Decision analysis Mixed exercise

1 The structure of the decision tree shows the decisions and possible outcomes using the following notation:

p/~*p*: play the game or not.

Q/T/O: card is a queen (Q), a 3, 4 or 5 (T) or anything else (O).

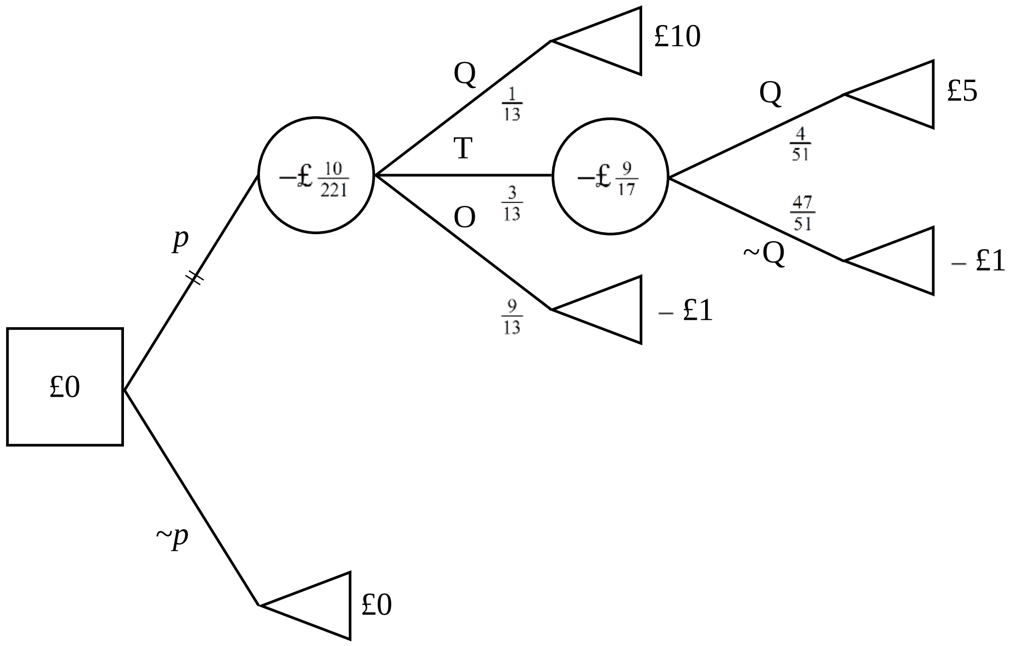
The probability of selecting a queen on the 1st selection is $\frac{4}{52}=\frac{1}{13}$.

The probability of selecting a 3, 4, or 5 on the 1st selection is $\frac{12}{52}=\frac{3}{13}$.

The probability of selecting any other card on the 1st selection is $1-\left(\frac{1}{13}+\frac{3}{13}\right)=\frac{9}{13}$.

If a 3, 4 or 5 are selected on the 1st selection, the probability of selecting a queen on the 2nd selection (without replacement) is $\frac{4}{51}$.

Example EMV: the far right EMV is $\left(\frac{4}{51}\times £5\right)+\left(\frac{47}{51}\times -£1\right)=-£\frac{9}{17}$.



Since $-£\frac{10}{221}$ < £0 Jacob should not play the game.

2 a The structure of the decision tree shows the decisions and possible outcomes using the following notation:

I/~I: company should offer insurance cover or not.

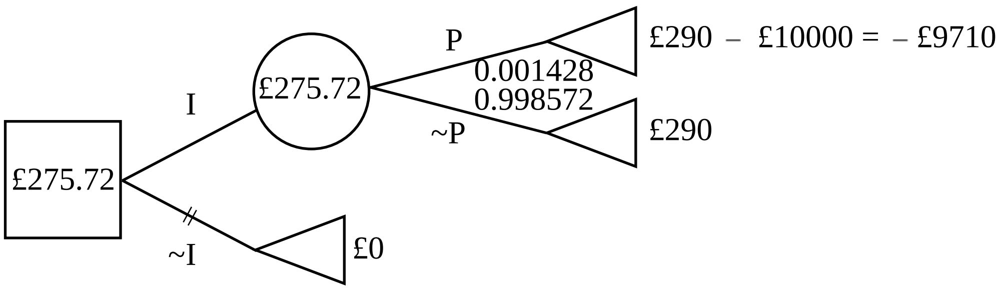
P/~P: pay-out or not.

