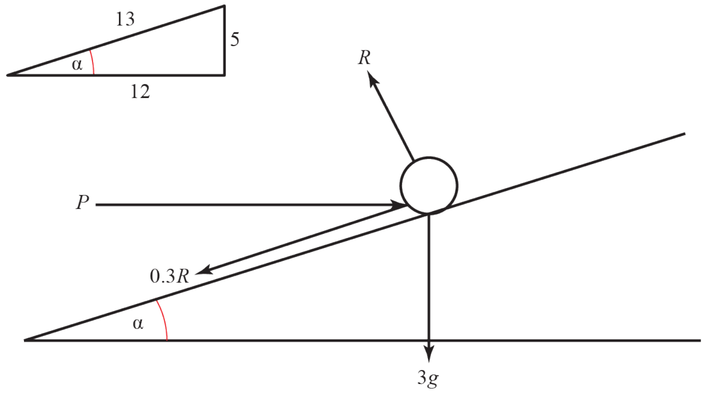
Review exercise 2

1 Since the particle is moving at constant velocity, the forces acting on it are balanced.

$\tan\alpha = \frac{5}{12} \Rightarrow \sin\alpha = \frac{5}{13}$ and $\cos\alpha = \frac{12}{13}$



$R(\nwarrow)$:

$R = 3g\cos\alpha + P\sin\alpha$

$R = \frac{3g\times 12}{13} + \frac{5P}{13}$

$R = \frac{36g + 5P}{13}$

$R(\nearrow)$:

$P\cos\alpha = \mu R + 3g\sin\alpha$

$\frac{12}{13}P = \frac{1}{5}\left(\frac{36g+5P}{13}\right) + \frac{3g\times 5}{13}$

$12P = \frac{36g}{5} + P + 15g$

$11P = \left(\frac{36}{5} + 15\right)g$

$P = \left(\frac{36}{5} + 15\right)\times\frac{9.8}{11}$

$= 19.778...$

P is 19.8 N (to 3 s.f.).

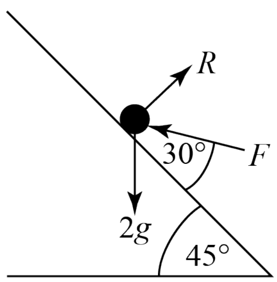
2 Using Newton's 2[nd] Law, $F = ma$ in the direction of the acceleration

$F\cos 30° - 2g\cos 45° = 2\times 2$

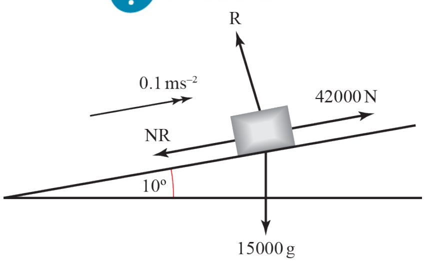
$\frac{\sqrt{3}}{2}F - \sqrt{2}g = 4$

$F = \frac{2}{\sqrt{3}}\left(4 + \sqrt{2}g\right)$

$= 20.6$ N (3 s.f.)



3 $m = 15\,000$ kg, $a = 0.1$ ms^{-2}



a $R(\nwarrow)$:

$R = 15\,000g\cos 10^\circ$

$R = 15\,000 \times 9.8\cos 10^\circ = 144\,767$

To the nearest whole newton, the reaction between the container and the slope is 144 767 N.

b Using Newton's second law of motion and resolving up the slope:

$$F = ma$$

$$42\,000 - \mu R - 15\,000g\sin 10^\circ = 15\,000 \times 0.1$$

$$\mu \times 144\,767 = 42\,000 - 1500 - (15\,000 \times 9.8\sin 10^\circ)$$

$$\mu = \frac{40\,500 - 25\,526.2}{144\,767}$$

$$= 0.103\,433...$$

The coefficient of friction between the container and the slope is 0.103 (3 s.f.).

