Review exercise 1

1 Equation of circle with centre (−3, 8) and radius r:

$(x + 3)^2 + (y - 8)^2 = r^2$

r = distance from (−3, 8) to (0, 9)
$r^2 = (0 + 3)^2 + (9 - 8)^2 = 9 + 1 = 10$

The equation for C is:
$(x + 3)^2 + (y - 8)^2 = 10$

2 a Rearranging:
$x^2 - 6x + y^2 + 2y = 10$

Completing the square:

$$(x - 3)^2 - 9 + (y + 1)^2 - 1 = 10$$
$$(x - 3)^2 + (y + 1)^2 = 20$$
$$a = 3,\ b = -1,\ r = \sqrt{20}$$

b The circle has centre (3, −1) and radius $\sqrt{20}$.

3 a Rearranging $3x + y = 14$:
$y = 14 - 3x$

Solving simultaneously using substitution:

$$\begin{aligned}(x - 2)^2 + (14 - 3x - 3)^2 &= 5\\ (x - 2)^2 + (-3x + 11)^2 &= 5\\ x^2 - 4x + 4 + 9x^2 - 66x + 121 - 5 &= 0\\ 10x^2 - 70x + 120 &= 0\\ x^2 - 7x + 12 &= 0\\ (x - 3)(x - 4) &= 0\\ \text{So } x = 3 \text{ and } x &= 4\end{aligned}$$

$x = 3$: $y = 14 - 3 \times 3 = 5$
$x = 4$: $y = 14 - 3 \times 4 = 2$
Point A is (3, 5) and point B is (4, 2).

b Using Pythagoras' theorem:

$$\text{Length } AB = \sqrt{(4-3)^2 + (2-5)^2}$$
$$= \sqrt{10}$$

4 The equation of the circle is $x^2 + y^2 = r^2$.

Solving simultaneously using substitution:

$$\begin{aligned} x^2 + (3x - 2)^2 &= r^2 \\ x^2 + 9x^2 - 12x + 4 - r^2 &= 0 \\ 10x^2 - 12x + 4 - r^2 &= 0 \end{aligned}$$

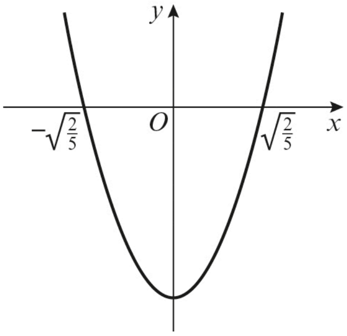
Using the discriminant for no solutions:

$$\begin{aligned} b^2 - 4ac &< 0 \\ (-12)^2 - 4(10)(4 - r^2) &< 0 \\ 144 - 160 + 40r^2 &< 0 \end{aligned}$$

$40r^2 - 16 < 0$

When $40r^2 - 16 = 0$

$$\begin{aligned} 8(5r^2 - 2) &= 0 \\ r^2 &= \frac{2}{5} \\ r &= \pm\sqrt{\frac{2}{5}} \end{aligned}$$



$-\sqrt{\frac{2}{5}} < r < \sqrt{\frac{2}{5}}$

However, the radius cannot be negative.

So $0 < r < \sqrt{\frac{2}{5}}$

5 a Equation of circle with centre (1, 5) and radius r:

$(x - 1)^2 + (y - 5)^2 = r^2$

r = distance from (1, 5) to (4, −2)

$$\begin{aligned} r^2 &= (4 - 1)^2 + (-2 - 5)^2 \\ &= 9 + 49 \\ &= 58 \end{aligned}$$

The equation for C is:

