Matrix algebra 5A

1 a $\mathbf{A}-\lambda\mathbf{I}=\begin{pmatrix}2&4\\1&5\end{pmatrix}-\begin{pmatrix}\lambda&0\\0&\lambda\end{pmatrix}=\begin{pmatrix}2-\lambda&4\\1&5-\lambda\end{pmatrix}$

$\begin{vmatrix}2-\lambda&4\\1&5-\lambda\end{vmatrix}=(2-\lambda)(5-\lambda)-4$

$=10-7\lambda+\lambda^2-4=\lambda^2-7\lambda+6=(\lambda-1)(\lambda-6)$

$\det(\mathbf{A}-\lambda\mathbf{I})=0\Rightarrow(\lambda-1)(\lambda-6)=0\Rightarrow\lambda=1,6$

The eigenvalues are 1 and 6.

For $\lambda=1$

$\begin{pmatrix}2&4\\1&5\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=1\begin{pmatrix}x\\y\end{pmatrix}$

$\begin{pmatrix}2x+4y\\x+5y\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$

Equating the upper elements

$2x+4y=x\Rightarrow x=-4y$

Let $y=1$, then $x=-4$

An eigenvector corresponding to the eigenvalue 1 is $\begin{pmatrix}-4\\1\end{pmatrix}$.

For $\lambda=6$

$\begin{pmatrix}2&4\\1&5\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=6\begin{pmatrix}x\\y\end{pmatrix}$

$\begin{pmatrix}2x+4y\\x+5y\end{pmatrix}=\begin{pmatrix}6x\\6y\end{pmatrix}$

Equating the upper elements

$2x+4y=6x\Rightarrow y=x$

Let $x=1$, then $y=1$

An eigenvector corresponding to the eigenvalue 6 is $\begin{pmatrix}1\\1\end{pmatrix}$.

1 b $\mathbf{A}-\lambda\mathbf{I}=\begin{pmatrix}4 & -1\\ -1 & 4\end{pmatrix}-\begin{pmatrix}\lambda & 0\\ 0 & \lambda\end{pmatrix}=\begin{pmatrix}4-\lambda & -1\\ -1 & 4-\lambda\end{pmatrix}$

$$\begin{vmatrix}4-\lambda & -1\\ -1 & 4-\lambda\end{vmatrix}=(4-\lambda)^2-1$$

$$=16-8\lambda+\lambda^2-1=\lambda^2-8\lambda+15=(\lambda-3)(\lambda-5)$$

$\det(\mathbf{A}-\lambda\mathbf{I})=0\Rightarrow(\lambda-3)(\lambda-5)=0\Rightarrow\lambda=3,5$

The eigenvalues are 3 and 5.

For $\lambda=3$

$$\begin{pmatrix}4 & -1\\ -1 & 4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=3\begin{pmatrix}x\\ y\end{pmatrix}$$

$$\begin{pmatrix}4x-y\\ -x+4y\end{pmatrix}=\begin{pmatrix}3x\\ 3y\end{pmatrix}$$

Equating the upper elements

$4x-y=3x\Rightarrow y=x$

Let $x=1$, then $y=1$

An eigenvector corresponding to the eigenvalue 3 is $\begin{pmatrix}1\\ 1\end{pmatrix}$.

For $\lambda=5$

$$\begin{pmatrix}4 & -1\\ -1 & 4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=5\begin{pmatrix}x\\ y\end{pmatrix}$$

$$\begin{pmatrix}4x-y\\ -x+4y\end{pmatrix}=\begin{pmatrix}5x\\ 5y\end{pmatrix}$$

Equating the upper elements

$4x+y=5x\Rightarrow y=-x$

Let $x=1$, then $y=-1$

An eigenvector corresponding to the eigenvalue 5 is $\begin{pmatrix}1\\ -1\end{pmatrix}$.

1 c $\mathbf{A}-\lambda\mathbf{I}=\begin{pmatrix}3 & -2\\ 0 & 4\end{pmatrix}-\begin{pmatrix}\lambda & 0\\ 0 & \lambda\end{pmatrix}=\begin{pmatrix}3-\lambda & -2\\ 0 & 4-\lambda\end{pmatrix}$

$\begin{vmatrix}3-\lambda & -2\\ 0 & 4-\lambda\end{vmatrix}=(3-\lambda)(4-\lambda)$

$\det(\mathbf{A}-\lambda\mathbf{I})=0\Rightarrow(3-\lambda)(4-\lambda)=0\Rightarrow\lambda=3,4$

The eigenvalues are 3 and 4.