P(pay-out) = $\frac{1}{6^7}$.

P(~pay-out) = $1 - \frac{1}{6^7}$.

If there are 400 throws, the P(someone wins) = $1-\left(1-\frac{1}{6^7}\right)^{400} = 0.001428$ (4 s.f.)

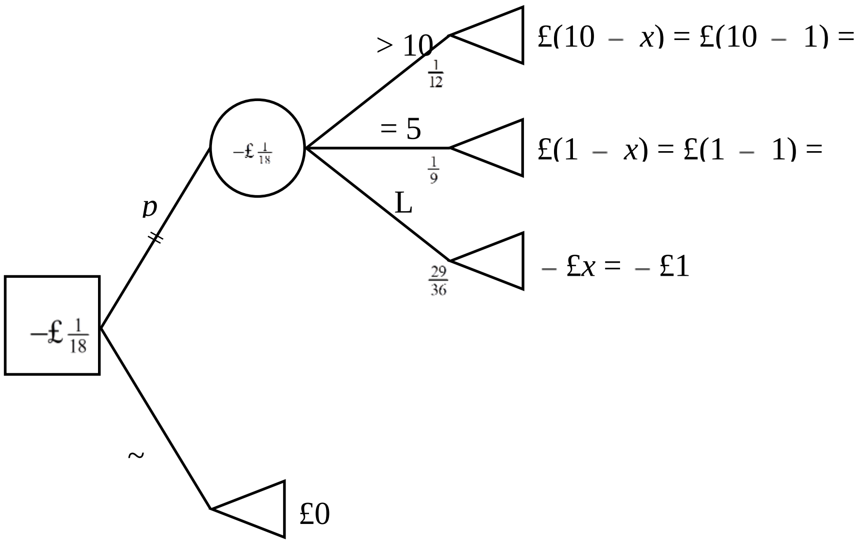
Example EMV: the top EMV is (0.001428 × –£9710) + (0.998572 × £290) = £275.72.



Since £275.72 > £0 the company should offer the cover.

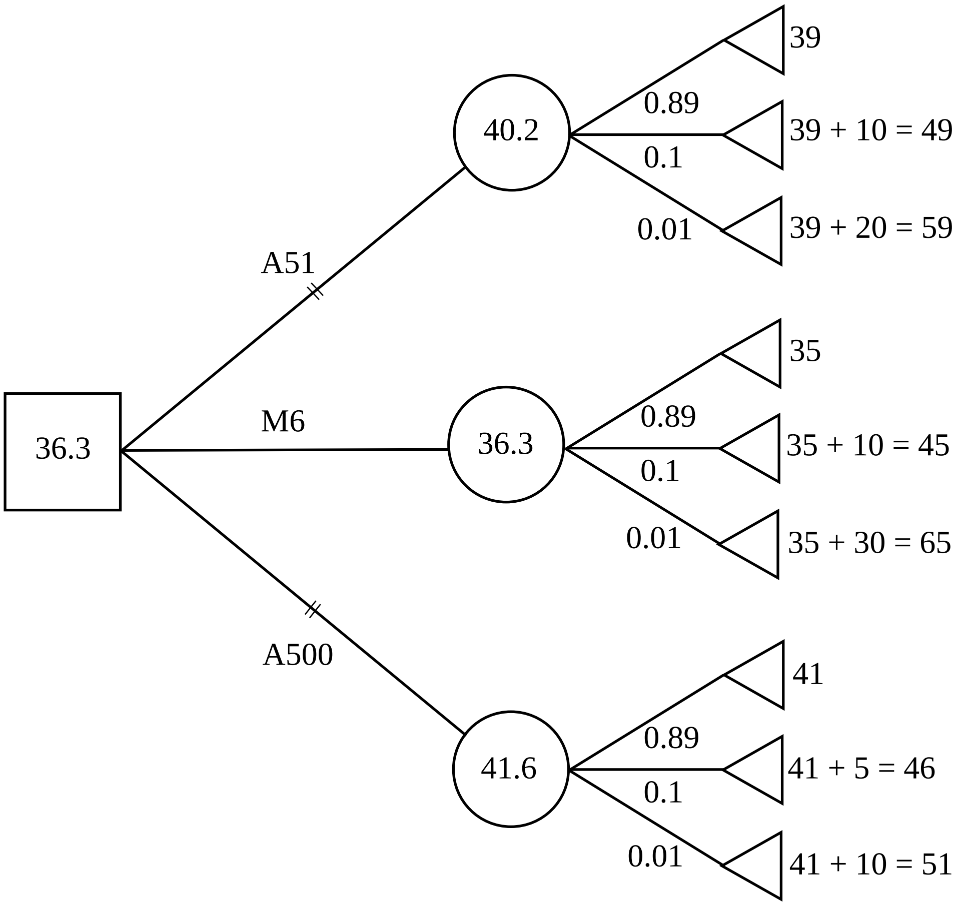
b The EMV criterion is appropriate here because it represents the long-term return to the company, which will be able to deal with the occasional loss.

