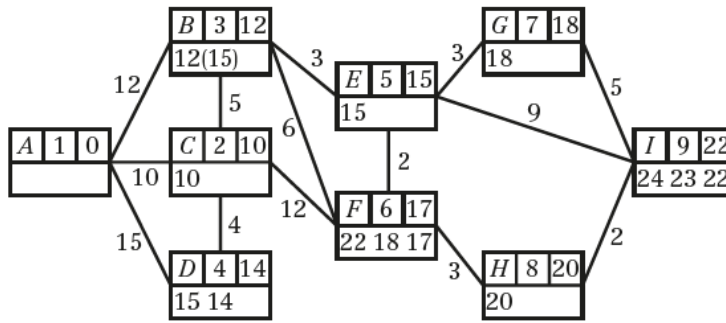


Review Exercise 2

- 1 a i Shortest path through A is $18 + y$ or 26 , both of which are greater than 17 .
Shortest path through C is 23 , which is greater than 17 . So shortest path cannot go through A or C .
- ii Shortest path must go through B
 $SBDT = 13 + x$
 $13 + x = 17$
 $x = 4$
- b If $y = 0$ shortest path is $SADT = 18$
 If $y = 5$ shortest path is $SCDT = 23$
 so range is 18 to 23 .
- c For example, a person seeking the quickest route from home to work through a city. The arcs are the roads that may be chosen, the number the time, in minutes, to journey along that road. The nodes represent junctions.
- 2 a Odd vertices are B_1, B_2, E, G
 $B_1B_2 + EG = 65 + 18 = 83$
 $B_1E + B_2G = 41 + 42 = 83$
 $B_1G + B_2E = 26 + 30 = 56$
 Repeat B_1D, DG, B_2A, AE
 Route: For example,
 $FAEAB_2ACEFGDHDGDB_1DF$
 (All correct routes have 17 letters in their 'word')
 Length = $129 + 56 = 185$ km
- b Now only the route between E and G needs repeating
 so repeat $EF + FG = 18$
 length of new route = $129 + 18$
 $= 147$ km
- 3 a All arcs are to be traversed twice, this is, in effect, repeating each arc. So all valencies are even.
- b e.g. $ABDGF GDCEAECAFEFBFABDCA$
 (all correct routes will have 23 letters in their name)
 length = $2 \times 6 = 12$ km

4 a



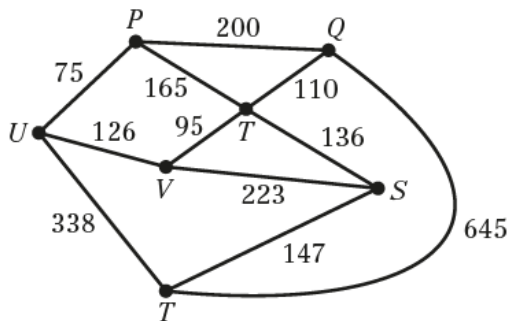
Shortest route is *ABEFHI* length 22 km

b i Odd vertices are *A* and *I* (only), so we need to repeat the shortest route from *A* to *I*. This was found in a. So repeat *AB, BE, EF, FH, HI*.

ii For example *ABCADCEHIHEFIGFEBFBA* (20 letters in route)

iii $91 + 22 = 113$ km

5



a Total length = 2260 m
 Odd nodes *P, Q, S, T, U, V*
T and *P* remain odd.
 $QS + UV = 246 + 126 = 372$ ← least weight
 $QU + SV = 275 + 223 = 498$
 $QV + SU = 205 + 349 = 554$
QS and *UV* gives the shortest pairing.
 Roads to be traversed twice: *QR, RS, UV*

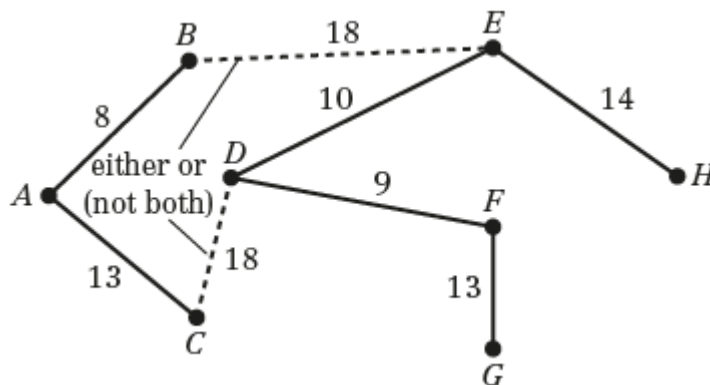
b Total of roads traversed twice $110 + 136 + 126 = 372$
 Shortest route is $2260 + 372 = 2632$ m

6 a Odd valencies are at *A, B, C, D, F, G*
 Route starts at *A* and finishes at *G* so these can remain odd.
 Choose pairings of remaining odd vertices *B, C, D, F*
 By inspection, these path lengths are:
 $BC + DF = 0.8 + 1.7 = 2.5$
 $BD + CF = 1.3 + 2.3 = 3.6$
 $BF + CD = 1.5 + 0.7 = 2.2$ ← least weight
 Repeating *BF* and *CD* minimises the total distance travelled.
 Length = $9.5 + 2.2 = 11.7$ km

b *ABCAGBDCDEFBFG* (14 letters in route)

- 6 c Repeating AC and $BF = 2.1$
 Minimum distance = 11.6 km
 The engineer is correct. His new route is 0.1 km shorter.
- 7 a In the *practical* T.S.P each vertex must be visited *at least once*
 In the *classical* T.S.P. each vertex must be visited *exactly once*

- b $AB, DF, DE, (reject\ EF), \left\{ \begin{matrix} FG \\ AC \end{matrix} \right\}, EH, \left\{ \begin{matrix} DC \\ or \\ BE \end{matrix} \right\}$



- c Initial upper bound = $2 \times 85 = 170$ km
- d When CD is part of tree
 Use GH (saving 26) and BD (saving 19) giving a new upper bound of 125 km
 Tour $ABDEHGFDCA$
 e.g. when BE is part of tree
 Use CG (saving 40) giving a new upper bound of 130 km
 Tour $ABEHEDFGCA$

8 a

	A	B	C	D	E	F
A	–	20	30	32	12	15
B	20	–	10	25	32	16
C	30	10	–	15	35	19
D	32	25	15	–	20	34
E	12	32	35	20	–	16
F	15	16	19	34	16	–

Each row shows the shortest route.