For $\lambda=3$

$\begin{pmatrix}3 & -2\\ 0 & 4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=3\begin{pmatrix}x\\ y\end{pmatrix}$

$\begin{pmatrix}3x-2y\\ 4y\end{pmatrix}=\begin{pmatrix}3x\\ 3y\end{pmatrix}$

Equating the lower elements

$4y=3y\Rightarrow y=0$

As x can take any non-zero value, let $x=1$.

An eigenvector corresponding to the eigenvalue 3 is $\begin{pmatrix}1\\ 0\end{pmatrix}$.

For $\lambda=4$

$\begin{pmatrix}3 & -2\\ 0 & 4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=4\begin{pmatrix}x\\ y\end{pmatrix}$

$\begin{pmatrix}3x-2y\\ 4y\end{pmatrix}=\begin{pmatrix}4x\\ 4y\end{pmatrix}$

Equating the upper elements

$3x-2y=4x\Rightarrow x=-2y$

Let $y=1$, then $x=-2$

An eigenvector corresponding to the eigenvalue 4 is $\begin{pmatrix}-2\\ 1\end{pmatrix}$.

2 a $M = \begin{pmatrix} -2 & 1 \\ -1 & 0 \end{pmatrix}$

$M - \lambda I = \begin{pmatrix} -2 & 1 \\ -1 & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} 2-\lambda & 1 \\ -1 & -\lambda \end{pmatrix}$

$\det(M - \lambda I) = \begin{vmatrix} -2-\lambda & 1 \\ -1 & -\lambda \end{vmatrix}$

$= (-2-\lambda)(-\lambda) + 1$

$= \lambda^2 + 2\lambda + 1 = (\lambda + 1)^2$

$\det(M - \lambda I) = 0 \Rightarrow \lambda = -1$

Hence $\lambda = -1$ is a repeated eigenvalue

$\begin{pmatrix} -2 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -\begin{pmatrix} x \\ y \end{pmatrix}$

Equating lower elements: $-x = -y$, or $x = y$

So the required eigenvector is a multiple of $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Therefore the simplest eigenvector is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

b $N = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$

$N - \lambda I = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} 4-\lambda & 0 \\ 0 & 4-\lambda \end{pmatrix}$

$\det(N - \lambda I) = \begin{vmatrix} 4-\lambda & 0 \\ 0 & 4-\lambda \end{vmatrix}$

$= (4-\lambda)^2$

$\det(N - \lambda I) = 0 \Rightarrow \lambda = 4$

Hence $\lambda = 4$ is a repeated eigenvalue.

$\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4x \\ 4y \end{pmatrix}$

Equating upper elements: $4x = 4x$

So you can choose two eigenvectors that are linearly independent

Therefore the simplest eigenvectors are $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

3 a $A=\begin{pmatrix}-3 & -1\\ 4 & -3\end{pmatrix}$

$A-\lambda I=\begin{pmatrix}-3 & -1\\ 4 & -3\end{pmatrix}-\begin{pmatrix}\lambda & 0\\ 0 & \lambda\end{pmatrix}=\begin{pmatrix}-3-\lambda & -1\\ 4 & -3-\lambda\end{pmatrix}$

$\det\left(A-\lambda I\right)=\begin{vmatrix}-3-\lambda & -1\\ 4 & -3-\lambda\end{vmatrix}$

$=\left(-3-\lambda\right)^2+4$

$\det\left(A-\lambda I\right)=0\Rightarrow=\left(-3-\lambda\right)^2+4=0$

$\lambda^2+6\lambda+13=0$

$\left(\lambda+3\right)^2+4=0$

$\lambda+3=\pm 2i$

$\lambda=-3\pm 2i$

Hence the eigenvalues are $\lambda=-3\pm 2i$

Taking $\lambda=-3+2i$

$\begin{pmatrix}-2 & 1\\ 2 & 4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\left(-3+2i\right)\begin{pmatrix}x\\ y\end{pmatrix}$

Equating upper elements:

$-3x-y=\left(-3+2i\right)x$

$-3x-y=-3x+2ix$

$x=-\frac{y}{2i}=\frac{iy}{2}$

Choosing $y=1$ gives $x=\frac{i}{2}$

So a required eigenvector is $\begin{pmatrix}\frac{i}{2}\\ 1\end{pmatrix}$

Taking $\lambda=-3-2i$

$\begin{pmatrix}-2 & 1\\ 2 & 4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\left(-3-2i\right)\begin{pmatrix}x\\ y\end{pmatrix}$