3 The structure of the decision tree shows the decisions and possible outcomes using the following notation:
p/~*p*: play the game or not.
> 10/= 5/L: a score of > 10, = 5 or a loss (L).
There are 36 outcomes in total, 3 of which result in a score of more than 10 (5, 6), (6, 5) and (6, 6).
Therefore, the probability of scoring more than 10 is $\frac{3}{36}=\frac{1}{12}$.
4 of the 36 outcomes result in a score of exactly 5 (1, 4), (2, 3), (3, 2) and (4, 1). Therefore, the probability of scoring exactly 5 is $\frac{4}{36}=\frac{1}{9}$.
The probability of losing = $1-\left(\frac{1}{12}+\frac{1}{9}\right)=\frac{29}{36}$.
Example EMV: the top EMV is
$$\left(\frac{1}{12}\times £(10-x)\right)+\left(\frac{1}{9}\times £(1-x)\right)+\left(\frac{29}{36}\times -£x\right)=£\left(\frac{17}{18}-x\right)=£\left(\frac{17-18x}{18}\right)=£\left(\frac{17-18(1)}{18}\right)=-£\frac{1}{18}.$$



Since $-£\frac{1}{18}$ < £0 Liam should not play the game if x = £1.

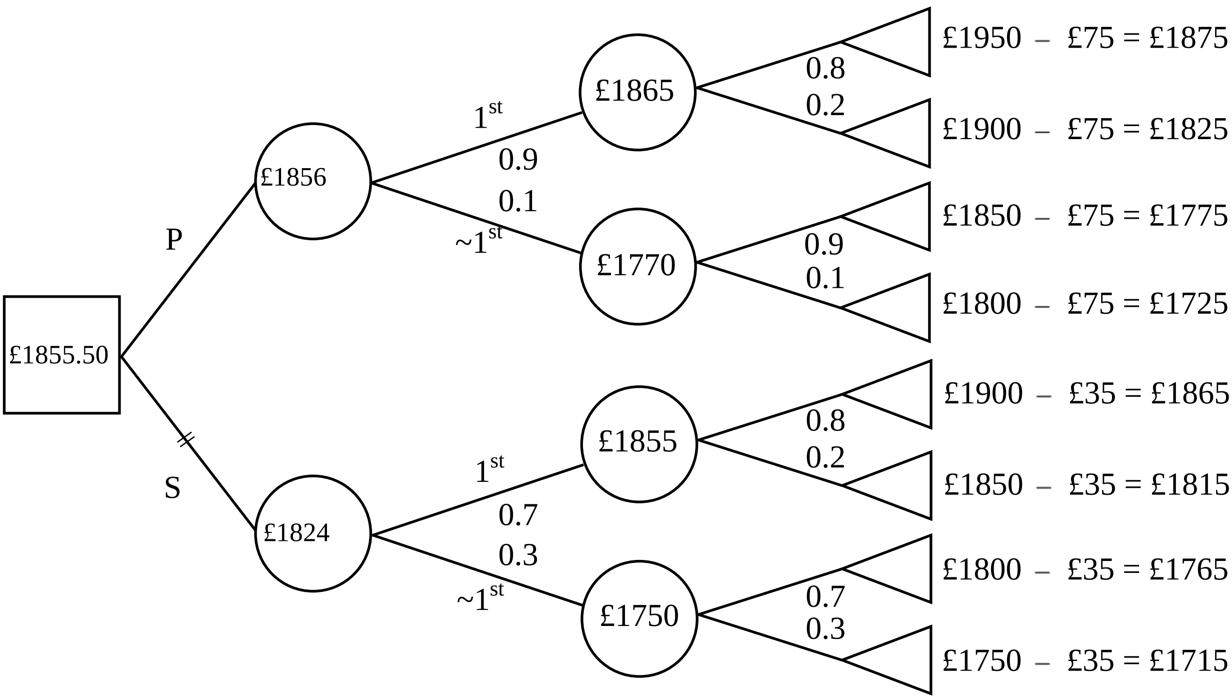
4 a The structure of the decision tree shows the decisions and possible outcomes using the following notation:
A51/M6/A500: take the A51, M6 or A500.
All pay-offs are journey times and are shown in minutes.
Example EMV: the top EMV is (0.89 × 39) + (0.1 × 49) + (0.01 × 59) = 40.2 minutes.