The first row shows the shortest route starting at A . There are direct routes from AB, AE and AF and these are the shortest routes. AC (30) is by observation using ABC and AD (32) is by observation using AED .

- b AE (12), EF (16), FB (16), BC (10), CD (15), DA (32)
 101 km tour $AEFBCDA$
- c In the original network AD is not a direct path. The tour becomes $AEFBCDEA$.

8 d e.g.

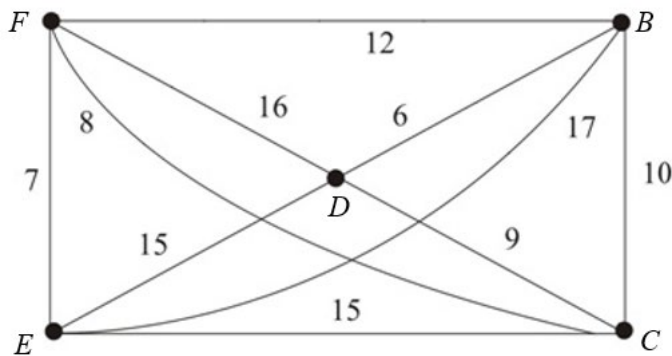
$$\left. \begin{array}{l} B C D E A F B \\ C B F A E D C \\ D C B F A E D \\ E A F B C D E \\ F A E D C B F \end{array} \right\} \text{ length 88}$$

9 a i Minimum connector using Prim: AC, CB, CD, CE
 length = $98 + 74 + 82 + 103 = 357$ $\{1, 3, 2, 4, 5\}$
 So upper bound = $2 \times 357 = 714$

ii $A(98) C(74) A(131) D(134) E(115)A$
 length = $98 + 74 + 131 + 134 + 115 = 552$

b Residual minimum connector is AC, CB, CD length 254
 Lower bound = $254 + 103 + 115 = 472$

c $472 \leq \text{solution} \leq 552$

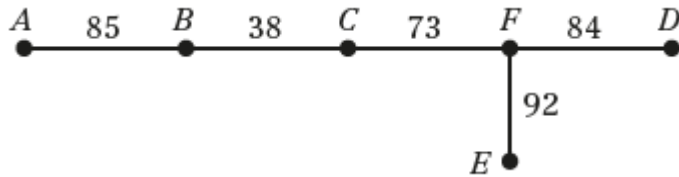
10 a Deleting vertex A we obtain

By Kruskal's algorithm an MST is $DB(6), EF(7), CF(8), DC(9)$ of weight 30
 The two edges of least weight at A are $AE(7)$ and $AD(8)$
 \therefore A lower bound is $30 + 8 + 7 = 45$

b i A – nearest neighbor $E(7)$
 E – nearest neighbor $F(7)$
 F – nearest neighbor $C(8)$
 C – nearest neighbor $D(9)$
 D – nearest neighbor $B(6)$
 Complete tour with $BA(12)$
 $AEFCDBA$ length 49

ii Choose a tour that does not use AB
 e.g. $DB(6) BC(10), CF(8), FE(4), EA(4)$
 Complete with $AD(8), DBCFEAD$.
 Total weight 46

11 a Order of arcs: AB, BC, CF, FD, FE



b i $2 \times 372 = 744$

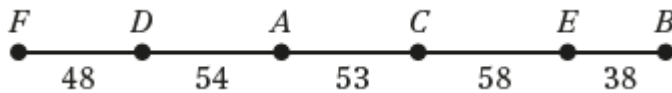
- ii e.g. AD saves 105 giving 639.
 or AE saves 180 giving 564.
 AF saves 96 giving 648.
 DE saves 66 giving 678.

c Residual M.S.T.

AB, BC, AE, ED

$$\begin{aligned} \text{Lower bound} &= 341 + 73 + 84 \\ &= 498 \end{aligned}$$

12 a $AC(53), AD(54), DF(48), CE(58), EB(38)$



b i M.S.T. $XZ = 251 \times 2 = 502$

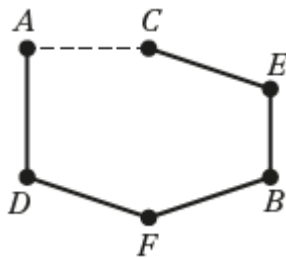
- ii Finding a shortcut to below 360, e.g. $FB = 100$ shortens by 151 so we get $251 + 100 = 351$.

c M.S.T. is DF, CE, EB length 244

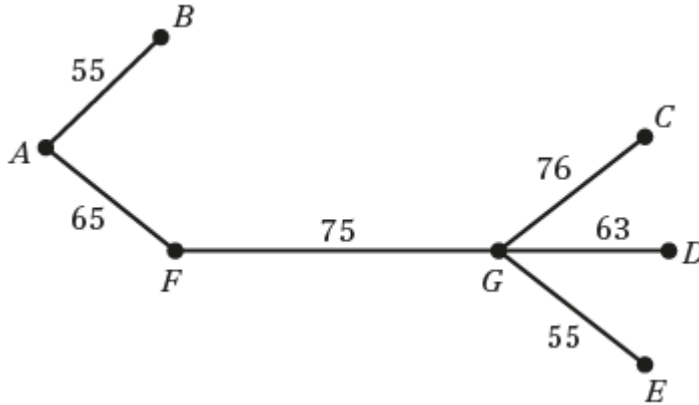
The 2 shortest arcs are AC (53) and AD (54) giving a total of 351

d The optimal solution is 351 and is $ACEBFDA$.

e



- 13 a** Label column A , delete row A .
 Scan all labelled columns and choose the least number.
 Add that new vertex to the tree.
 Label the new vertex's column and delete its row.
 Repeat the 3 steps until all vertices added.
 Applying algorithm order of vertex selection A, B, F, G, E, D, C .



