatrix}$$

$$=\begin{pmatrix}\frac{26}{7}\\ \frac{18}{7}\end{pmatrix}$$

a Distance from PS is $\frac{26}{7}$ cm

b Distance from PQ is $\frac{18}{7}$ cm

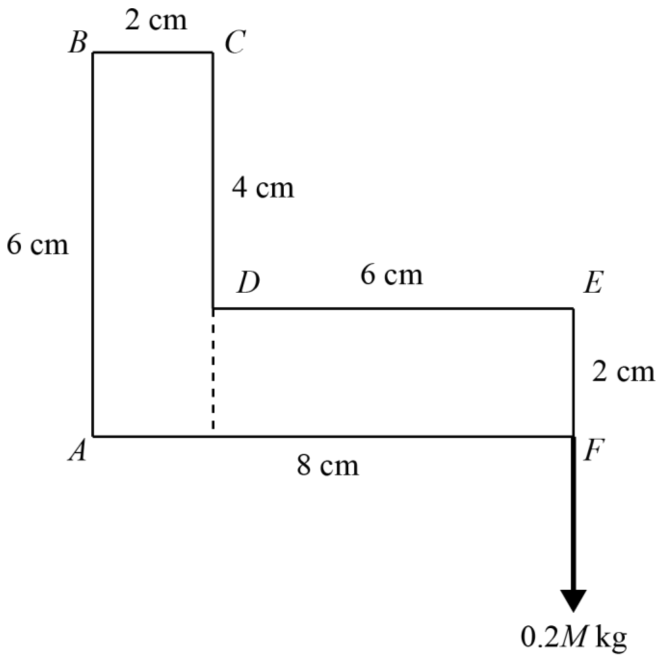
4 c



$$\tan\theta = \frac{\frac{18}{7}}{10-\frac{26}{7}}$$

$\theta = 22.249\ldots$
$\theta = 22.2°$ (3 s.f.)

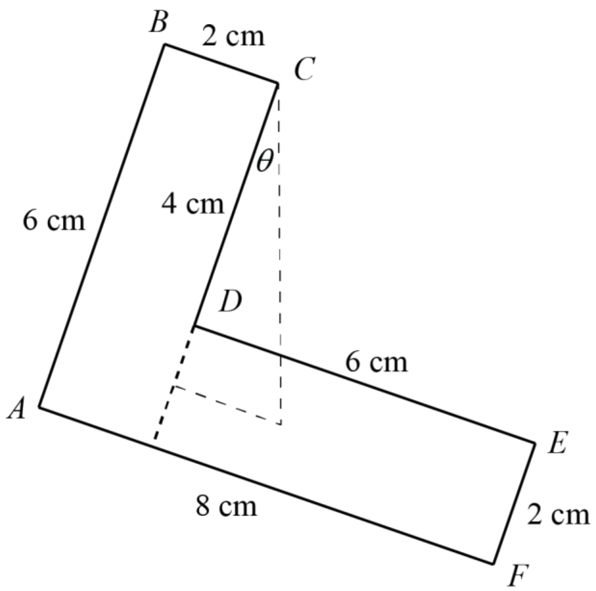
5 Let C be the origin and BC is the negative x-axis.



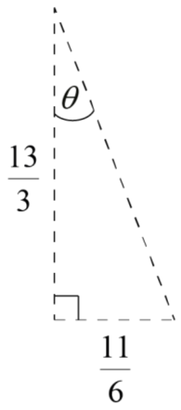
$$0.5\begin{pmatrix}-1\\-3\end{pmatrix}+0.5\begin{pmatrix}3\\-5\end{pmatrix}+0.2\begin{pmatrix}6\\-6\end{pmatrix}=1.2\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=\frac{1}{1.2}\begin{pmatrix}2.2\\-5.2\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=\begin{pmatrix}\frac{11}{6}\\-\frac{13}{3}\end{pmatrix}$$