b Minimum expected time = 36.3 minutes on the M6.

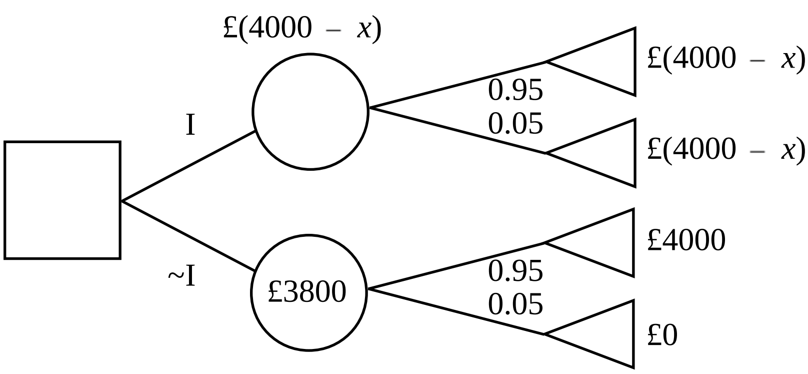
c If it is important to arrive on time, then it may be better to choose a route where the maximum expected delay is minimised.

5 a The structure of the decision tree shows the decisions and possible outcomes using the following notation:
P/S: premium or standard package.
1st/~1st: car sells in 1st week or not.
Example EMV: the top right EMV is (0.8 × £1875) + (0.2 × £1825) = £1865.



b Optimum EMV = £1856 for the premium package.

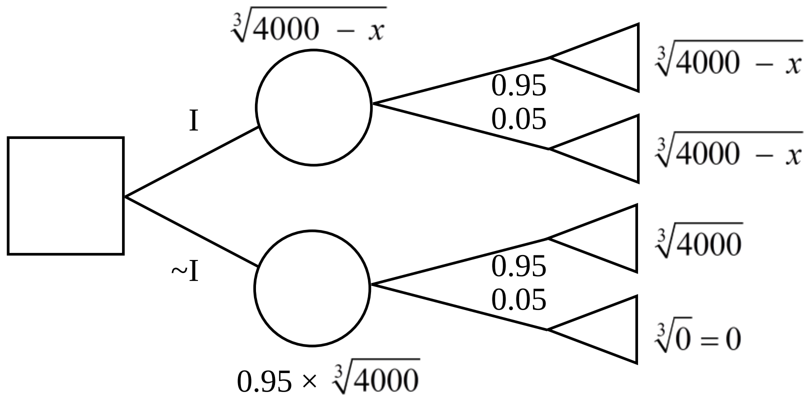
6 a The structure of the decision tree shows the decisions and possible outcomes using the following notation:
I/~I: insure the camera or not.
£x = the cost of the insurance premium.
Example EMV: the bottom EMV is (0.95 × £4000) + (0.05 × £0) = £3800.



b The maximum that Zoe should pay for insurance is given by solving the following inequality for x.
$4000 - x > 3800$.
$-x > 3800 - 4000$.
$-x > -200$.
$x <$ £200.
Therefore, Zoe should pay a maximum of £200 for insurance.