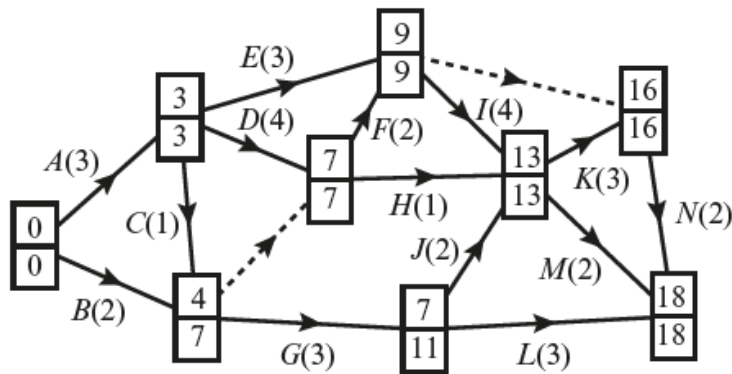
- b** Initial upper bound = $2 \times 389 = 778$ km
- c** Reducing upper bound by short cuts
 e.g. Using $BC = 82$ instead of $BA + AF + FG + GC$ leaves an upper bound of 589
 Lists new route e.g. $ABCGDGEGFA$
 States revisited vertices e.g. G

d



- e** Lower bound = $352 + GD + GE$
 $= 352 + 63 + 55$
 $= 470$ km
- f** e.g. Use GE and GF (rather than GD)
 length = $352 + 55 + 75 = 482$ km
 Route $ABCDEGFA$

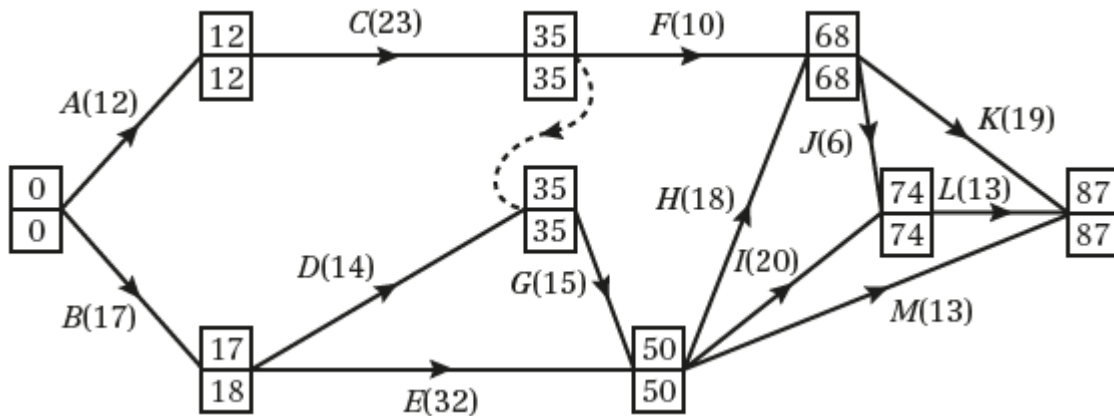
14 a



b 18 days

c ADFIKN

15 a



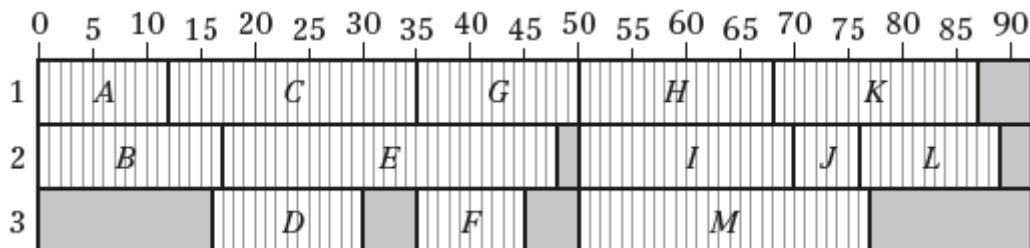
b A, C, G, H, J, K and L

All critical activities have a zero total float.

c Total float = 35 - 17 - 14 = 4

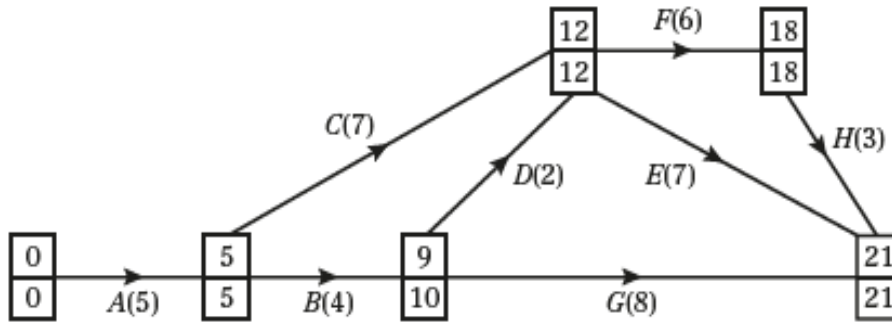
d Either $\frac{226}{87} = 2.6$ (1 d.p.) so at least 3 workers needed (here 226 is the total number of hours required for all the activities) or 69 hours into the project activities J, K, I and M must be happening so at least 4 workers will be needed.

e



New shortest time is 89 hours.