$(x - 1)^2 + (y - 5)^2 = 58$

5 b Gradient of the radius of the circle at P

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-5}{4-1} = -\frac{7}{3}$$

Gradient of the tangent = $\frac{3}{7}$

Equation of the tangent at P:

$$y - y_1 = m(x - x_1)$$
$$y + 2 = \tfrac{3}{7}(x - 4)$$
$$3x - 7y - 26 = 0$$

6 a $AB^2 = (6 - 2)^2 + (5 - 1)^2$
$= 4^2 + 4^2 = 32$
$BC^2 = (8 - 6)^2 + (3 - 5)^2$
$= 2^2 + 2^2 = 8$
$AC^2 = (8 - 2)^2 + (3 - 1)^2$
$= 6^2 + 2^2 = 40$

Using Pythagoras' theorem:
$AB^2 + BC^2 = 32 + 8 = 40 = AC^2$
Therefore, $\angle ABC$ is 90°.

b As triangle ABC is a right-angled triangle,
AC is a diameter of the circle.

c AC is a diameter of the circle, so the midpoint of AC is the centre.

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{2+8}{2}, \frac{1+3}{2}\right)$$
$$= (5, 2)$$

$$\text{Radius} = \tfrac{1}{2} \times AC$$
$$= \tfrac{1}{2} \times \sqrt{40}$$
$$= \tfrac{1}{2} \times 2\sqrt{10}$$
$$= \sqrt{10}$$

The equation of the circle is:
$(x - 5)^2 + (y - 2)^2 = 10$

7
$$\begin{aligned}\frac{2x^2+20x+42}{224x+4x^2-4x^3} &= \frac{x^2+10x+21}{112x+2x^2-2x^3}\\ &= \frac{(x+3)(x+7)}{-2x(x^2-x-56)}\\ &= \frac{(x+3)(x+7)}{-2x(x+7)(x-8)}\\ &= \frac{(x+3)}{-2x(x-8)}\end{aligned}$$

$a=3,\ b=-2,\ c=-8$

8 a Using the factor theorem:

$$\begin{aligned}f(\tfrac{1}{2}) &= 2(\tfrac{1}{2})^3-7(\tfrac{1}{2})^2-17(\tfrac{1}{2})+10\\ &= \tfrac{1}{4}-\tfrac{7}{4}-\tfrac{17}{2}+10\\ &= 0\end{aligned}$$

So $(2x-1)$ is a factor of $2x^3-7x^2-17x+10$.

b
$$\begin{array}{r} x^2-3x-10 \\ 2x-1\overline{)2x^3-7x^2-17x+10} \\ \underline{2x^3-x^2} \\ -6x^2-17x \\ \underline{-6x^2+3x} \\ -20x+10 \\ \underline{-20x+10} \\ 0 \end{array}$$

$$\begin{aligned}2x^3-7x^2-17x+10 &= (2x-1)(x^2-3x-10)\\ &= (2x-1)(x-5)(x+2)\end{aligned}$$

c $(2x-1)(x-5)(x+2)=0$

So $x=\frac{1}{2}$, $x=5$ or $x=-2$

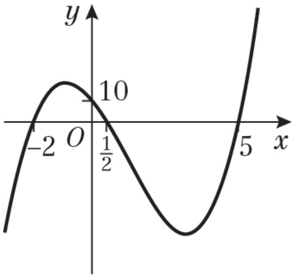
So the curve crosses the x-axis at $(\frac{1}{2}, 0)$, $(5, 0)$ and $(-2, 0)$.

When $x=0$, $y=-1\times-5\times2=10$

So the curve crosses the y-axis at $(0, 10)$.

$x\to\infty,\ y\to\infty$

$x\to-\infty,\ y\to-\infty$



9 $f(x) = 3x^3 + x^2 - 38x + c$

a $f(3) = 0$

$3(3)^3 + (3)^2 - 38(3) + c = 0$

$3 \times 27 + 9 - 114 + c = 0$

$c = 24$

b $f(x) = 3x^3 + x^2 - 38x + 24$

$f(3) = 0$, so $(x - 3)$ is a factor of $3x^3 + x^2 - 38x + 24$.

