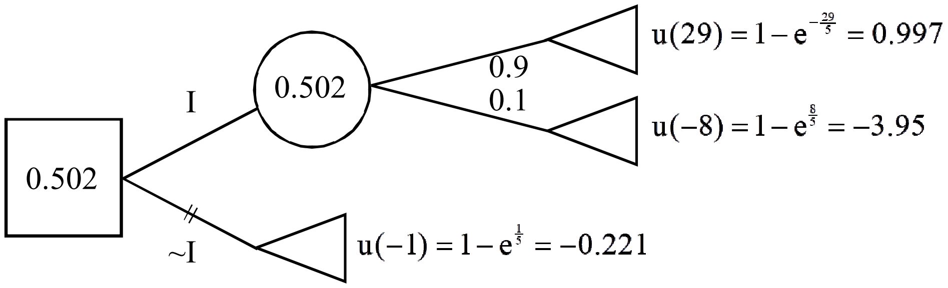
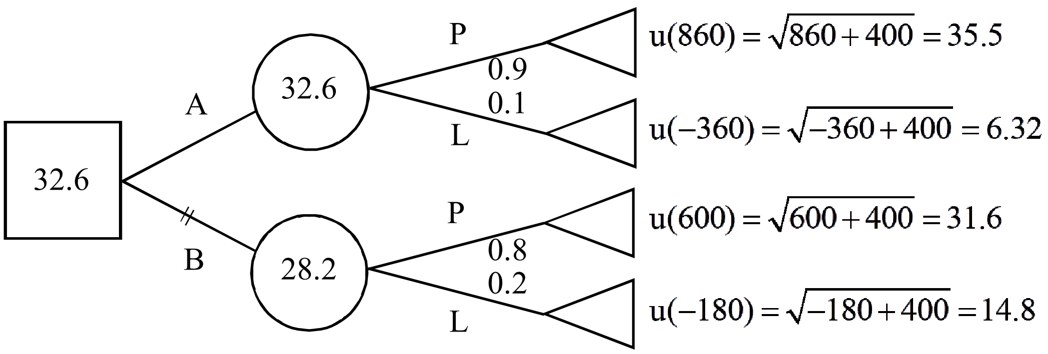
Decision analysis 8B

1 The structure of the decision tree shows the decisions and possible outcomes using the following notation:
I/~I: invest or not.
All pay-offs are in units of utilities (utils) and are calculated to 3 significant figures.
Example expected utility: (0.9 × 0.997) + (0.1 × – 3.95) = 0.502 utils (3 s.f.).



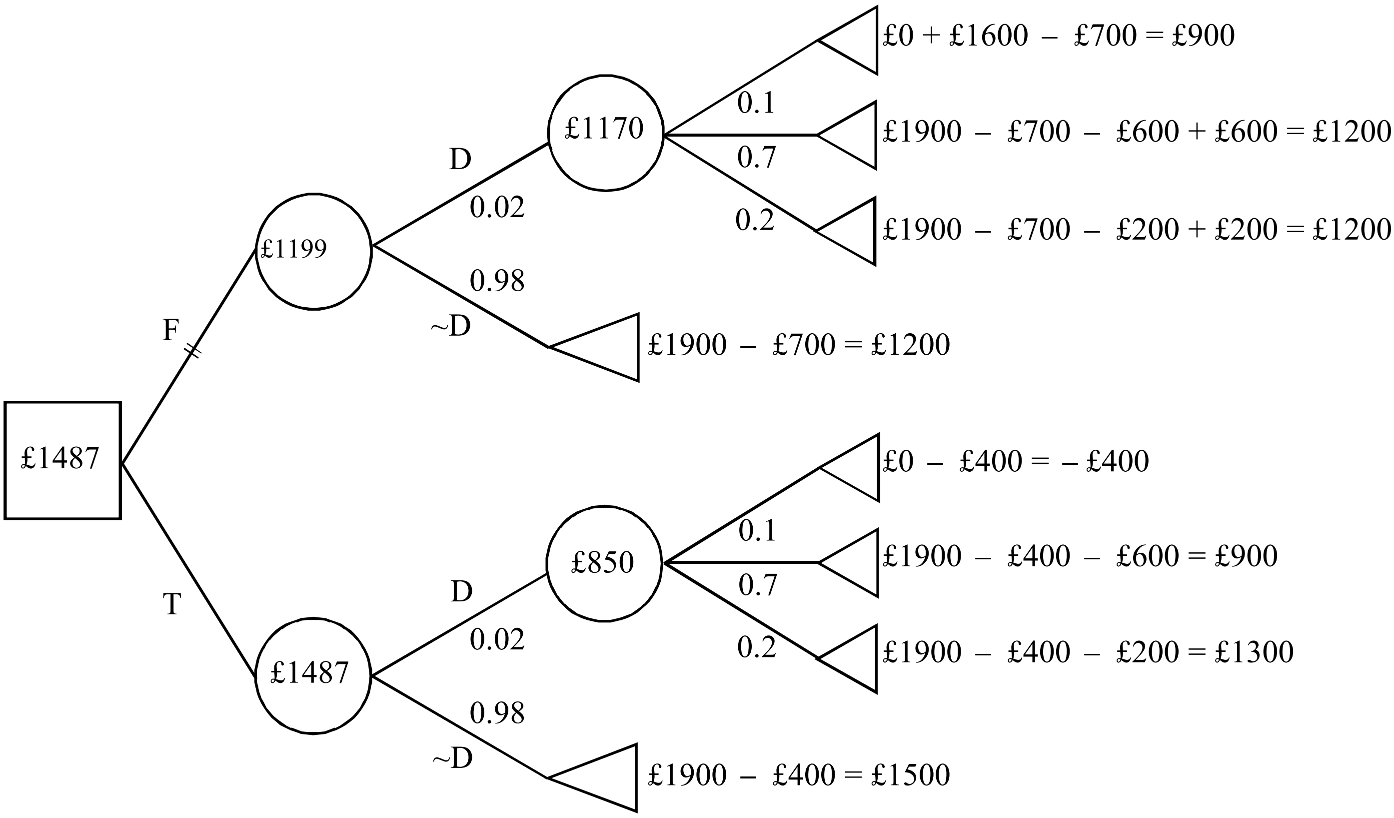
Since 0.502 > – 0.221 the investment should be made.