c Using Newton's second law of motion and resolving down the slope after winch stops working:

$$F = ma$$

$$\mu R + 15\,000g\sin 10^\circ = 15\,000a$$

$$144\,767 \times 0.103\,433 + 15\,000g\sin 10^\circ = 15\,000a$$ (using results from **a** and **b**)

$$a = \frac{40\,500}{15\,000}$$

$$= 2.7$$

So the container accelerates down the slope at 2.7 ms^{-2}

So: $u = -2$ ms^{-1}, $v = 0$ ms^{-1}, $a = 2.7$ ms^{-2}, $t = ?$

$v = u + at$

$0 = -2 + 2.7t$

$t = \frac{2}{2.7}$

$= 0.74074...$

The container takes 0.740 s (3 s.f.) to come to rest.

d Once the container comes to rest, the container will tend to move down the slope and hence the frictional force will act up the slope. The container will therefore move back down if the component of weight down the slope is greater than the frictional force; i.e. if

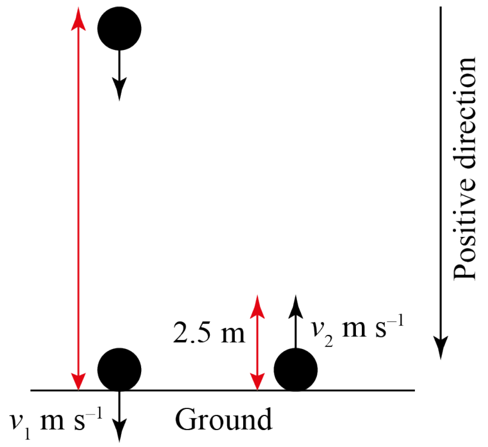
$mg\sin 10^\circ > \mu R$

$15\,000g\sin 10^\circ > 144\,767 \times 0.103\,433$

$25\,526 > 14\,974$

Since this inequality is true, the container will start to slide back down the slope.

4



As ball descends

$u = 0, a = 9.8, s = 10, v = v_1$

$v^2 = u^2 + 2as$

$v_1^2 = 0^2 + 2 \times 10 \times 9.8 = 196$

$v_1 = \sqrt{196} = 14$

The ball is released from rest 10 m above the ground. The first step is to calculate the speed with which the ball strikes the ground.

4 After rebound

$v = 0, a = 9.8, s = -2.5, u = v_2$

$v^2 = u^2 + 2as$

$0^2 = v_2^2 + 2 \times 9.8 \times (-2.5) \Rightarrow v_2^2 = 49$

$v_2 = -\sqrt{49} = -7$

$I = mv_2 - mv_1$

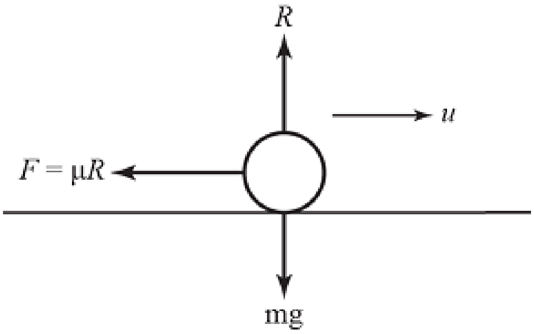
$= 0.3 \times (-7) - 0.3 \times 14 = -6.3$

You must then use the fact that the ball reaches a maximum height of 2.5 m to find the velocity with which it rebounds from the ground.

As it rebounds from the ground, the ball is moving upwards. That is in the negative direction. You must take the negative square root of 49, which is –7.

The magnitude of the impulse is 6.3 N.

5 **a** $m = 0.250$ kg, $mu = 2$ Ns, $\mu = 0.2$, $v = 0$ ms^{-1}, $s = ?$



Resolving vertically:
$R = mg$

Friction is limiting, so $F = \mu R = \mu mg$

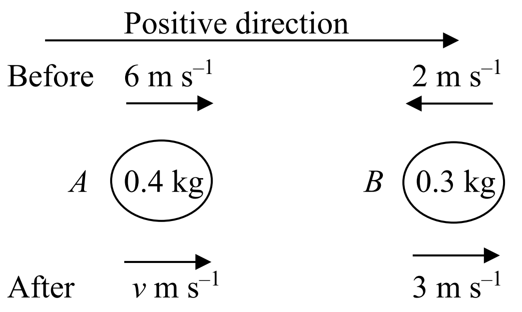
Impulse on car = change of momentum of car:

$$Ft = mv - mu$$
$$\mu mgt = 0 - (-2)$$
$$t = \frac{2}{\mu mg}$$
$$s = \tfrac{1}{2}(u + v)t$$
$$s = \frac{1}{2}\left(\frac{2}{m} + 0\right)\frac{2}{\mu mg}$$
$$s = \frac{2}{\mu m^2 g} = \frac{2}{0.2 \times 0.25^2 \times 9.8} = 16.3265...$$