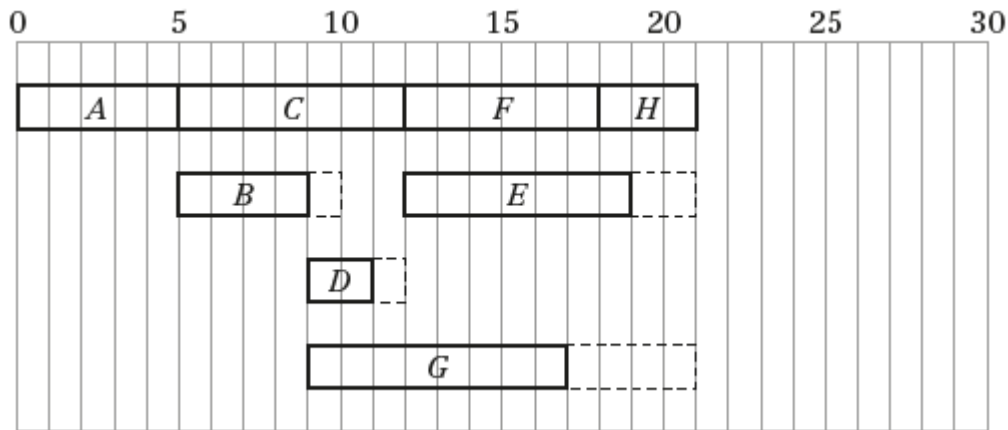
16 a



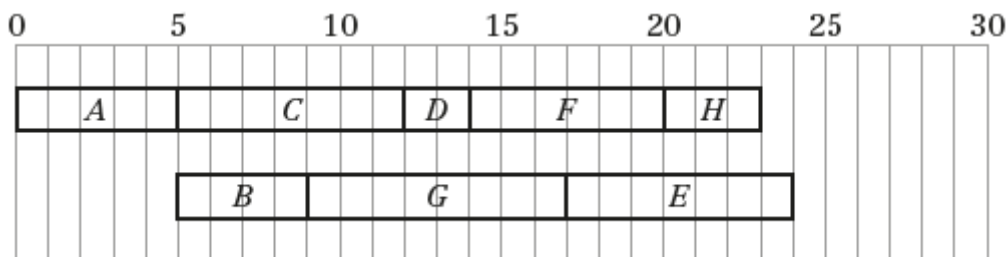
b Critical activities: *A, C, F* and *H*; length of critical path = 21

c Total float on *B* = $10 - 5 - 4 = 1$ Total float on *E* = $21 - 12 - 7 = 2$
 Total float on *D* = $12 - 9 - 2 = 1$ Total float on *G* = $21 - 9 - 8 = 4$

d



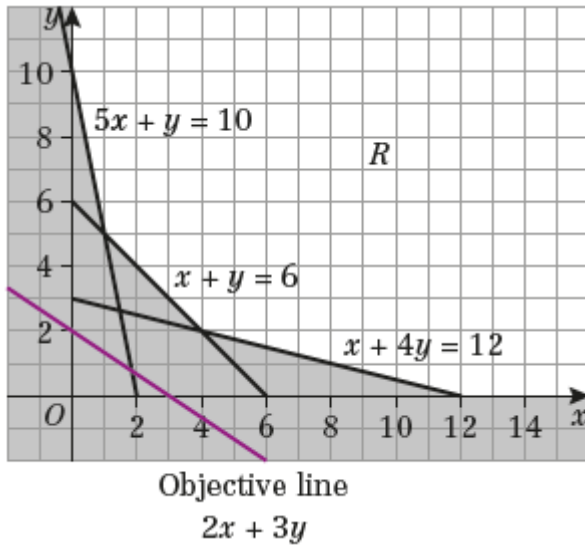
e For example;



Minimum time for 2 workers is 24 days.

- 17 a Chemical *A* $5x + y \geq 10$
 Chemical *B* $2x + 2y \geq 12$ $[x + y \geq 6]$
 Chemical *C* $\frac{1}{2}x + 2y \geq 6$ $[x + 4y \geq 12]$
 $x, y \geq 0$

17 b



c $T = 2x + 3y$

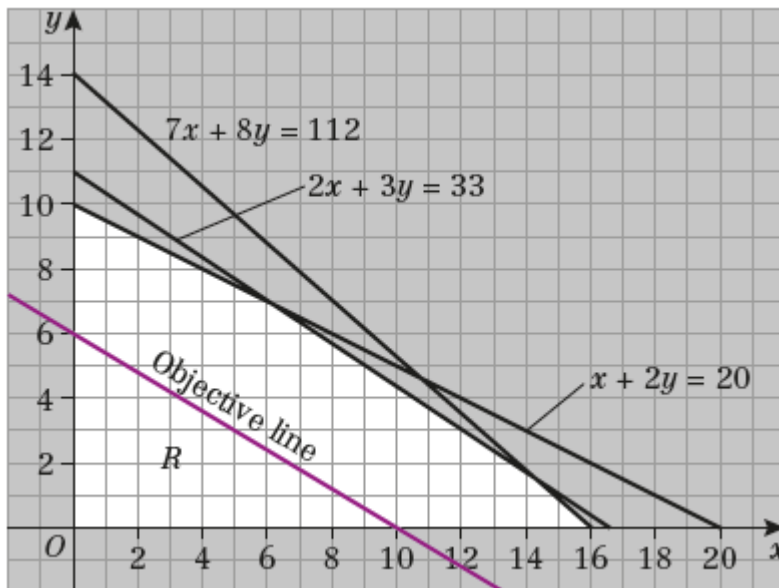
d $(x, y) = (4, 2) \quad T = 14$

18 a Maximise $P = 300x + 500y$

b Finishing $3.5x + 4y \leq 56 \Rightarrow 7x + 8y \leq 112$ (o.e.)

Packing $2x + 4y \leq 40 \Rightarrow x + 2y \leq 20$ (o.e.)

c

d For example, *point testing*

- Test all corner points in feasible region.
- Find profit at each and select point yielding maximum.

profit line

- Draw profit lines.
- Select point on profit line furthest from the origin.

18 e Using a correct, complete method.

Making 6 Oxford and 7 York gives a profit = £5300

$(6, 7) \rightarrow 5300$ $(14.4, 1.4) \xrightarrow{\text{integer}} (14, 1) \rightarrow 4700$ $(16, 0) \rightarrow 4800$

$(0, 10) \rightarrow 5000$

f The line $3.5x + 4y = 49$ passes through $(6, 7)$ so reduce *finishing* by 7 hours.