$$\begin{array}{r} 3x^2+10x-8 \\ x-3\,\overline{)\,3x^3+x^2-38x+24} \\ \underline{3x^3-9x^2}\qquad\qquad \\ 10x^2-38x\qquad \\ \underline{10x^2-30x}\qquad \\ -8x-24 \\ \underline{-8x+24} \\ 0 \end{array}$$

$$3x^3 + x^2 - 38x + 24 = (x - 3)(3x^2 + 10x - 8)$$
$$= (x - 3)(3x - 2)(x + 4)$$

10 a $g(x) = x^3 - 13x + 12$

$g(3) = (3)^3 - 13(3) + 12$

$= 27 - 39 + 12$

$= 0$

So $(x - 3)$ is a factor of $g(x)$.

b

$$\begin{array}{r} x^2+3x-4 \\ x-3\,\overline{)\,x^3-0x^2-13x+12} \\ \underline{x^3-3x^2}\qquad\qquad \\ 3x^2-13x\qquad \\ \underline{3x^2-9x}\qquad \\ -4x+12 \\ \underline{-4x+12} \\ 0 \end{array}$$

$g(x) = x^3 - 13x + 12$

$= (x - 3)(x^2 + 3x - 4)$

$= (x - 3)(x + 4)(x - 1)$

11 a $\mathrm{f}(x) = x^3 + ax^2 + bx + 8$

$\mathrm{f}(3) = 2$, so

$(3)^3 + a(3)^2 + b(3) + 8 = 2$

$27 + 9a + 3b + 8 = 2$

$9a + 3b = -33$ **(1)**

$\mathrm{f}(-1) = -2$ so

$(-1)^3 + a(-1)^2 + b(-1) + 8 = -2$

$-1 + a - b + 8 = -2$

$a - b = -9$ **(2)**

Multiply equation **(2**) by 3 and add to equation **(1)**

$$\begin{aligned} 9a + 3b &= -33 \\ \underline{3a - 3b} &\underline{= -27} \\ 12a \quad &= -60 \end{aligned}$$

$a = -5$

When $a = -5$, $b = 4$

b $\mathrm{f}(x) = x^3 - 5x^2 + 4x + 8$

$$\begin{aligned} \mathrm{f}(2) &= (2)^3 - 5(2)^2 + 4(2) + 8 \\ &= 8 - 20 + 8 + 8 \\ &= 4 \end{aligned}$$

12 a $\mathrm{f}(x) = 2x^3 + ax^2 + bx + 6$

$\mathrm{f}(1) = 0$, so

$2(1)^3 + a(1)^2 + b(1) + 6 = 0$

$2 + a + b + 6 = 0$

$a + b = -8$ **(1)**

$\mathrm{f}(-1) = 10$, so

$2(-1)^3 + a(-1)^2 + b(-1) + 6 = 10$

$-2 + a - b + 6 = 10$

$a - b = 6$ **(2)**

Add equations **(1)** and **(2)**

$$\begin{aligned} a + b &= -8 \\ \underline{a - b} &\underline{= \ 6} \\ 2a \quad &= -2 \end{aligned}$$

$a = -1$

When $a = -1$, $b = -7$

12 b $\mathrm{f}(x)=2x^3-x^2-7x+6$

$(x-1)$ is a factor, so

$$\begin{array}{rl}
 & 2x^2+x-6 \\
x-1 & \overline{)2x^3-x^2-7x+6} \\
 & \underline{2x^3-2x^2} \\
 & \phantom{)2x^3-{}}x^2-7x \\
 & \phantom{)2x^3-{}}\underline{x^2-x} \\
 & -6x+6 \\
 & \underline{-6x+6} \\
 & \phantom{)2x^3-x^2-6x+{}}0
\end{array}$$