The racing car travels a distance of 16 m (2 s.f.) past point A before coming to a stop.

b The car stops in a shorter distance because there will be additional frictional forces acting on it (e.g. air resistance) which will increase the deceleration.

6



The total linear momentum before impact must equal the total linear momentum after impact.

a Conservation of linear momentum

$$0.4 \times 6 + 0.3 \times (-2) = 0.4 \times v + 0.3 \times 3$$
$$2.4 - 0.6 = 0.4v + 0.9$$
$$0.4v = 2.4 - 0.6 - 0.9 = 0.9$$
$$v = \frac{0.9}{0.4} = 2.25$$

The velocity of B before impact is in the negative direction so it must be entered as –2 in any equations involving linear momentum.

The velocity of A is positive (2.25 m s^{-1}) after impact and it was positive (6 m s^{-1}) before impact. So the direction of motion of A is unchanged.

The speed of A after the collision is 2.25 m s^{-1}

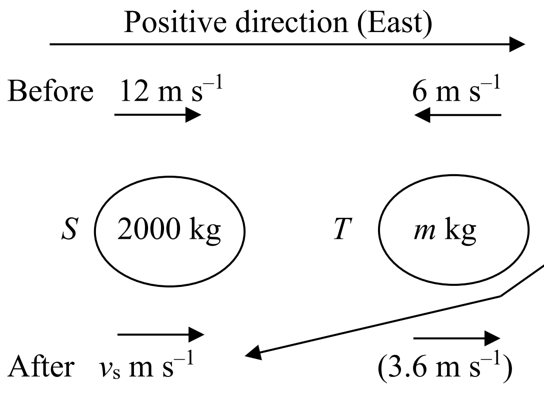
The direction of motion of A is unchanged.

b For $B, \mathbf{I} = m\mathbf{v} - m\mathbf{u}$

$$I = 0.3 \times 3 - 0.3 \times (-2)$$
$$= 0.9 + 0.6 = 1.5$$

The magnitude of the impulse exerted on B is 1.5 N s

7



You do not know which direction S will be moving in after the impact. Mark the unknown velocity as v m s^{-1} in the positive direction. After you have worked out v, the sign of v will tell you the direction in which S is moving.

a For $S, \mathbf{I} = m\mathbf{v} - m\mathbf{u}$

$$-28\,800 = 2000 \times v_s - 2000 \times 12$$
$$2000 v_s = -28\,800 + 24\,000 = -4800$$
$$v_s = -\frac{4800}{2000} = -2.4$$

The sign of v is negative, so S is moving in the negative direction. In this solution, the positive direction has been taken as east, so S is now moving west.

The speed of S immediately after the collision is 2.4 m s^{-1}

b Immediately after the collision S is moving due west.