19 a Let badge 1 be x and badge 2 be y

$$P = 30x + 40y$$

$$x + y \geq 200$$

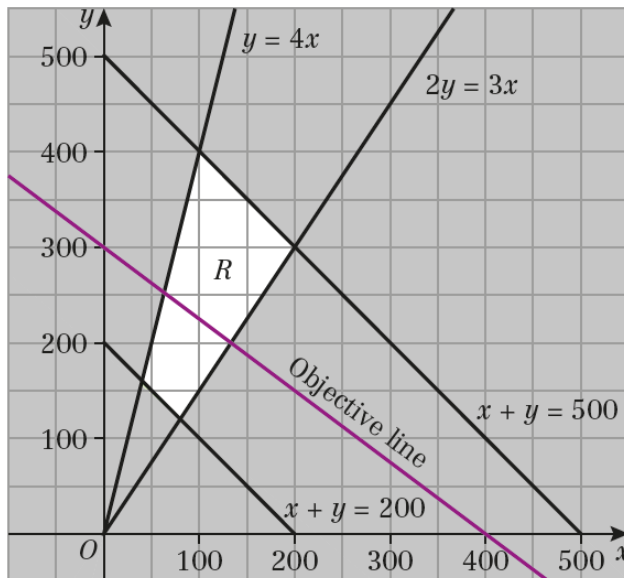
$$x + y \leq 500$$

$$x \geq 0.2(x + y) \Rightarrow 4x \geq y$$

$$x \leq 0.4(x + y) \Rightarrow 3x \leq 2y$$

$$x \geq 0, y \geq 0$$

b



c Testing the vertices:

$$(40, 160) = 7600$$

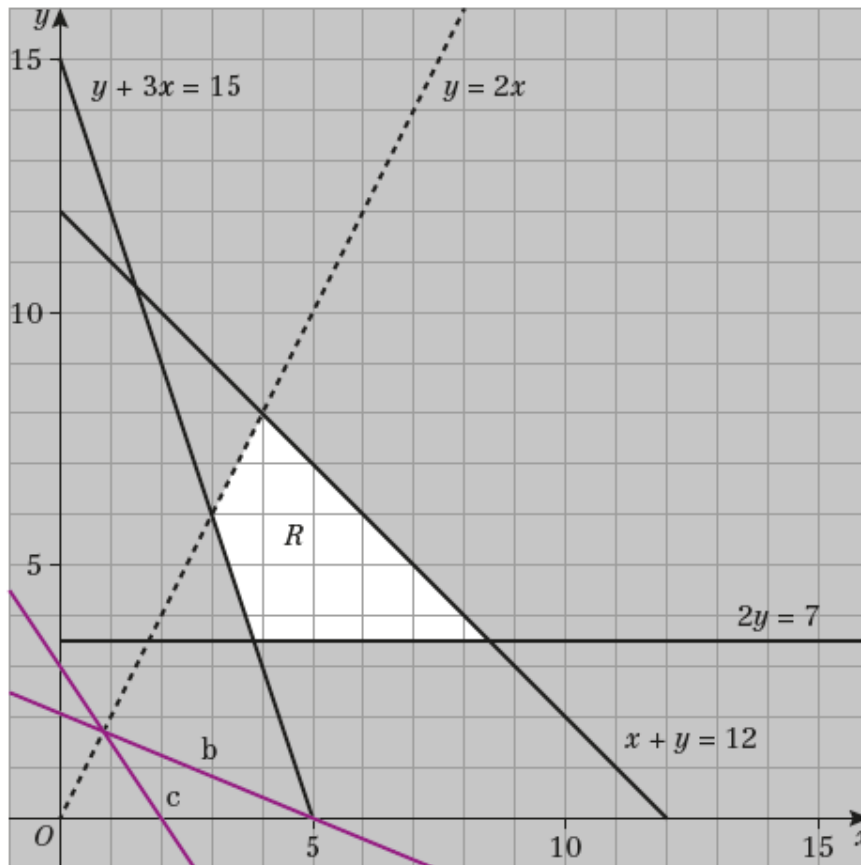
$$(80, 120) = 7200$$

$$(100, 400) = 19\,000$$

$$(200, 300) = 18\,000$$

Maximum profit of £190 at $(100, 400)$ so they should make 100 of badge 1 and 400 of badge 2.

20 a



b Visible use of objective line method – objective line drawn or vertex testing.

$$\left[\left(3\frac{5}{6}, 3\frac{1}{2} \right) \rightarrow 25\frac{1}{6} \left(8\frac{1}{2}, 3\frac{1}{2} \right) \rightarrow 34\frac{1}{2} (4, 8) \rightarrow 48 (3, 6) \rightarrow 36 \right]$$

Optimal point $\left(3\frac{5}{6}, 3\frac{1}{2} \right)$ with value $25\frac{1}{6}$

c Visible use of objective line method – objective line drawn, or vertex testing – all 4 vertices tested.

$$\left(3\frac{5}{6}, 3\frac{1}{2} \right) \text{ not an integer try } (4, 4) \rightarrow 20 \quad (4, 8) \rightarrow 28$$

$$\left(8\frac{1}{2}, 3\frac{1}{2} \right) \text{ not an integer try } (8, 4) \rightarrow 32 \quad (3, 6) \rightarrow 21$$

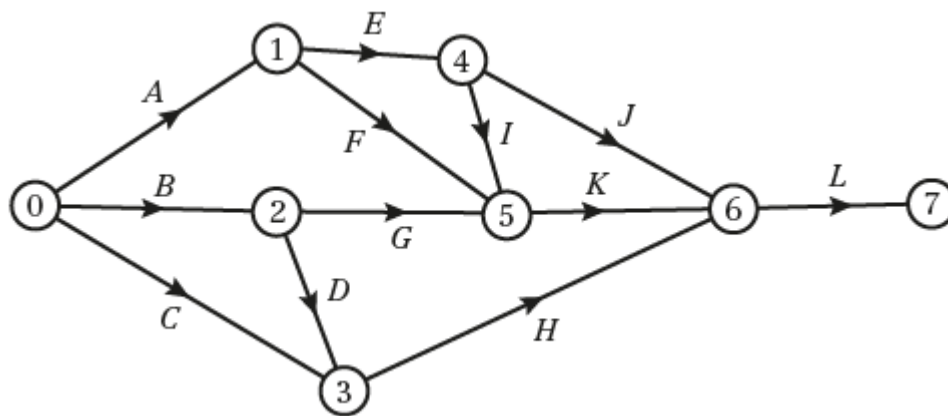
Optimal point $(8, 4)$ with value £32, so Becky should use 4 kg of bird feeder and 3.5 kg of bird table food.