$2x^3-x^2-7x+6=(x-1)(2x^2+x-6)$

Now factorise the quadratic

$(2x^2+x-6)=(2x-3)(x+2)$

so

$2x^3-x^2-7x+6=(x-1)(2x-3)(x+2)$

So $x=1,\ x=\dfrac{3}{2}$, or $x=-2$

13 a $\mathrm{f}(x)=x^4+5x^3+ax+b$

$\mathrm{f}(2)=\mathrm{f}(-1)$, so

$(2)^4+5(2)^3+a(2)+b=(-1)^4+5(-1)^3+a(-1)+b$

$16+40+2a=1-5-a$

$3a=-60$

$a=-20$

b $\mathrm{f}(x)=x^4+5x^3-20x+b$

$(x+3)$ is a factor so by the factor theorem $\mathrm{f}(-3)=0$

$(-3)^4+5(-3)^3-20(-3)+b=0$

$81-135+60+b=0$

$b=-6$

14 a Example:

When $a = 0$ and $b = 0$, $0^2 + 0^2 = (0 + 0)^2$.

b $(a + b)^2 = a^2 + 2ab + b^2$

When $a > 0$ and $b > 0$, $2ab > 0$

Therefore $a^2 + b^2 < (a + b)^2$

When $a < 0$ and $b < 0$, $2ab > 0$

Therefore $a^2 + b^2 < (a + b)^2$

When $a > 0$ and $b < 0$, $2ab < 0$

Therefore $a^2 + b^2 > (a + b)^2$

When $a < 0$ and $b > 0$, $2ab < 0$

Therefore $a^2 + b^2 > (a + b)^2$

The conditions are $a > 0$ and $b > 0$ or $a < 0$ and $b < 0$.

15 a $p = 5$: $5^2 = 25 = 24 + 1$
$p = 7$: $7^2 = 49 = 2(24) + 1$
$p = 11$: $11^2 = 121 = 5(24) + 1$
$p = 13$: $13^2 = 169 = 7(24) + 1$
$p = 17$: $17^2 = 289 = 12(24) + 1$
$p = 19$: $19^2 = 361 = 15(24) + 1$

b $3(24) + 1 = 73$ and 73 is not a square number.

16 a Rearranging:
$x^2 - 10x + y^2 - 8y = -32$
Completing the square:
$$(x - 5)^2 - 25 + (y - 4)^2 - 16 = -32$$
$$(x - 5)^2 + (y - 4)^2 = 9$$
$$(x - 5)^2 + (y - 4)^2 = 3^2$$
$a = 5$, $b = 4$, $r = 3$

b Centre of circle C is (5, 4).
Centre of circle D is (0, 0).
Using Pythagoras' theorem:
Distance = $\sqrt{(5-0)^2+(4-0)^2} = \sqrt{41}$

c Radius of circle $C = 3$
Radius of circle $D = 3$
Distance between the centres = $\sqrt{41}$
$3 + 3 < \sqrt{41}$
Therefore, the circles C and D do not touch.

17 a $5^x = 0.75$
$$x\log 5 = \log 0.75$$
$$x = \frac{\log 0.75}{\log 5}$$
$$= -0.179 \text{ (3 s.f.)}$$

b $2\log_5 x - \log_5 3x = 1$
$$\log_5 x^2 - \log_5 3x = 1$$
$$\log_5\left(\frac{x^2}{3x}\right) = 1$$
$$\frac{x^2}{3x} = 5^1$$
$$x^2 = 15x$$
$$x^2 - 15x = 0$$
$$x(x-15) = 0$$
$x = 0$ or $x = 15$
since $x \neq 0$, $x = 15$

18 a $3^{2x-1} = 10$

$(2x-1)\log 3 = \log 10$

$(2x-1) = \frac{\log 10}{\log 3}$

$x = \frac{1}{2}\left(\frac{\log 10}{\log 3} + 1\right)$

$= 1.55$ (3 s.f.)