Equating upper elements:

$-3x-y=\left(-3-2i\right)x$

$-3x-y=-3x-2ix$

$y=2ix$

$x=\frac{y}{2i}=-\frac{iy}{2}$

Choosing $y=1$ gives $x=-\frac{i}{2}$

So a required eigenvector is $\begin{pmatrix}-\frac{i}{2}\\ 1\end{pmatrix}$

3 b $B=\begin{pmatrix}2 & -1\\ 2 & 4\end{pmatrix}$

$B-\lambda I=\begin{pmatrix}2 & -1\\ 2 & 4\end{pmatrix}-\begin{pmatrix}\lambda & 0\\ 0 & \lambda\end{pmatrix}=\begin{pmatrix}2-\lambda & -1\\ 2 & 4-\lambda\end{pmatrix}$

$\det(N-\lambda I)=\begin{vmatrix}2-\lambda & -1\\ 2 & 4-\lambda\end{vmatrix}$

$=(2-\lambda)(4-\lambda)+2=0$

$\det(N-\lambda I)=0$

$\lambda^2-6\lambda+10=0$

$(\lambda-3)^2+1=0$

$\lambda=3\pm i$

Hence the eigenvalues are $\lambda=3\pm i$

Taking $\lambda=3+i$

$\begin{pmatrix}-2 & 1\\ 2 & 4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=(3+i)\begin{pmatrix}x\\ y\end{pmatrix}$

Equating upper elements:

$2x-y=(3+i)x$

$2x-y=3x+ix$

$-y=x(1+i)$

$x=-\dfrac{y}{1+i}=\dfrac{-y+i}{2}$

Choosing $y=1$ gives $x=\dfrac{-1+i}{2}$

So a required eigenvector is $\begin{pmatrix}\dfrac{-1+i}{2}\\ 1\end{pmatrix}$

Taking $\lambda=3-i$

$\begin{pmatrix}-2 & 1\\ 2 & 4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=(3-i)\begin{pmatrix}x\\ y\end{pmatrix}$

Equating upper elements:

$2x-y=(3-i)x$

$2x-y=3x-ix$

$-y=x(1-i)$

$x=-\dfrac{y}{1-i}=\dfrac{-y-i}{2}$

Choosing $y=1$ gives $x=\dfrac{-1-i}{2}$

So a required eigenvector is $\begin{pmatrix}\dfrac{-1-i}{2}\\ 1\end{pmatrix}$

4 a $\mathbf{A}-\lambda\mathbf{I}=\begin{pmatrix}3 & 4\\ -2 & 9\end{pmatrix}-\begin{pmatrix}\lambda & 0\\ 0 & \lambda\end{pmatrix}=\begin{pmatrix}3-\lambda & 4\\ -2 & 9-\lambda\end{pmatrix}$

$$\begin{vmatrix}3-\lambda & 4\\ -2 & 9-\lambda\end{vmatrix}=(3-\lambda)(9-\lambda)+8$$

$$=27-12\lambda+\lambda^2+8=\lambda^2-12\lambda+35=(\lambda-5)(\lambda-7)$$

$$\det(\mathbf{A}-\lambda\mathbf{I})=0\Rightarrow(\lambda-5)(\lambda-7)=0\Rightarrow\lambda=5,7$$

The eigenvalues of A are 5 and 7.

b For $\lambda=5$

$$\begin{pmatrix}3 & 4\\ -2 & 9\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=5\begin{pmatrix}x\\ y\end{pmatrix}$$

$$\begin{pmatrix}3x+4y\\ -2x+9y\end{pmatrix}=\begin{pmatrix}5x\\ 5y\end{pmatrix}$$

Equating the upper elements

$$3x+4y=5x\Rightarrow 4y=2x\Rightarrow y=\frac{1}{2}x$$

For $\lambda=7$

$$\begin{pmatrix}3 & 4\\ -2 & 9\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=7\begin{pmatrix}x\\ y\end{pmatrix}$$

$$\begin{pmatrix}3x+4y\\ -2x+9y\end{pmatrix}=\begin{pmatrix}7x\\ 7y\end{pmatrix}$$

Equating the upper elements

$$3x+4y=7x\Rightarrow 4y=4x\Rightarrow y=x$$

Cartesian equations of the invariant lines are $y=\frac{1}{2}x$ and $y=x$.