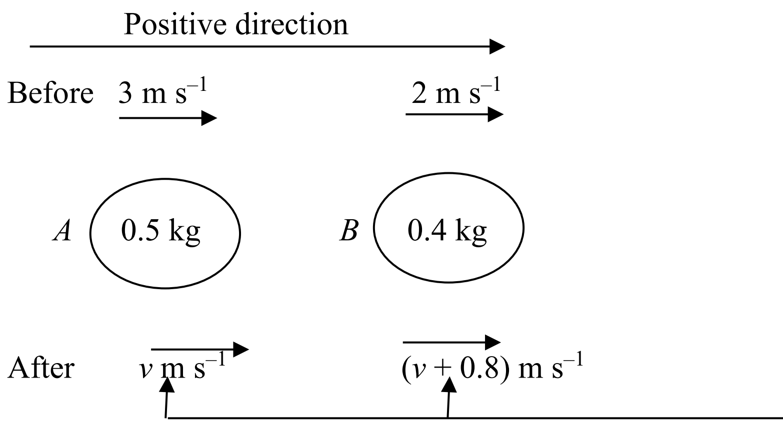
7 c Conservation of linear momentum

$$2000 \times 12 + m \times (-6) = 2000 \times (-2.4) + m \times 3.6$$

$$9.6m = 24\,000 + 4800 = 28\,800 \Rightarrow m = \frac{28\,800}{9.6} = 3000$$

The mass of T is 3000 kg

8



You need to translate the statement that 'the speed of B is 0.8 m s^{-1} greater than the speed of A' into algebra. If the speed of A after the collision is v m s^{-1} then the speed of B is 0.8 m s^{-1} greater; that is $(v + 0.8)$m s^{-1}

a Conservation of linear momentum

$$0.5 \times 3 + 0.4 \times 2 = 0.5 \times v + 0.4(v + 0.8)$$

$$1.5 + 0.8 = 0.5v + 0.4v + 0.32$$

$$0.9v = 1.5 + 0.8 - 0.32 = 1.98$$

$$v = \frac{1.98}{0.9} = 2.2$$

All velocities in this part are in the positive direction.

The speed of A after the collision is 2.2 m s^{-1}

The speed of B after the collision is (2.2 + 0.8) m s^{-1} = 3 m s^{-1}

To find the speed of B add 0.8 m s^{-1} to the speed of A.

b The momentum of A before the collision is given by

$$mu = 0.5 \times 3\,\text{N s} = 1.5\,\text{N s}$$

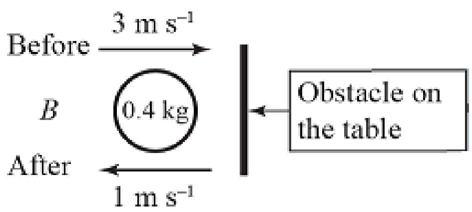
The momentum of A after the collision is given by

$$mv = 0.5 \times 2.2\,\text{N s} = 1.1\,\text{N s}$$

The momentum of a particle is its mass times its velocity. Momentum is a vector quantity.

A loses a momentum of $(1.5 - 1.1)\,\text{N s} = 0.4\,\text{N s}$, as required.

8 c



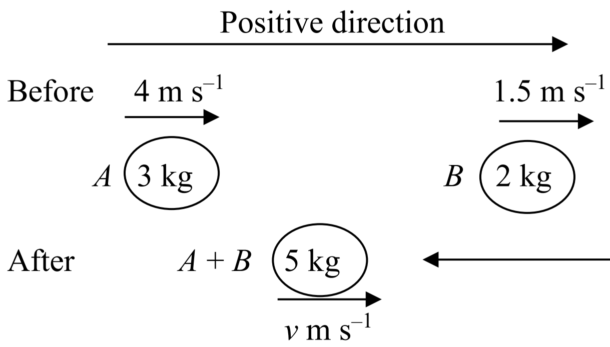
For B, before and after the second impact

$$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$$
$$= 0.4 \times (-1) - 0.4 \times 3$$
$$= -1.6$$

Left to right has been taken as the positive direction throughout the question. The impulse on B is negative as, as the situation is drawn here, the impulse on B is in the direction from right to left.

The magnitude of the impulse received by B in this second impact is 1.6 Ns

9



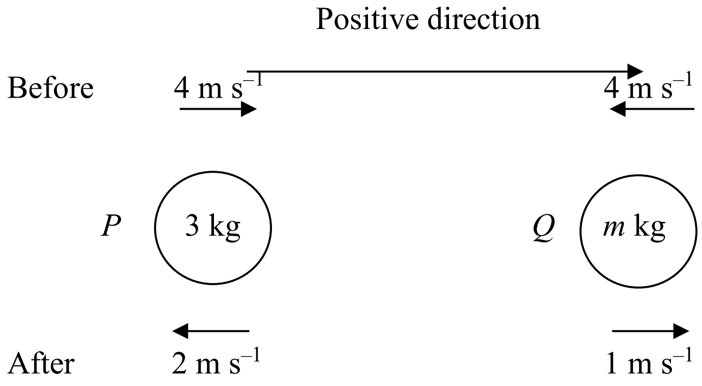
After the collision A (of mass 3 kg) and B (of mass 2 kg) combine to form a single particle. That particle will have the mass which is the sum of the two individual masses, 5 kg.