b $\log_2 x + \log_2(9-2x) = 2$

$\log_2\left(x(9-2x)\right) = 2$

$x(9-2x) = 2^2$

$2x^2 - 9x + 4 = 0$

$(2x-1)(x-4) = 0$

$x = \frac{1}{2}$ or $x = 4$

19 a $\log_p 12 - \left(\frac{1}{2}\log_p 9 + \frac{1}{3}\log_p 8\right)$

$= \log_p 12 - \left(\log_p 9^{\frac{1}{2}} + \log_p 8^{\frac{1}{3}}\right)$

$= \log_p 12 - \left(\log_p 3 + \log_p 2\right)$

$= \log_p 12 - \log_p 6$

$= \log_p\left(\frac{12}{6}\right)$

$= \log_p 2$

b $\log_4 x = -1.5$

$x = 4^{-1.5}$

$= \frac{1}{8}$

20 $\log_x 64 + 3\log_4 x - \log_x 4 = 5$

$$\log_x 64 + \frac{3}{\log_x 4} - \log_x 4 = 5$$

$$\log_x 4^3 - \log_x 4 + \frac{3}{\log_x 4} = 5$$

$$\log_x \left(\frac{4^3}{4}\right) + \frac{3}{\log_x 4} = 5$$

$$\log_x 4^2 + \frac{3}{\log_x 4} = 5$$

$$2\log_x 4 + \frac{3}{\log_x 4} = 5$$

Let $y = \log_x 4$, so

$$2y + \frac{3}{y} = 5$$

$$2y^2 + 3 = 5y$$

$$2y^2 - 5y + 3 = 0$$

$$(2y-3)(y-1) = 0$$

$$y = \frac{3}{2} \text{ or } y = 1$$

Therefore

$$\log_x 4 = \frac{3}{2} \text{ or } \log_x 4 = 1$$

When $\log_x 4 = \frac{3}{2}$

$$x^{\frac{3}{2}} = 4$$

$$x = 2.52 \ \text{(3 s.f.)}$$

When $\log_x 4 = 1$

$$x^1 = 4$$

$$x = 4$$

21 $\log_2 x + 6\log_x 2 = 7$

$$\log_2 x + \frac{6}{\log_2 x} = 7$$

Let $y = \log_2 x$, so

$$y + \frac{6}{y} = 7$$

$$y^2 - 7y + 6 = 0$$

$$(y-1)(y-6) = 0$$

$y = 1$ or $y = 6$

Therefore

$\log_2 x = 1$ or $\log_2 x = 6$

When $\log_2 x = 1$

$x = 2$

When $\log_2 x = 6$

$x = 2^6 = 64$

22 $\log_3 9t = \log_9 \left(\frac{12}{t}\right)^2 + 2$

$$\frac{\log 9t}{\log 3} = \frac{\log\left(\frac{12}{t}\right)^2}{\log 9} + 2$$

$$\frac{\log 9t}{\log 3} = \frac{2\log\left(\frac{12}{t}\right)}{2\log 3} + 2$$

$$\log 9t = \log\left(\frac{12}{t}\right) + 2\log 3$$

$$\log 9t = \log\left(\frac{108}{t}\right)$$

$$9t = \frac{108}{t}$$

$$t^2 = 12$$

$$t = 2\sqrt{3}$$

$t \neq -2\sqrt{3}$, since it's not valid.

23 a $(1-2x)^{10}$

$$= 1^{10} + \binom{10}{1}1^9(-2x) + \binom{10}{2}1^8(-2x)^2 + \binom{10}{3}1^7(-2x)^3 + \ldots$$

$$= 1 + 10(-2x) + \frac{10(9)}{2}(-2x) + \frac{10(9)(8)}{6}(-2x)^3 + \ldots$$

$$= 1 - 20x + 180x^2 - 960x^3 + \ldots$$

23 b $(0.98)^{10}$

$= (1-2(0.01))^{10}$

$= 1 - 20(0.01) + 180(0.01)^2 - 960(0.01)^3 + \ldots$

$= 0.817$ (3 d.p.)