6 c The revised decision tree using utilities is as follows.

Example expected utility: the bottom expected utility is $(0.95 \times \sqrt[3]{4000}) + (0.05 \times 0) = 0.95 \times \sqrt[3]{4000}$.



The maximum that Zoe should pay for insurance is given by solving the following inequality for x.

$\sqrt[3]{4000 - x} > 0.95 \times \sqrt[3]{4000}$.

$4000 - x > (0.95)^3 \times 4000$.

$4000 - x > 3429.5$.

$-x > -570.5$.

$x <$ £570.50.

Therefore, Zoe should pay a maximum of £570.50 for insurance.

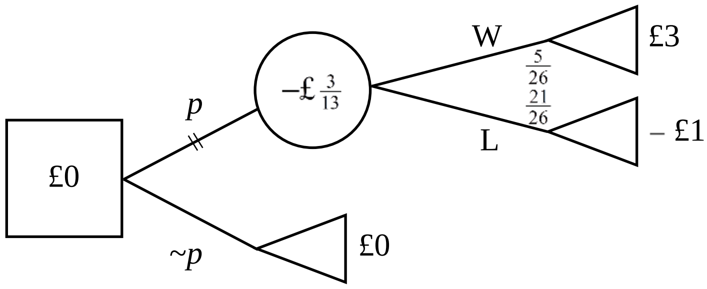
7 a The structure of the decision tree shows the decisions and possible outcomes using the following notation:

p/~p: play the game or not.

W/L: Joe wins (W) or loses (L) the game.

P(winning) $= \frac{10}{52} = \frac{5}{26}$ as 10 of the 52 cards are red even numbers (2, 4, 6, 8 and 10 in the hearts and diamonds suits).

Example EMV: the top EMV is $\left(\frac{5}{26} \times £3\right) + \left(\frac{21}{26} \times -£1\right) = -£\frac{3}{13}$.



Since $-£\frac{3}{13} <$ £0 Joe should not play the game.

7 b The prize amount is given as £x. If Joe plays the game and wins, he will then have £(x + 1). If he plays and loses, he will have £0. If he chooses not to play, then he will still have £1.

The corresponding utility values are $\sqrt[3]{(x+1)^2}$, 0, $\sqrt[3]{1^2}=1$.

The probability of winning the game = $\frac{5}{26}$. The expected utility of playing is $\frac{5}{26}\times\sqrt[3]{(x+1)^2}>1$.

$(x+1)^2>\left(\frac{26}{5}\right)^3$.

$x>\sqrt{\left(\frac{26}{5}\right)^3}-1$.

$x>$ £10.86.

Therefore, the minimum value of x (winning amount) that makes the game worthwhile to Joe is £10.86.

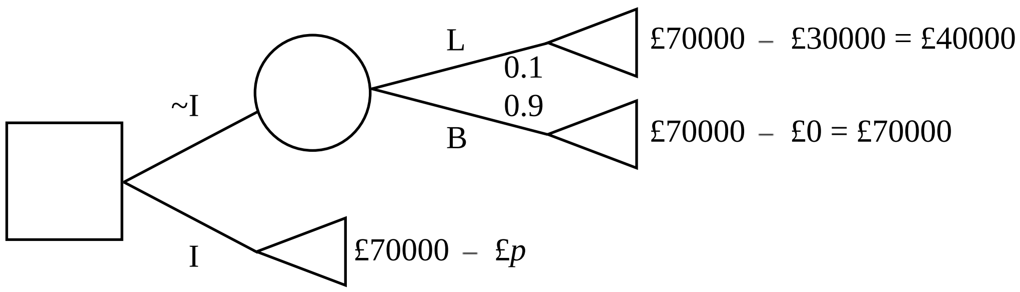
8 a The purpose of a utility function, in decision analysis, is to provide a customisable way to compare the value of outcomes taking into account, for example, the degree of aversion to risk.

b The structure of the decision tree shows the decisions and possible outcomes using the following notation:

I/~I: company should pay for insurance or not.