Conservation of linear momentum

$$4 \times 3 + 2 \times 1.5 = 5 \times v$$
$$12 + 3 = 5v \Rightarrow v = \frac{15}{5} = 3$$

The speed of C immediately after the collision is 3 m s^{-1}

10



a Conservation of linear momentum

$$3\times4+m\times(-4)=3\times(-2)+m\times1$$
$$12-4m=-6+m\Rightarrow5m=18$$
$$m=\frac{18}{5}=3.6$$

In the equation for the conservation of momentum, you must give the velocities in the negative direction a negative sign.

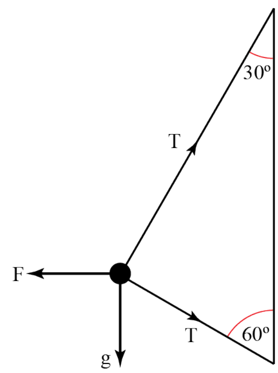
b For $Q, \mathbf{I}=m\mathbf{v}-m\mathbf{u}$

$$I=3.6\times1-3.6\times(-4)$$
$$=3.6+14.4=18$$

The magnitude of the impulse exerted on Q in the collision is 18 N s

As the magnitude of the impulse exerted on P is the same as the magnitude of the impulse exerted on Q, you could equally correctly work out the change in linear momentum of P. The working then would be $I=3\times(-2)-3\times4=-18,$ which gives the same magnitude, 18 Ns

11 The system is in equilibrium.



a Resolving vertically:

$$T\cos60^\circ+g=T\cos30^\circ$$
$$\frac{T}{2}+g=\frac{T\sqrt{3}}{2}$$
$$2g=T\left(\sqrt{3}-1\right)$$
$$T=\frac{2g}{\sqrt{3}-1}\quad\text{as required.}$$

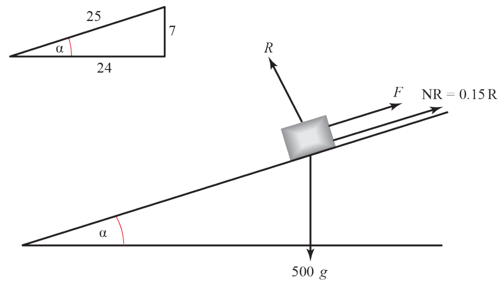
b Resolving horizontally:

$$F=T\sin60^\circ+T\sin30^\circ$$
$$F=T\left(\sin60^\circ+\sin30^\circ\right)$$
$$F=\left(\frac{2g}{\sqrt{3}-1}\right)\left(\frac{\sqrt{3}+1}{2}\right)$$
$$F=\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)g$$

c We model the bead as smooth in order to assume there is no friction between it and the string.

12 $\tan\alpha = \frac{7}{24} \Rightarrow \sin\alpha = \frac{7}{25}$ and $\cos\alpha = \frac{24}{25}$

The system is in equilibrium.



a $R(\nwarrow)$:

$R = 500g\cos\alpha$

$R = \frac{24}{25} \times 500g = 480g$

The normal reaction of the hill on the crate is $480g$ N, as required.

b Minimum value of F occurs when the crate is on the point of sliding down the hill. Frictional force then acts up the hill.

$R(\nearrow)$:

$F + \mu R = 500g\sin\alpha$

$F = \left(\frac{7}{25} \times 500g\right) - \left(\frac{3}{20} \times 480g\right)$ (Using $\mu = \frac{3}{20}$, and $R = 480g$ from **a**)

$F = (140 - 72)g$

$= 68g$

The minimum value of F required to maintain equilibrium is $68g$ N.