- 21 a** Objective: maximise $P = 0.4x + 0.2y$ ($P = 40x + 20y$)
 subject to:
 $x \leq 6.5$
 $y \leq 8$
 $x + y \leq 12$
 $y \leq 4x$
 $x, y \geq 0$

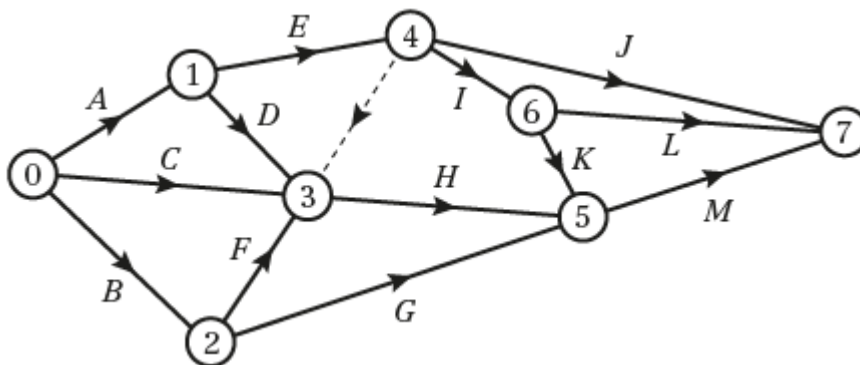
- b** Visible use of objective line method – objective line drawn (e.g. from (2, 0) to (0, 4)) or all 5 points tested.
 vertex testing
 $[(0, 0) \rightarrow 0; (2, 8) \rightarrow 2.4; (4, 8) \rightarrow 3.2; (6.5, 5.5) \rightarrow 3.7; (6.5, 0) \rightarrow 2.6]$
 Optimal point is $(6.5, 5.5) \Rightarrow 6500$ type X and 5500 type Y

c $P = 0.4(6500) + 0.2(5500) = \text{£}3700$

22

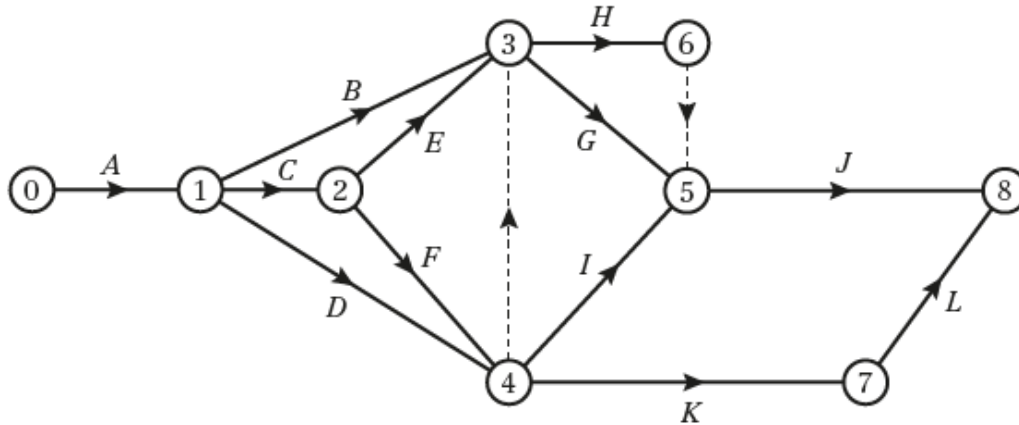


23 a



- b** Here we have that I and J depend only on E , whereas H depends on C, D, E and F . Hence we need separate nodes with a dummy.

24 a



b *D* will only be critical if it lies on the longest path

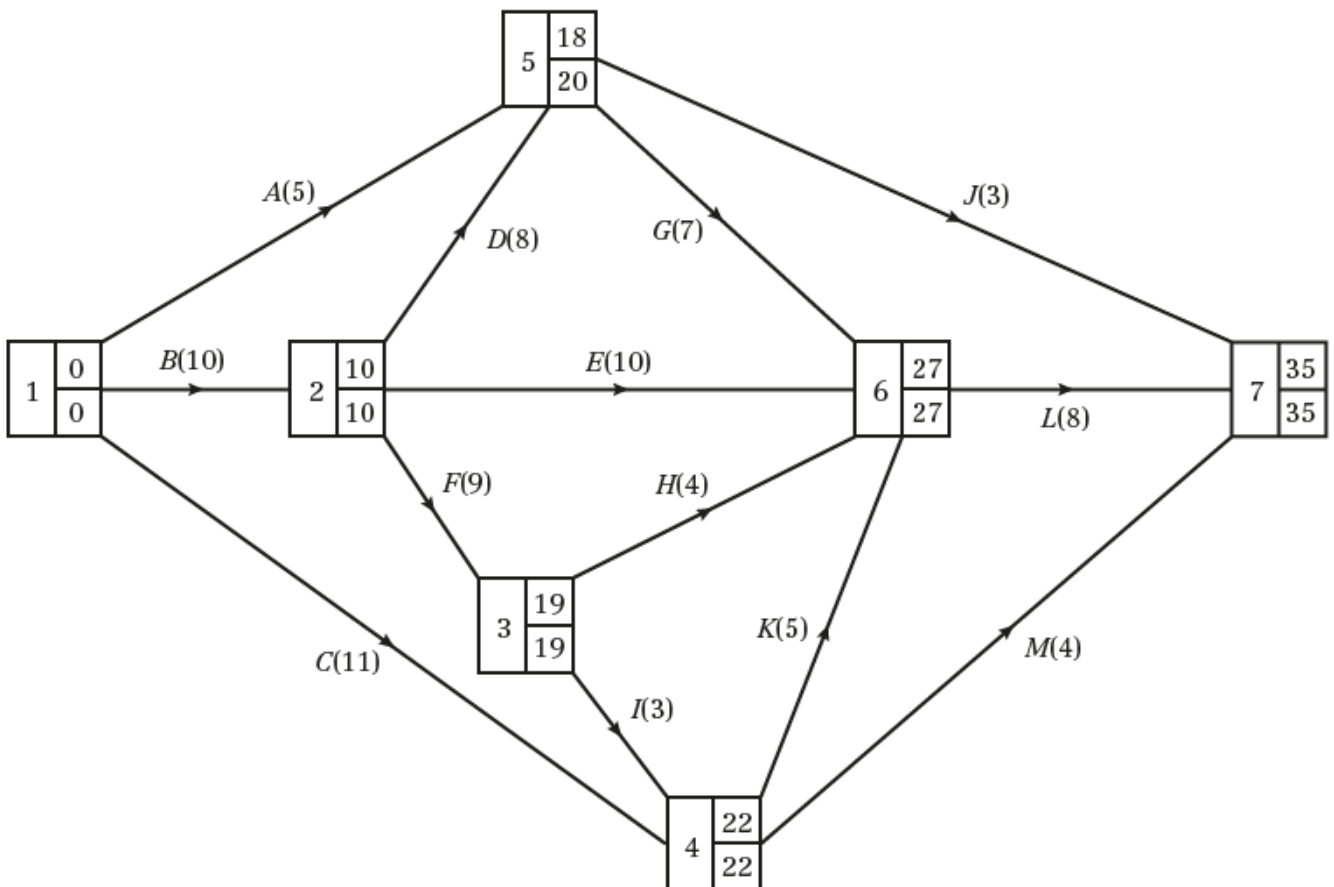
Path <i>A</i> to <i>G</i>	Length
<i>ABEG</i>	14
<i>ACFG</i>	15
<i>ACDEG</i>	$13 + x$