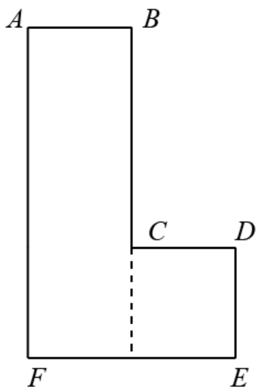
5 (continued)



Required angle is;

$$\theta = \tan^{-1}\left(\frac{\frac{11}{6}}{\frac{13}{3}}\right) = 67.1^{\circ} \text{ (3 s.f.)}$$

6



Let F be the origin and FE be the positive x-axis.

$$16\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=12\begin{pmatrix}1\\ 3\end{pmatrix}+4\begin{pmatrix}3\\ 1\end{pmatrix}$$

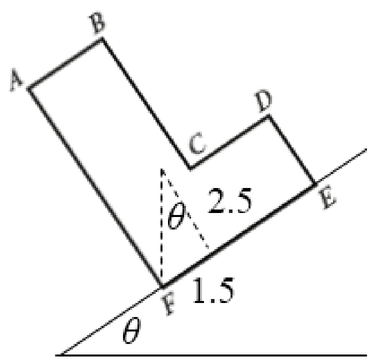
$$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\frac{1}{16}\begin{pmatrix}24\\ 40\end{pmatrix}$$

$$=\begin{pmatrix}\frac{3}{2}\\ \frac{5}{2}\end{pmatrix}$$

a Therefore centre of mass lies 1.5 cm from FA

b Therefore centre of mass lies 2.5 cm from EF

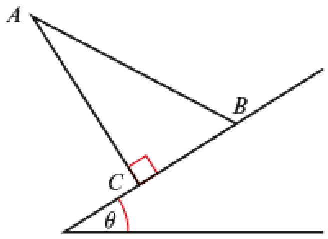
c



$$\tan\theta=\frac{1.5}{2.5}$$

θ = 30.963…
= 31.0° (3 s.f.)

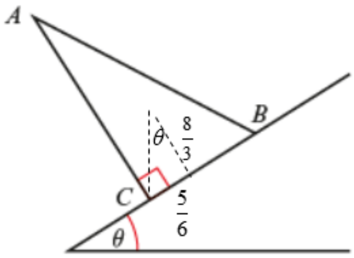
7



Let C be the origin and CB be the positive x-axis.
The centre of mass of the lamina lies at the point (1, 2)

$$1.2M\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = M\begin{pmatrix}1\\2\end{pmatrix} + 0.2M\begin{pmatrix}0\\6\end{pmatrix}$$

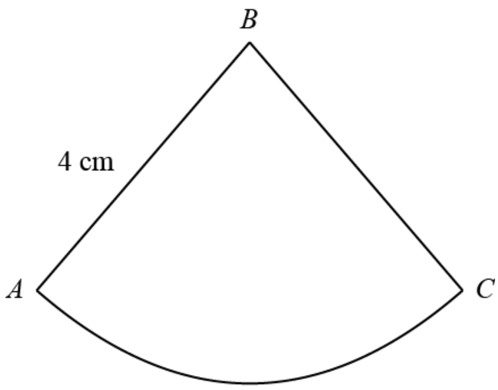
$$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}\frac{5}{6}\\ \frac{8}{3}\end{pmatrix}$$



$$\tan\theta = \frac{\frac{5}{6}}{\frac{8}{3}}$$

$\theta = 17.354\ldots$
$\quad = 17.4°$ (3 s.f.)

8



Let B be the origin.