2 **a** The structure of the decision tree shows the decisions and possible outcomes using the following notation:
A/B: projects A and B.
P/L: profit (P) and loss (L).
All pay-offs are in units of utilities (utils) and are calculated to 3 significant figures.
Example expected utility: top E(U) is (0.9 × 35.5) + (0.1 × 6.32) = 32.6 utils (3 s.f.).



b Since 32.6 > 28.2 it is best to invest in project A.

3 a The structure of the decision tree shows the decisions and possible outcomes using the following notation:
F/T: fully comp (F) or third-party (T) insurance.
D/~D: damage or no damage.
EMV example: The top EMV is (0.1 × £900) + (0.7 × £1200) + (0.2 × £1200) = £1170.

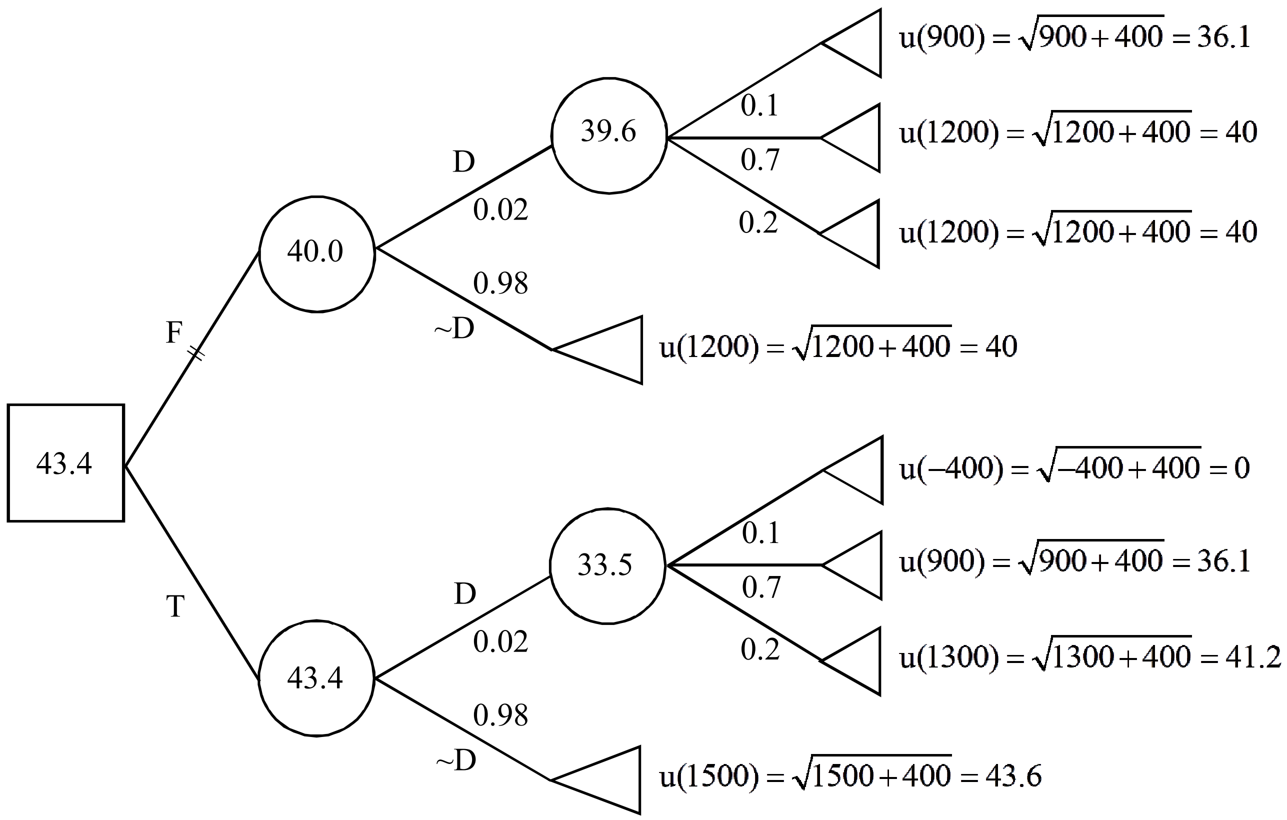


b Since £1487 > £1199.40 third-party insurance is the better option.

c The EMV represents the average or long-term view. However, there is a risk that Sally could lose her car and may not be able to replace it and this is reflected in the EMV. The worst-case scenario is that Sally takes out the third-party insurance and damage beyond repair occurs to the car. This would result in an EMV value of – £400.

3 d The modified decision tree below shows expected utilities.

All pay-offs are in units of utilities (utils) and are calculated to 3 significant figures.
Example expected utility: top E(U) is (0.1 × 36.1) + (0.7 × 40) + (0.2 × 40) = 39.6 utils (3 s.f.).



Since 43.4 > 40 third-party insurance is also the better option using expected utility.

4 The prize amount is given as £x.
If Amy plays the game and wins, she will then have £(x + 2).
If she plays and loses, she will have £0.
If she chooses not to play, then she will still have £2.

The corresponding utility values are $\sqrt[3]{(x+2)^2}$, 0, $\sqrt[3]{2^2} = \sqrt[3]{4}$.