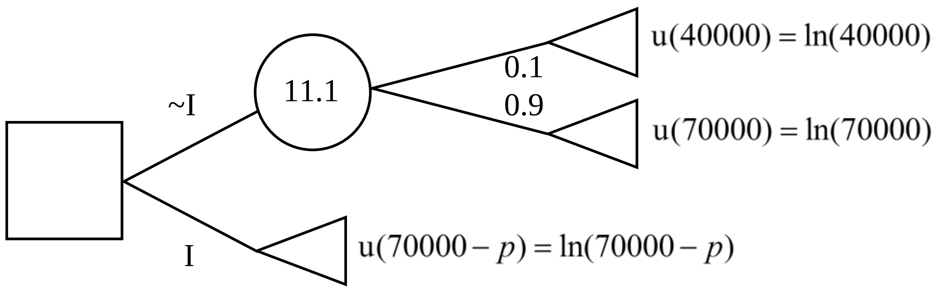
L/B: loss (L) or break-even (B).

£p = the cost of the insurance.



The revised decision tree using utilities is as follows:

The expected utility is: (0.1 × ln(40 000)) + (0.9 × ln(70 000)) = 11.1 utils (3 s.f.).



The maximum the company should pay for insurance is given by solving the following inequality for p.

$\ln(70\,000-p)>11.1$.

$70\,000-p>e^{11.1}$.

$-p>-3809.72$.

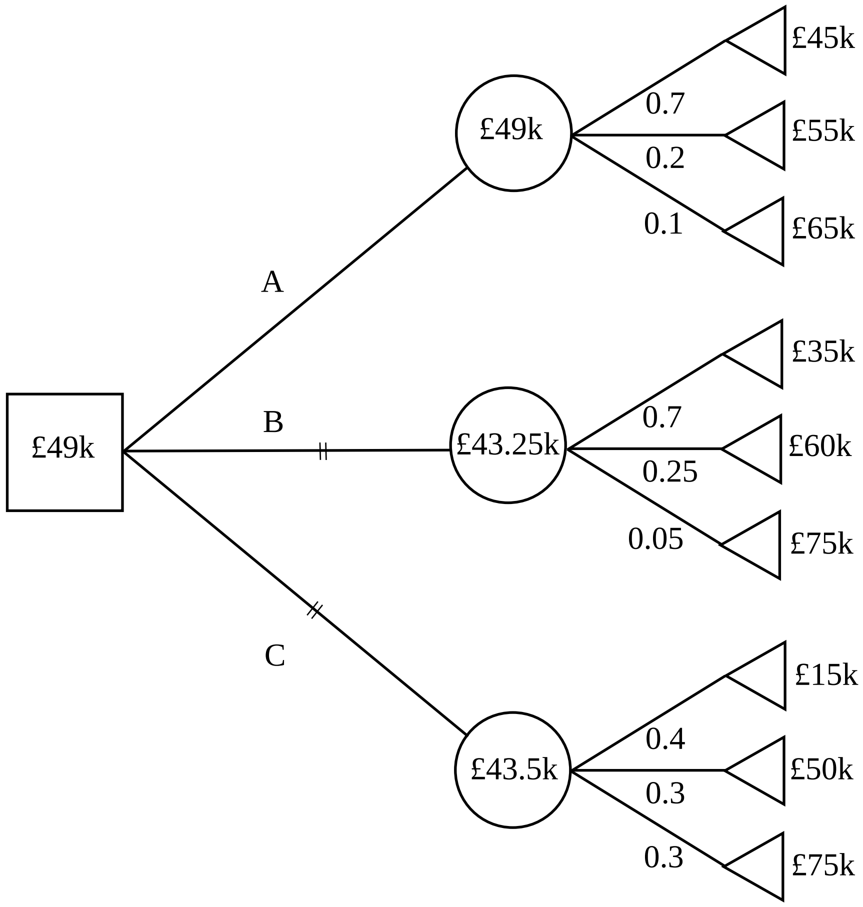
$p<$ £3810.

Therefore, the company should pay a maximum of £3810 for insurance.

Challenge

a The utility function is designed to prevent the possibility of a very high profit having too much influence on decisions.

b The structure of the decision tree shows the decisions and possible outcomes using the following notation:
A/B/C: projects A, B and C.
The pay-offs are shown as utilities.
Example expected utility: top E(U) is (0.7 × £45k) + (0.2 × £55k) + (0.1 × 65k) = £49k.



Optimum expected utility = £49 000 for project A.