5 $M=\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$

$$\begin{vmatrix}1-\lambda & 1\\ 0 & 1-\lambda\end{vmatrix}=0$$

$$(1-\lambda)^2=0$$

$$\lambda=1$$

Therefore $\lambda=1$ is a repeated eigenvalue.

$$\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\\ y\end{pmatrix}$$

Equating upper elements:

$x+y=x\Rightarrow y=0$

Choosing $x=1,$ the corresponding eigenvector will therefore be $\begin{pmatrix}1\\ 0\end{pmatrix}$

6 $A = \begin{pmatrix} 3 & k \\ 1 & -1 \end{pmatrix}$

$\begin{vmatrix} 3-\lambda & k \\ 1 & -1-\lambda \end{vmatrix} = 0$

$(3-\lambda)(-1-\lambda) - k = 0$

$\lambda^2 - 2\lambda - (3+k) = 0$

Repeated eigenvalue implies $b^2 - 4ac = 0$

$4 + 4(3+k) = 0$

$4k + 16 = 0$

$k = -4$

7 $M = \begin{pmatrix} 1 & -1 \\ k & -3 \end{pmatrix}$

$\begin{vmatrix} 1-\lambda & -1 \\ k & -3-\lambda \end{vmatrix} = 0$

$(1-\lambda)(-3-\lambda) + k = 0$

$\lambda^2 + 2\lambda + (k-3) = 0$

Complex eigenvalue implies $b^2 - 4ac < 0$

$4 - 4(k-3) < 0$

$4 - 4k + 12 < 0$

$16 < 4k$

$k > 4$

8 $A = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

$\begin{vmatrix} a-\lambda & b \\ -b & a-\lambda \end{vmatrix} = 0$

$(a-\lambda)^2 + b^2 = 0$

$(a-\lambda)^2 = -b^2$

$a - \lambda = \pm bi$

$\lambda = a \pm bi$ as required

9 Let T be the transformation matrix.

Then $T\begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -15 \\ -6 \end{pmatrix} = -3\begin{pmatrix} 5 \\ 2 \end{pmatrix}$

$T = \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$

and an eigenvector is $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and its corresponding eigenvalue is -3

10 a $\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} 2-\lambda & -1 \\ 1 & 2-\lambda \end{pmatrix}$

$\begin{vmatrix} 2-\lambda & -1 \\ 1 & 2-\lambda \end{vmatrix} = 0$

$= (2-\lambda)^2 + 1 = 0$

$2 - \lambda = \pm i$

$\lambda = 2 \pm i,$ so there are no real eigenvalues.

b To find the eigenvectors:

Taking $\lambda = 2 + i$

$\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = (2+i)\begin{pmatrix} x \\ y \end{pmatrix}$

Equating upper elements:

$2x - y = (2+i)x$

$y = -ix$

Choosing $y = 1$ gives $x = \frac{-1}{i} = i$

So a required eigenvector is $\begin{pmatrix} i \\ 1 \end{pmatrix}$

Taking $\lambda = 2 - i$

$\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = (2+i)\begin{pmatrix} x \\ y \end{pmatrix}$

Equating upper elements:

$2x - y = (2-i)x$

$y = ix$

Choosing $y = 1$ gives $x = \frac{1}{i} = -i$

So a required eigenvector is $\begin{pmatrix} -i \\ 1 \end{pmatrix}$

Neither eigenvector corresponds to a straight line in $\mathbb{R}^2$, so the transformation has no invariant lines.

11 a $M=\begin{pmatrix}-\frac{3}{5} & \frac{4}{5}\\ \frac{4}{5} & \frac{3}{5}\end{pmatrix}$

$M-\lambda I=\begin{pmatrix}-\frac{3}{5} & \frac{4}{5}\\ \frac{4}{5} & \frac{3}{5}\end{pmatrix}-\begin{pmatrix}\lambda & 0\\ 0 & \lambda\end{pmatrix}=\begin{pmatrix}-\frac{3}{5}-\lambda & \frac{4}{5}-\lambda\\ \frac{4}{5}-\lambda & \frac{3}{5}-\lambda\end{pmatrix}$