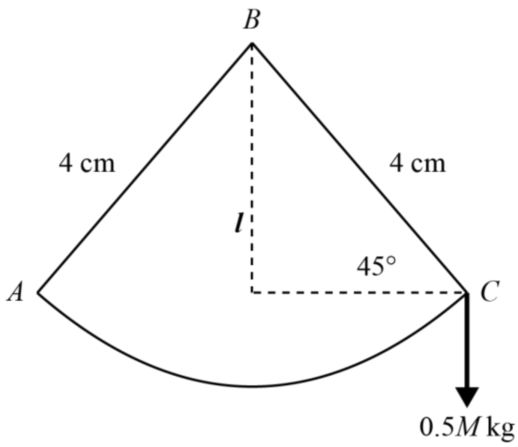
The centre of mass of the lamina lies at the point $\left(0, -\frac{2r\sin\alpha}{3\alpha}\right)$

$\alpha = \frac{\pi}{4}$ and $r = 4$ therefore:

$$\frac{2r\sin\alpha}{3\alpha} = \frac{2\times4\times\sin\left(\frac{\pi}{4}\right)}{3\times\frac{\pi}{4}}$$

$$= \frac{16\sqrt{2}}{3\pi}$$

Therefore the centre of mass of the lamina lies at the point $\left(0, -\frac{16\sqrt{2}}{3\pi}\right)$



$\sin 45 = \frac{l}{4}$

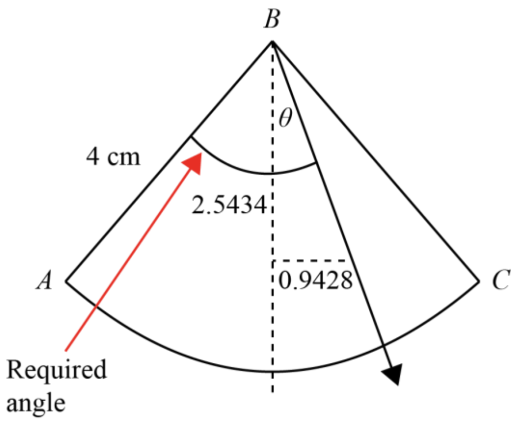
$l = 2\sqrt{2}$

Therefore C lies at the point $\left(2\sqrt{2}, -2\sqrt{2}\right)$

8 (continued)

$$1.5M\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = M\begin{pmatrix}0\\ -\dfrac{16\sqrt{2}}{3\pi}\end{pmatrix} + 0.5M\begin{pmatrix}2\sqrt{2}\\ -2\sqrt{2}\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}0.9428\\ -2.5434\end{pmatrix}$$

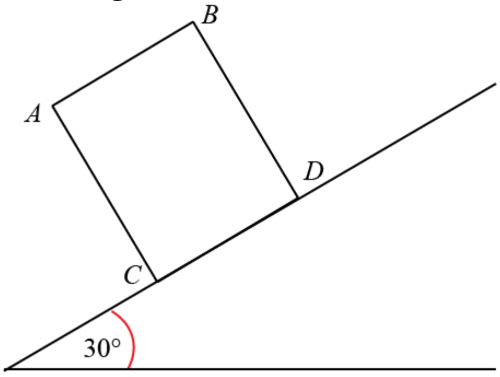


$$\theta = \tan^{-1}\left(\frac{0.9428}{2.5434}\right) = 20.3362...°$$

So the required angle that *AB* makes with the vertical is:

45° + 20.3° = 65.3° (3 s.f.)

Challenge



Let C be the origin and CD be the positive x-axis.

The centre of mass of the lamina lies at the point (2, 3)

A lies at the point (0, 6).

$$(M+kM)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=M\begin{pmatrix}2\\3\end{pmatrix}+kM\begin{pmatrix}0\\6\end{pmatrix}$$

$$(1+k)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\begin{pmatrix}2\\3\end{pmatrix}+k\begin{pmatrix}0\\6\end{pmatrix}$$

$$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\begin{pmatrix}\dfrac{2}{1+k}\\ \dfrac{3+6k}{1+k}\end{pmatrix}$$

$$\tan 30=\frac{\dfrac{2}{1+k}}{\dfrac{3+6k}{1+k}}$$

$$\sqrt{3}(3+6k)=6$$

$$3+6k=\frac{6}{\sqrt{3}}$$

$$k=\frac{1}{\sqrt{3}}-\frac{1}{2}\quad \text{as required}$$