So we need $13 + x$ to be the longest, or equal longest

$$13 + x \geq 15$$

$$x \geq 2$$

25 a



- 25 b** Total float on $A = 20 - 0 - 5 = 15$
 Total float on $B = 10 - 0 - 10 = 0$
 Total float on $C = 22 - 0 - 11 = 11$
 Total float on $D = 20 - 10 - 8 = 2$
 Total float on $E = 27 - 10 - 10 = 7$
 Total float on $F = 19 - 10 - 9 = 0$
 Total float on $G = 27 - 18 - 7 = 2$

- Total float on $H = 27 - 19 - 4 = 4$
 Total float on $I = 22 - 19 - 3 = 0$
 Total float on $J = 35 - 18 - 3 = 14$
 Total float on $K = 27 - 22 - 5 = 0$
 Total float on $L = 35 - 27 - 8 = 0$
 Total float on $M = 35 - 22 - 4 = 9$

- c** Critical activities: B, F, I, K and L
 length of critical path is 35 days
- d** New critical path is $B - F - H - L$
 length of new critical path is 36 days

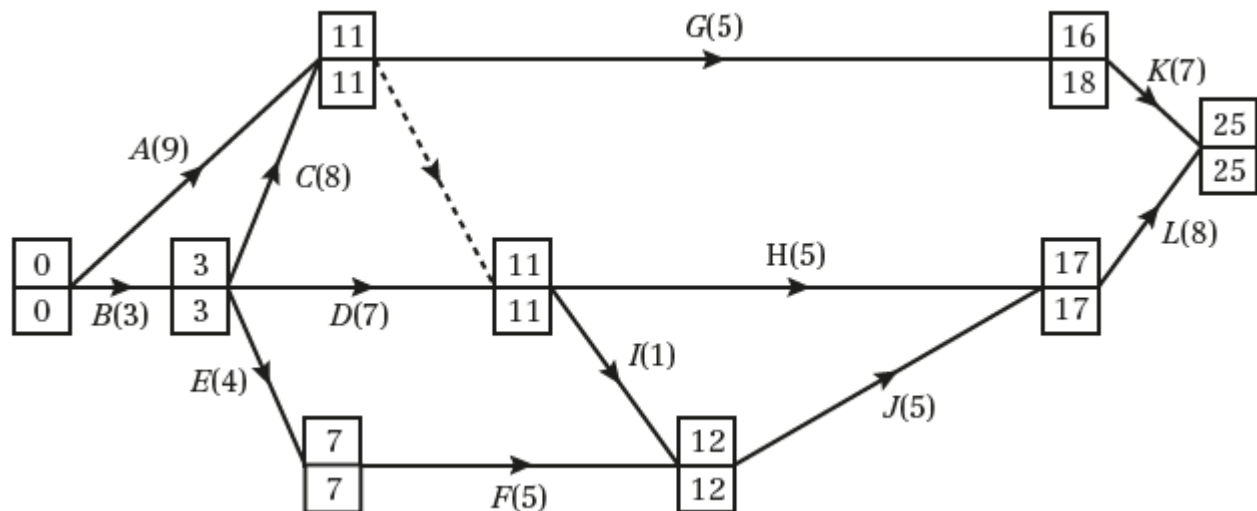
- 26 a** $x = 0$
 $y = 7$ [latest out of $(3 + 2)$ and $(5 + 2)$]
 $z = 9$ [Earliest out of $(13 - 4)$ and $(19 - 7)$ and $(16 - 2)$]

- b** Length is 22
 Critical activities: B, D, E and L

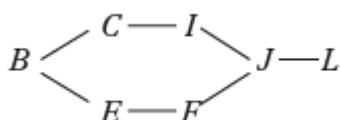
- c i** Total float on $N = 22 - 14 - 3 = 5$
ii Total float on $H = 16 - 5 - 3 = 8$

- 27 a** For example, it shows dependence but it is not an activity. G depends on A and C only but H and I depend on A, C and D .