24 $(1+2x)^5 = 1^5 + \binom{5}{1}1^4(2x) + \binom{5}{2}1^3(2x)^2 + \ldots$

$$= 1 + 5(2x) + \frac{5(4)}{2}(2x)^2 + \ldots$$

$$= 1 + 10x + 40x^2 + \ldots$$

$$\begin{aligned}(2-x)(1+2x)^5 &= (2-x)(1+10x+40x^2+\ldots)\\ &= 2 + 20x + 80x^2 + \ldots - x - 10x^2 + \ldots\\ &= 2 + 19x + 70x^2 + \ldots\\ &\approx 2 + 19x + 70x^2\end{aligned}$$

$a = 2$, $b = 19$, $c = 70$

25 $(2-4x)^q$

x term $= \binom{q}{q-1}2^{q-1}(-4x)^1$

$= q \times 2^{q-1} \times -4x$

$= -4 \times 2^{q-1}qx$

$-4 \times 2^{q-1}q = -32q$

$2^{q-1} = 8$

$q - 1 = 3$

$q = 4$

26 a Using the binomial expansion

$$(1+4x)^{\frac{3}{2}} = 1+\left(\frac{3}{2}\right)(4x)+\frac{\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)}{2!}(4x)^2+\frac{\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{3!}(4x)^3+\ldots$$

$$= 1+6x+6x^2-4x^3+\ldots$$

b $$\left(1+4\left(\frac{3}{100}\right)\right)^{\frac{3}{2}} = \left(\frac{112}{100}\right)^{\frac{3}{2}}$$

$$= \left(\sqrt{\frac{112}{100}}\right)^3$$

$$= \frac{112\sqrt{122}}{1000}$$

c $$1+6\left(\frac{3}{100}\right)+6\left(\frac{3}{100}\right)^2-4\left(\frac{3}{100}\right)^3 = 1.185292$$

So $\dfrac{112\sqrt{112}}{1000} \approx 1.185292$

$$\Rightarrow \sqrt{112} \approx \frac{1185.292}{112} = 10.582962857\ldots = 10.58296 \text{ (5 d.p.)}$$

d Using a calculator $\sqrt{112} = 10.5830052$ (7 d.p.)

$$\text{Percentage error } = \frac{10.5830052-10.5829643}{10.5830052}\times 100 = 0.00039\% \text{ (5 d.p.)}$$

Note, you will get different answers if you use values rounded to 5 d.p. in calculating the percentage error.

Challenge

1 a $\mathrm{f}(x)=2x^4+ax^3-23x^2+bx+24$

x^2+x-6 is a factor so

$$\begin{array}{r}
2x^2+(a-2)x-(9+a) \\
x^2+x-6\,\overline{)\,2x^4+ax^3-23x^2+bx+24} \\
\underline{2x^4+2x^3-12x^2} \\
(a-2)x^3-11x^2+bx \\
\underline{(a-2)x^3+(a-2)x^2-6(a-2)x} \\
(-9-a)x^2+(6a+b-12)x+24 \\
\underline{(-9-a)x^2+(-9-a)x-6(-9-a)} \\
(7a+b-3)x+(-30-6a)
\end{array}$$

Since x^2+x-6 is a factor

$-30-6a=0 \Rightarrow a=-5$

$7a+b-3=0 \Rightarrow b=38$

Alternative solution: by factorisation:

$2x^4+ax^3-23x^2+bx+24$

$(x^2+x-6)(2x^2-7x-4)$

x^3 coefficient: $a=2-7=-5$

x coefficient: $b=-6\times-7-4=38$

b Substitute for *a* and *b* into

$2x^2+(a-2)x-(9+a)=0$

$2x^2-7x-4=0$

$(2x+1)(x-4)=0$

Therefore

$$\begin{aligned}2x^4+ax^3-23x^2+bx+24&=(x^2+x-6)(2x+1)(x-4)\\&=(x+3)(x-2)(2x+1)(x-4)\end{aligned}$$