$\det(M-\lambda I)=\begin{vmatrix}-\frac{3}{5}-\lambda & \frac{4}{5}-\lambda\\ \frac{4}{5}-\lambda & \frac{3}{5}-\lambda\end{vmatrix}$

$=\left(-\frac{3}{5}-\lambda\right)\left(\frac{3}{5}-\lambda\right)-\frac{16}{25}$

$\det(M-\lambda I)=0\Rightarrow\left(-\frac{3}{5}-\lambda\right)\left(\frac{3}{5}-\lambda\right)-\frac{16}{25}=0$

$\lambda^2-1=0$

$\lambda^2=1$

$\lambda=\pm1$

Hence the eigenvalues are $\lambda=\pm1$

To find the eigenvectors:

Taking $\lambda=1$

$\begin{pmatrix}-\frac{3}{5} & \frac{4}{5}\\ \frac{4}{5} & \frac{3}{5}\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\\ y\end{pmatrix}$

Equating upper elements:

$-\frac{3x}{5}+\frac{4y}{5}=x$

$y=2x$

Choosing $x=1$ gives $y=2$

So a required eigenvector is $\begin{pmatrix}1\\ 2\end{pmatrix}$

Taking $\lambda=-1$

$\begin{pmatrix}-\frac{3}{5} & \frac{4}{5}\\ \frac{4}{5} & \frac{3}{5}\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=-\begin{pmatrix}x\\ y\end{pmatrix}$

Equating upper elements:

$-\frac{3x}{5}+\frac{4y}{5}==x$

$x=2y$

Choosing $x=2$ gives $y=1$

So a required eigenvector is $\begin{pmatrix}-2\\ 1\end{pmatrix}$

11 b $\begin{pmatrix}1\\2\end{pmatrix}.\begin{pmatrix}-2\\1\end{pmatrix}=-2+2=0$

Therefore the eigenvectors are perpendicular

c $y=2x$ corresponds to the eigenvector $\begin{pmatrix}1\\2\end{pmatrix}$, and $\lambda=1$

so every point on the line $y=2x$ is invariant

d $M^2=\begin{pmatrix}-\frac{3}{5} & \frac{4}{5}\\ \frac{4}{5} & \frac{3}{5}\end{pmatrix}\begin{pmatrix}-\frac{3}{5} & \frac{4}{5}\\ \frac{4}{5} & \frac{3}{5}\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, the identity matrix.

Therefore M must be a reflection in the line $y=2x$.

12 Suppose λ is an eigenvalue of matrix A and x is its associated eigenvector.
Then $Ax=\lambda x$

Therefore $A^2x=A(Ax)=A(\lambda x)=\lambda(Ax)=\lambda(\lambda x)=\lambda^2 x$

Therefore λ^2 is an eigenvalue of A^2

13 Let $M=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ and let $\mathbf{x}=\begin{pmatrix}x\\y\end{pmatrix}$

You are given $M\mathbf{x}=0$

So $\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=0$

Equating upper elements: $ax+by=0\Rightarrow y=-\frac{ax}{b}$

Equating lower elements: $cx+dy=0\Rightarrow y=-\frac{cx}{d}$

Therefore $\frac{a}{b}=\frac{c}{d}$

$ad=bc$

$ad-bc=0$

Therefore $\det M=0$

Therefore M is singular

Challenge

$T = \begin{pmatrix} -1 & 0 \\ -2 & 1 \end{pmatrix}$

$\begin{vmatrix} -1-\lambda & 0 \\ -2 & 1-\lambda \end{vmatrix} = 0$

$(-1-\lambda)(1-\lambda) = 0$

Therefore the eigenvalues are $\lambda = 1$ and $\lambda = -1$

Taking $\lambda = 1$

$\begin{pmatrix} -1 & 0 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$

Equating upper elements:

$-x = x \Rightarrow x = 0$

Choosing $y = 1$, the corresponding eigenvector will therefore be $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Therefore all points on the y-axis are invariant.

Taking $\lambda = -1$

$\begin{pmatrix} -1 & 0 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -\begin{pmatrix} x \\ y \end{pmatrix}$

Equating lower elements:

$-2x + y = -y \Rightarrow x = y$

Choosing $y = 1$, the corresponding eigenvector will therefore be $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Therefore all lines parallel to $y = x$ will stay parallel to $y = x$ under T

Since every line will cross the $y-$axis at one point, and this point is invariant under T,

every line of the form $y = x + k$ is an invariant line of T, and there are infinitely many of these.