The probability of winning the game is P(rolling a 5 or 6) = $\frac{2}{6} = \frac{1}{3}$.

The expected utility of playing is $\frac{1}{3} \times \sqrt[3]{(x+2)^2} > \sqrt[3]{4}$.

$(x+2)^2 > 3^3 \times 4$.

$x > \sqrt{108} - 2$.

$x > 8.39$ to 3 significant figures.
Therefore, the minimum value of x is 8.39.

Challenge

a Expected profit = expected winnings – cost of the lottery.

Expected winnings

$$= \left(\frac{1}{56} \times £10\right) + \left(\frac{1}{1032} \times £50\right) + \left(\frac{1}{55\,419} \times £2000\right) + \left(\frac{1}{2\,330\,636} \times £100\,000\right)$$
$$+ \left(\frac{1}{13\,983\,816} \times £15\,000\,000\right) = £1.379$$

to 4 significant figures.

Expected profit = £1.379 – £1 = £0.379 to 3 significant figures.
As the expected profit is > £0 Daniel should play the lottery game.

b Utility function given by:

$$u(x) = \begin{cases} 2x & 0 \leqslant x \leqslant 50\,000 \\ 50\,000 + x & 50\,000 \leqslant x \leqslant 1\,000\,000 \\ 1\,000\,000 & x \geqslant 1\,000\,000 \end{cases}$$

c Expected winnings converted to utilities:

$u(10) = 2(10) = 20$ utils.
$u(50) = 2(50) = 100$ utils.
$u(2000) = 2(2000) = 4000$ utils.
$u(100\,000) = 50\,000 + 100\,000 = 150\,000$ utils.
$u(15\,000\,000) = 1\,000\,000$ utils.

Cost of playing the lottery converted to utilities:
$u(1) = 2(1) = 2$ utils.

Expected utility

$$= \left(\frac{1}{56} \times 20\right) + \left(\frac{1}{1032} \times 100\right) + \left(\frac{1}{55\,419} \times 4000\right) + \left(\frac{1}{2\,330\,636} \times 150\,000\right) + \left(\frac{1}{13\,983\,816} \times 1\,000\,000\right)$$
$$= 0.6621$$

utils to 4 significant figures.

Expected utility profit = 0.6621 – 2 = – 1.34 to 3 significant figures.
As the expected utility profit is negative Daniel should not play the lottery game.