13 For the 3 kg mass

R(↑) $T = 3g$ **(1)**

For the 12 kg mass

R(→) $T = \mu R$ **(2)**

R(↑) $R = 12g$ **(3)**

Equating **(1)** and **(2)** gives

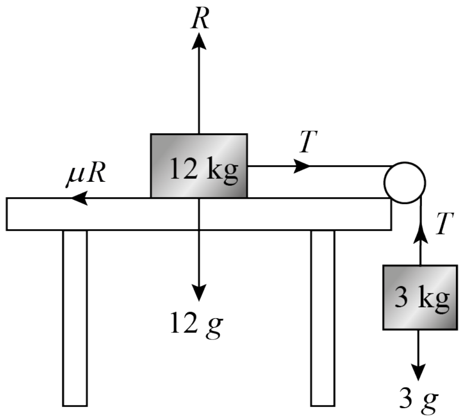
$\mu R = 3g$

$\mu = \frac{3g}{R}$

Then substituting $R = 12g$ gives

$\mu = \frac{3g}{12g}$

$= 0.25$



14 Assuming that the box of mass m_1 is on the point of sliding down the slope and the box of mass m_2 is on the point of sliding up the slope.

For box of mass m_1

R(↗) $T + \mu R_1 = m_1 g\sin\theta_1$

R(↖) $R_1 = m_1 g\cos\theta_1$

So $T = m_1 g\sin\theta_1 - \mu m_1 g\cos\theta_1$ **(1)**

For box of mass m_2

R(↖) $T = m_2 g\sin\theta_2 + \mu R_2$

R(↗) $R_2 = m_2 g\cos\theta_2$

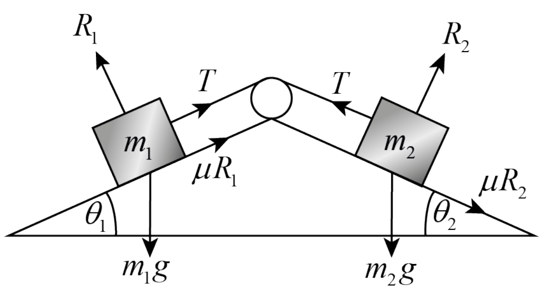
So $T = m_2 g\sin\theta_2 - \mu m_2 g\cos\theta_2$ **(2)**

Equating **(1)** and **(2)** gives

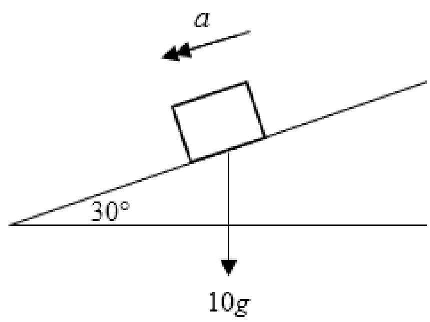
$m_1 g\sin\theta_1 - \mu m_1 g\cos\theta_1 = m_2 g\sin\theta_2 + \mu m_2 g\cos\theta_2$

$m_1 g\sin\theta_1 - m_2 g\sin\theta_2 = \mu m_1 g\cos\theta_1 + \mu m_2 g\cos\theta_2$

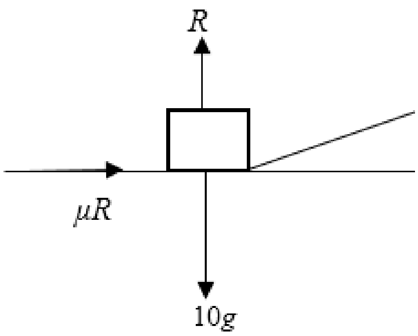
$\mu = \frac{m_1\sin\theta_1 - m_2\sin\theta_2}{m_1\cos\theta_1 + m_2\cos\theta_2}$



