

## Exercise 5A

1 a  $(x_1, y_1) = (4, 2), (x_2, y_2) = (6, 3)$

$$\begin{aligned}\frac{y_2 - y_1}{x_2 - x_1} &= \frac{3 - 2}{6 - 4} \\ &= \frac{1}{2}\end{aligned}$$

b  $(x_1, y_1) = (-1, 3), (x_2, y_2) = (5, 4)$

$$\begin{aligned}\frac{y_2 - y_1}{x_2 - x_1} &= \frac{4 - 3}{5 - (-1)} \\ &= \frac{1}{6}\end{aligned}$$

c  $(x_1, y_1) = (-4, 5), (x_2, y_2) = (1, 2)$

$$\begin{aligned}\frac{y_2 - y_1}{x_2 - x_1} &= \frac{2 - 5}{1 - (-4)} \\ &= -\frac{3}{5}\end{aligned}$$

d  $(x_1, y_1) = (2, -3), (x_2, y_2) = (6, 5)$

$$\begin{aligned}\frac{y_2 - y_1}{x_2 - x_1} &= \frac{5 - (-3)}{6 - 2} \\ &= \frac{8}{4} \\ &= 2\end{aligned}$$

e  $(x_1, y_1) = (-3, 4), (x_2, y_2) = (7, -6)$

$$\begin{aligned}\frac{y_2 - y_1}{x_2 - x_1} &= \frac{-6 - 4}{7 - (-3)} \\ &= \frac{-10}{10} \\ &= -1\end{aligned}$$

f  $(x_1, y_1) = (-12, 3), (x_2, y_2) = (-2, 8)$

$$\begin{aligned}\frac{y_2 - y_1}{x_2 - x_1} &= \frac{8 - 3}{-2 - (-12)} \\ &= \frac{1}{2}\end{aligned}$$

g  $(x_1, y_1) = (-2, -4), (x_2, y_2) = (10, 2)$

$$\begin{aligned}\frac{y_2 - y_1}{x_2 - x_1} &= \frac{2 - (-4)}{10 - (-2)} \\ &= \frac{6}{12} \\ &= \frac{1}{2}\end{aligned}$$

h  $(x_1, y_1) = (\frac{1}{2}, 2), (x_2, y_2) = (\frac{3}{4}, 4)$

$$\begin{aligned}\frac{y_2 - y_1}{x_2 - x_1} &= \frac{4 - 2}{\frac{3}{4} - \frac{1}{2}} \\ &= \frac{2}{\frac{1}{4}} \\ &= 8\end{aligned}$$

i  $(x_1, y_1) = (\frac{1}{4}, \frac{1}{2}), (x_2, y_2) = (\frac{1}{2}, \frac{2}{3})$

$$\begin{aligned}\frac{y_2 - y_1}{x_2 - x_1} &= \frac{\frac{2}{3} - \frac{1}{2}}{\frac{1}{2} - \frac{1}{4}} \\ &= \frac{\frac{1}{6}}{\frac{1}{4}} \\ &= \frac{2}{3}\end{aligned}$$

j  $(x_1, y_1) = (-2.4, 9.6), (x_2, y_2) = (0, 0)$

$$\begin{aligned}\frac{y_2 - y_1}{x_2 - x_1} &= \frac{0 - 9.6}{0 - (-2.4)} \\ &= \frac{-9.6}{2.4} \\ &= -4\end{aligned}$$

k  $(x_1, y_1) = (1.3, -2.2), (x_2, y_2) = (8.8, -4.7)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-4.7 - (-2.2)}{8.8 - 1.3}$$

$$\begin{aligned} 2 \text{ k } \frac{y_2 - y_1}{x_2 - x_1} &= \frac{-2.5}{7.5} \\ &= -\frac{1}{3} \end{aligned}$$

$$\begin{aligned} 1 \text{ } (x_1, y_1) &= (0, 5a), (x_2, y_2) = (10a, 0) \\ \frac{y_2 - y_1}{x_2 - x_1} &= \frac{0 - 5a}{10a - 0} \\ &= \frac{-5a}{10a} \\ &= \frac{-5}{10} \\ &= -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{m } (x_1, y_1) &= (3b, -2b), (x_2, y_2) = (7b, 2b) \\ \frac{y_2 - y_1}{x_2 - x_1} &= \frac{2b - (-2b)}{7b - 3b} \\ &= \frac{4b}{4b} \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{n } (x_1, y_1) &= (p, p^2), (x_2, y_2) = (q, q^2) \\ \frac{y_2 - y_1}{x_2 - x_1} &= \frac{q^2 - p^2}{q - p} \\ &= \frac{(q - p)(q + p)}{q - p} \\ &= q + p \end{aligned}$$