b



c

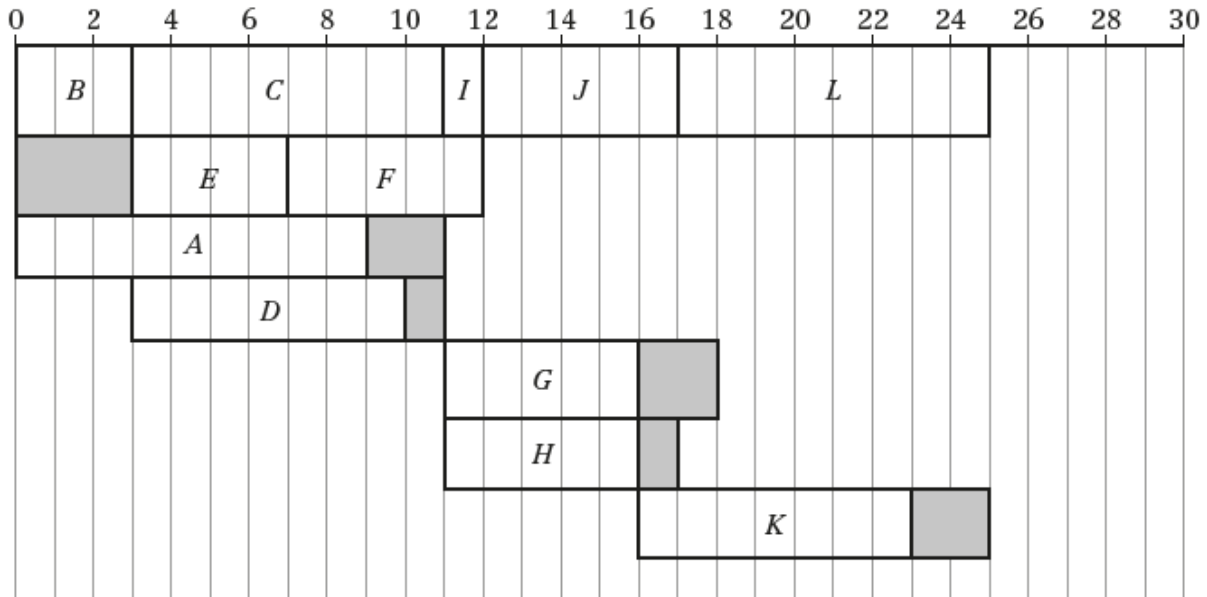


So B, C, E, F, I, J and L

- d** Total float on $A = 11 - 0 - 9 = 2$
 Total float on $D = 11 - 3 - 7 = 1$
 Total float on $G = 18 - 11 - 5 = 2$

- Total float on $H = 17 - 11 - 5 = 1$
 Total float on $K = 25 - 16 - 7 = 2$

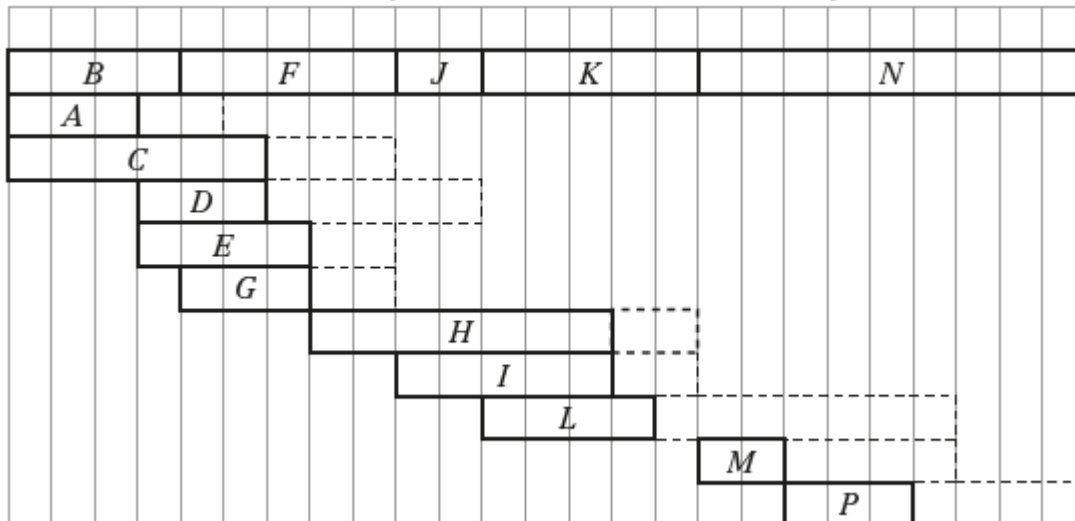
27 e



28 a Critical activities are B, F, J, K and N length of critical path is 25 hours
I is not critical.

- | | | |
|---|-----------------------------------|------------------------------------|
| b | Total float on A = 5 - 0 - 3 = 2 | Total float on H = 16 - 7 - 7 = 2 |
| | Total float on C = 9 - 0 - 6 = 3 | Total float on I = 16 - 9 - 5 = 2 |
| | Total float on D = 11 - 3 - 3 = 5 | Total float on L = 22 - 11 - 4 = 7 |
| | Total float on E = 9 - 3 - 4 = 2 | Total float on M = 22 - 16 - 2 = 4 |
| | Total float on G = 9 - 4 - 3 = 2 | Total float on P = 25 - 18 - 3 = 4 |

c
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25



d Look at 6.5 in the chart in c: F, E and G

Challenge

1 a $9\frac{1}{2}x - 26$

- b The only vertices of odd order are B and C , we have to repeat the shortest path between B and C .

If $x \geq 9$ the shortest path is BC (direct)

Weight of network + $BC = 100$

$$(9\frac{1}{2}x - 26) + x = 100 \Rightarrow x = 12$$

If $x < 9$ the shortest path is BAC of length $2x - 9$

$$(9\frac{1}{2}x - 26) + 2x - 9 = 100 \Rightarrow x = 11\frac{17}{23} \geq 9$$

so inconsistent and hence $BA + AC \geq BC$

- c The only vertices of odd order are B and C , we have to repeat the shortest path between B and C .

If $x \geq 9$ the shortest path is BC (direct).

Weight of network + $BC = 100$

$$(9\frac{1}{2}x - 26) + x = 100 \Rightarrow x = 12$$

If $x < 9$ the shortest path is BAC of length $2x - 9$

$$(9\frac{1}{2}x - 26) + 2x - 9 = 100 \Rightarrow x = 11\frac{17}{23} \geq 9 \text{ so inconsistent and hence } x = 12$$

2 a Minimum spanning tree = 751

Initial upper bound = $2 \times 751 = 1502$

Taking shortcut AH saves $120 + 131 - 144 = 107$

Tour length = 1395

Tour route: $ABACAEHFGDGFHA$

- b Delete G and use Prim's algorithm starting at A

RMST = 672

Lower bound by deleting $G = 672 + 144 + 155 = 971$

Route is not a tour, so does not give an optimal solution.