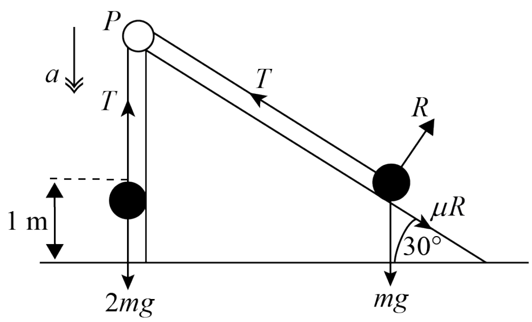
15 a (↙) Newton's 2nd law, $F = ma$
$10g \sin 30 = 10a$
$a = 0.5g$
$u = 0$, $a = 0.5g$ and $s = 10$
Using $v^2 = u^2 + 2as$ gives
$v^2 = 2(0.5g)(10)$
$= 10g$
$v = 7\sqrt{2}$ m s^{-1}
$= 9.90$ m s^{-1} (3 s.f.)



b R(↑) $R = 10g$
(←) Newton's 2nd law, $F = ma$
$-\mu R = 10a$
$10a = -0.2(10g)$
$a = -0.2g = -1.96$ m s^{-2}
Using $v^2 = u^2 + 2as$ gives
When the box comes to rest $v = 0$, so
$0 = 10g - 2(0.2g)s$
$s = \frac{10g}{0.4g}$
$= 25$
So the total distance travelled by the box is 10 m + 25 m = 35 m



16 a For the $2m$ mass
(↓) Newton's 2nd law, $F = ma$
$2mg - T = 2ma$
$T = 2mg - 2ma$ **(1)**
For the m mass
R(↗) $R = mg \cos 30$
$= \frac{\sqrt{3}}{2}mg$
(↖) Newton's 2nd law, $F = ma$
$T - mg \sin 30 - \mu R = ma$
Since $\mu = \frac{1}{5}$
$T - \frac{1}{2}mg - \frac{1}{5}\left(\frac{\sqrt{3}}{2}mg\right) = ma$
$T = ma + \frac{5+\sqrt{3}}{10}mg$ **(2)**
Equating **(1)** and **(2)** gives
$2mg - 2ma = ma + \frac{5+\sqrt{3}}{10}mg$
$3a = 2g - \frac{5+\sqrt{3}}{10}g$
$a = \frac{15-\sqrt{3}}{30}g$
$= 4.33$ m s^{-2} (3 s.f.)



16 b To find speed of A at the point B hits the ground:

$u = 0$, $s = 1$ and $a = \frac{15-\sqrt{3}}{30}g$

using $v^2 = u^2 + 2as$ gives

$$v^2 = 2\left(\frac{15-\sqrt{3}}{30}g\right)$$

$$= \frac{15-\sqrt{3}}{15}g$$

Once B has hit the ground, there will be no tension in the string.

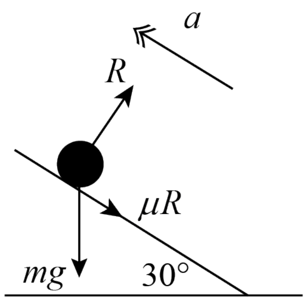
R(↗) $R = mg\cos 30$

$$= \frac{\sqrt{3}}{2}mg$$

(↘) Newton's 2nd law, $F = ma$

$Mg\sin 30 + \mu R = -ma$

Since $\mu = \frac{1}{5}$



$$\frac{1}{2}mg + \frac{1}{5}\left(\frac{\sqrt{3}}{2}mg\right) = -ma$$

$$a = -\left(\frac{1}{2}g + \frac{1}{5}\left(\frac{\sqrt{3}}{2}g\right)\right)$$

$$= -\left(\frac{5+\sqrt{3}}{10}\right)g$$

To find the distance travelled by A after B hits the ground, use $v^2 = u^2 + 2as$

$$0^2 = \left(\frac{15-\sqrt{3}}{15}\right)g - 2\left(\frac{5+\sqrt{3}}{10}\right)gs$$

$$\left(\frac{5+\sqrt{3}}{5}\right)s = \left(\frac{15-\sqrt{3}}{15}\right)$$

$$s = \frac{\left(15-\sqrt{3}\right)}{3\left(5+\sqrt{3}\right)}$$

Finally add on the 1 m that A travelled before B hit the ground to find the total distance travelled by A to be

$$1 + \frac{\left(15-\sqrt{3}\right)}{3\left(5+\sqrt{3}\right)}$$

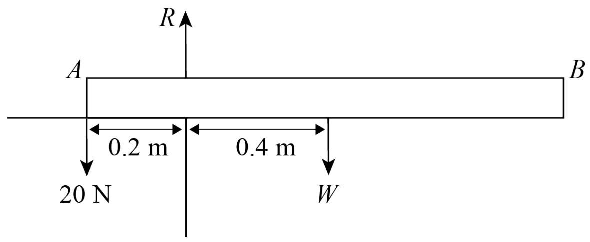
$= 1.66$ m (3 s.f.)