2 $\mathrm{f}(x) = ax^3 + bx^2 + cx + d$

$\mathrm{f}(-2) = -11 \Rightarrow -8a + 4b - 2c + d = -11$ **(1)**

$\mathrm{f}(1) = 4 \Rightarrow a + b + c + d = 4$ **(2)**

$(x+2)(x-1) = x^2 + x - 2$

$$
\begin{array}{r}
ax + b + a \\
x^2 + x - 2 \overline{\smash{\big)}\, ax^3 + bx^2 + cx + d} \\
\underline{ax^3 + ax^2 - 2ax} \\
bx^2 - ax^2 + 2ax + cx \\
\underline{bx^2 + bx - 2b} \\
-ax^2 + 2ax - bx + cx + 2b + d \\
\underline{ax^2 + ax - 2a} \\
3ax - bx + cx - 2a + 2b + d \\
x(3a - b + c) - 2a + 2b + d
\end{array}
$$

So the remainder is:
$x(3a - b + c) - 2a + 2b + d$

Equation **(1)** – equation **(2)** gives:
$-9a + 3b - 3c = -15$
$3a - b + c = 5$ **(3)**

Equation **(1)** – 2 × equation **(2)** gives:
$-6a + 6b + 3d = -3$
$-2a + 2b + d = -1$ **(4)**

Substituting equations **(3)** and **(4)** into the remainder gives:
$5x - 1$

So the remainder is $5x - 1$.

3 Rearranging $x^2 + y^2 + 8x - 10y = 59$:

$x^2 + 8x + y^2 - 10y = 59$
Completing the square:
$(x + 4)^2 - 16 + (y - 5)^2 - 25 = 59$
$(x + 4)^2 + (y - 5)^2 = 100$
Both circles have the same centre at $(-4, 5)$. The radius of one circle is 8 and the other is 10, so $(x + 4)^2 + (y - 5)^2 = 8^2$ lies completely inside $x^2 + y^2 + 8x - 10y = 59$.

4

$$8^{2y+1}=4^{2x-2}$$

$$(2y+1)\log 8=(2x-2)\log 4$$

$$3(2y+1)\log 2=4(x-1)\log 2$$

$$6y+3=4x-4$$

$$y=\frac{4x-7}{6}$$

$$\log_2 y=1+\log_4 x$$

$$\log_2\left(\frac{4x-7}{6}\right)=1+\frac{\log_2 x}{\log_2 4}$$

$$\log_2\left(\frac{4x-7}{6}\right)=1+\frac{1}{2}\log_2 x$$

$$\log_2\left(\frac{4x-7}{6}\right)=1+\log_2 x^{\frac{1}{2}}$$

$$\log_2\left(\frac{4x-7}{6x^{\frac{1}{2}}}\right)=1$$

$$\frac{4x-7}{6x^{\frac{1}{2}}}=2$$

$$4x-12x^{\frac{1}{2}}-7=0$$

Let $z=x^{\frac{1}{2}}$

$$4z^2-12z-7=0$$

$$(2z+1)(2z-7)=0$$

$$z=-\frac{1}{2}\text{ or }z=\frac{7}{2}$$

So

$$x^{\frac{1}{2}}=-\frac{1}{2}\text{ or }x^{\frac{1}{2}}=\frac{7}{2}$$

$$x=\frac{1}{4}\text{ or }x=\frac{49}{4}$$

When $x=\frac{1}{4}$

$$y=\frac{4\left(\frac{1}{4}\right)-7}{6}=-1$$

When $x=\frac{49}{4}$

$$y=\frac{4\left(\frac{49}{4}\right)-7}{6}=7$$

5 $$\text{LHS} = \binom{n}{k} + \binom{n}{k+1}$$

$$= \frac{n!}{k!(n-k)!} + \frac{n!}{(k+1)!(n-k-1)!}$$

$$= \frac{n!(k+1)}{(k+1)!(n-k)!} + \frac{n!(n-k)}{(k+1)!(n-k)!}$$

$$= \frac{n!((k+1)+(n-k))}{(k+1)!(n-k)!}$$

$$= \frac{n!(n+1)}{(k+1)!(n-k)!}$$

$$= \frac{(n+1)!}{(k+1)!(n-k)!}$$

$$= \binom{n+1}{k+1}$$

$$= \text{RHS}$$