$$\begin{aligned} 2 \text{ } (x_1, y_1) &= (3, -5), (x_2, y_2) = (6, a) \\ \frac{y_2 - y_1}{x_2 - x_1} &= 4 \\ \text{So } \frac{a - (-5)}{6 - 3} &= 4 \\ \Rightarrow \frac{a + 5}{3} &= 4 \\ \Rightarrow a + 5 &= 12 \\ \Rightarrow a &= 7 \end{aligned}$$

$$\begin{aligned} 3 \text{ } (x_1, y_1) &= (5, b), (x_2, y_2) = (8, 3) \\ \frac{3 - b}{8 - 5} &= -3 \\ \frac{3 - b}{3} &= -3 \\ 3 - b &= -9 \\ b &= 12 \end{aligned}$$

$$\begin{aligned} 4 \text{ } (x_1, y_1) &= (c, 4), (x_2, y_2) = (7, 6) \\ \frac{6 - 4}{7 - c} &= \frac{3}{4} \\ \frac{2}{7 - c} &= \frac{3}{4} \\ 2 &= \frac{3}{4}(7 - c) \\ 8 &= 3(7 - c) \\ 8 &= 21 - 3c \\ -13 &= -3c \\ c &= \frac{-13}{-3} \\ &= \frac{13}{3} \\ &= 4\frac{1}{3} \end{aligned}$$

$$\begin{aligned} 5 \text{ } (x_1, y_1) &= (-1, 2d), (x_2, y_2) = (1, 4) \\ \frac{4 - 2d}{1 - (-1)} &= -\frac{1}{4} \\ \frac{4 - 2d}{2} &= -\frac{1}{4} \\ 2 - d &= -\frac{1}{4} \\ 2\frac{1}{4} - d &= 0 \\ d &= 2\frac{1}{4} \end{aligned}$$

6  $(x_1, y_1) = (-3, -2), (x_2, y_2) = (2e, 5)$

$$\frac{5 - (-2)}{2e - (-3)} = 2$$

$$\frac{7}{2e + 3} = 2$$

$$7 = 2(2e + 3)$$

$$7 = 4e + 6$$

$$4e = 1$$

$$e = \frac{1}{4}$$

7  $(x_1, y_1) = (7, 2), (x_2, y_2) = (f, 3f)$

$$\frac{3f - 2}{f - 7} = 4$$

$$3f - 2 = 4(f - 7)$$

$$3f - 2 = 4f - 28$$

$$-2 = f - 28$$

$$28 - 2 = f$$

$$f = 26$$

8  $(x_1, y_1) = (3, -4), (x_2, y_2) = (-g, 2g)$

$$\frac{2g - (-4)}{-g - 3} = -3$$

$$\frac{2g + 4}{-g - 3} = -3$$

$$2g + 4 = -3(-g - 3)$$

$$2g + 4 = 3g + 9$$

$$4 = g + 9$$

$$g = -5$$

9 The gradient of  $AB$  is:

$$\frac{4 - 3}{4 - 2} = \frac{1}{2}$$

The gradient of  $AC$  is:

$$\begin{aligned} \frac{7 - 3}{10 - 2} &= \frac{4}{8} \\ &= \frac{1}{2} \end{aligned}$$

The gradients are equal and there is a point in common between the two line segments so the points can be joined by a straight line.

10 If the points  $A(-2a, 5a)$ ,  $B(0, 4a)$  and  $C(6a, a)$  are collinear, then they all lie on the same straight line.

The gradient of  $AB$  is:

$$\begin{aligned} \frac{4a - 5a}{0 - (-2a)} &= \frac{-a}{2a} \\ &= -\frac{1}{2} \end{aligned}$$

The gradient of  $AC$  is:

$$\begin{aligned} \frac{a - 5a}{6a - (-2a)} &= \frac{-4a}{8a} \\ &= -\frac{1}{2} \end{aligned}$$

The gradients are both  $-\frac{1}{2}$  and there is a point in common between the two line segments so the points are collinear.