17 The rod is on the point of tipping so the edge of the table is acting as a pivot and the reaction force is acting vertically upwards at that point. Taking moments about the edge of the table,

$20 \times 0.2 = 0.4W$

$W = \dfrac{20 \times 0.2}{0.4}$

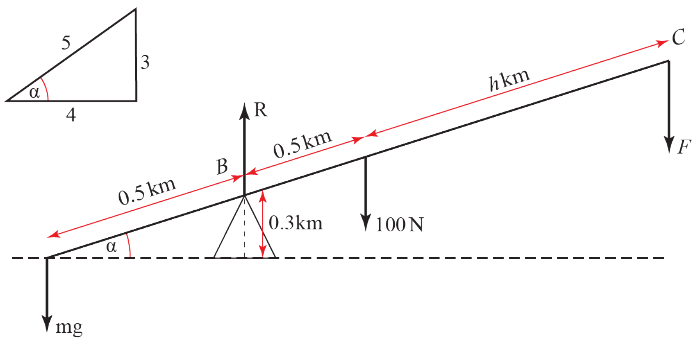
$= 10$ N as required



Challenge

1 The rod makes an angle of $\alpha°$ with the horizontal where

$\sin\alpha = \dfrac{0.3}{0.5} = \dfrac{3}{5} \Rightarrow \cos\alpha = \dfrac{4}{5}$



To lift the mass, total clockwise moments about B must exceed total anticlockwise moments about B:

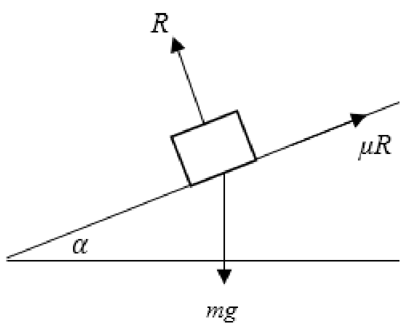
$$(F \times 1.5k\cos\alpha) + (100 \times 0.5k\cos\alpha) > mg \times 0.5k\cos\alpha$$

$$1.5F + 50 > 0.5mg$$

$$\frac{3}{2}F > \frac{1}{2}mg - 50$$

$$F > \frac{1}{3}(mg - 100) \quad \text{as required.}$$

Challenge 2



R(↖) $R = mg\cos\alpha$

(↙) Newton's 2nd law, $F = ma$

$ma = mg\sin\alpha - \mu R$

Therefore

$ma = mg\sin\alpha - \mu mg\cos\alpha$

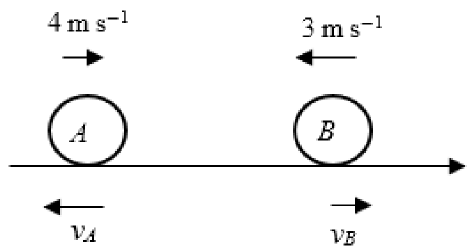
$a = g(\sin\alpha - \mu\cos\alpha)$

Using $s = ut + \frac{1}{2}at^2$ with $u = 0$ and $a = g(\sin\alpha - \mu\cos\alpha)$ gives

$$s = \frac{1}{2}gt^2(\sin\alpha - \mu\cos\alpha) \text{ as required}$$

Challenge 3

Let A be moving in the positive direction and let both balls have a mass of m.
By the conservation of momentum,



momentum before = momentum after

$4m - 3m = -mv_A + mv_B$
$v_B - v_A = 1$

To find the speed of B after the collision use $v = \frac{s}{t}$

$v_A = \frac{5}{10}$
$= 0.5$

Since $v_B - v_A = 1$
$v_B = 1.5 \text{ m s}^{-1}$