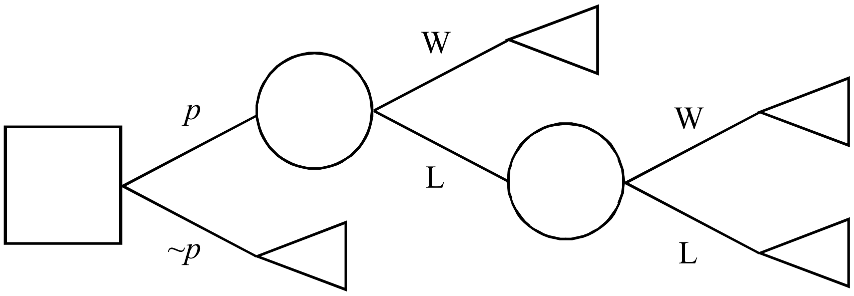
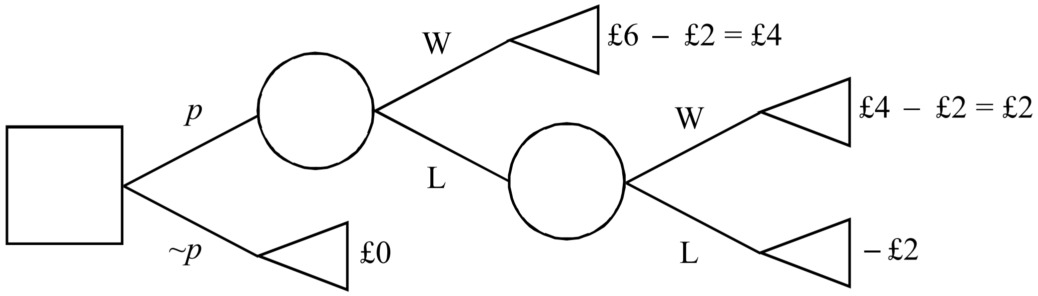
Decision analysis 8A

1 a The structure of the decision tree shows the decisions and possible outcomes using the following notation:
$p/{\sim}p$: play the game or not.
W: all three coins land the same way up (a win).
L: all three coins do not land the same way up (a loss).

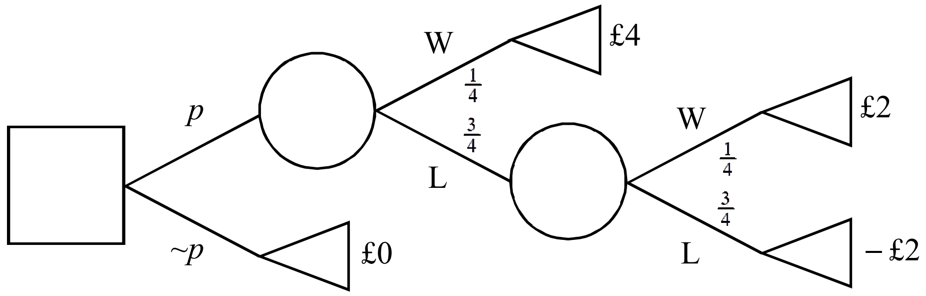


Note that a second decision node is not required after losing the first game as the second attempt is free. Therefore, you would always play again.
The pay-offs can now be added to the decision tree (note that pay-off is given by profit = winnings − amount paid):



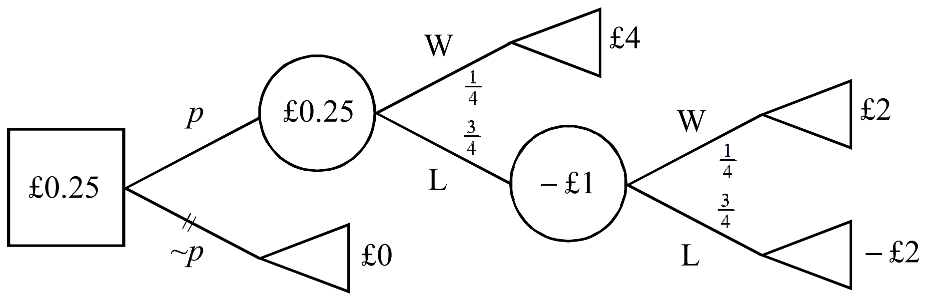
Probabilities are now added to the branches to the right of each chance node.

The probability of winning is given by spinning 3 heads or 3 tails = $\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)=\frac{1}{4}$.



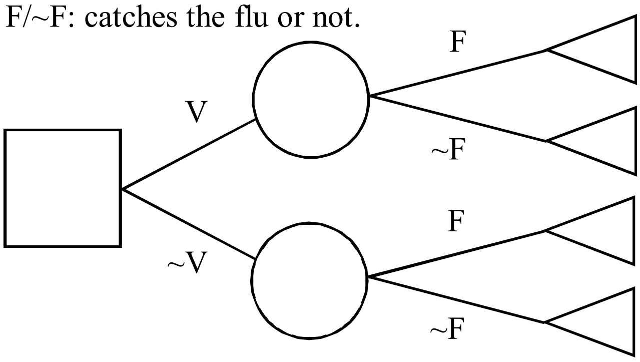
Add EMVs to the chance nodes to complete the decision tree.

EMVs from right to left: $\left(\frac{1}{4}\times £2\right)+\left(\frac{3}{4}\times -£2\right)=-£1$, $\left(\frac{1}{4}\times £4\right)+\left(\frac{3}{4}\times -£1\right)=£0.25$.

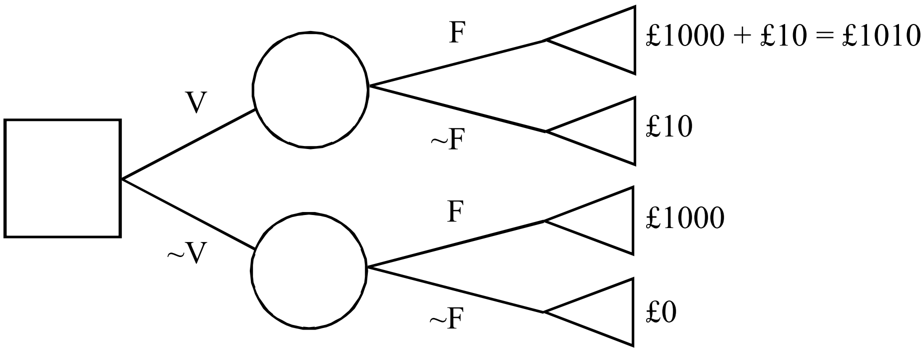


1 b Since £0.25 > £0 Claire should play the game.

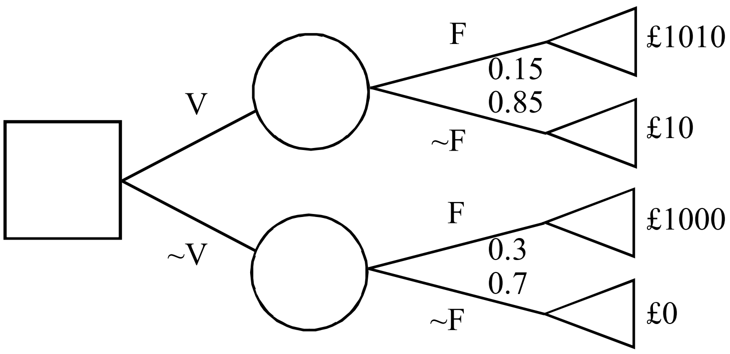
2 a The structure of the decision tree shows the decisions and possible outcomes using the following notation:
V/~V: have the flu vaccination or not.
F/~F: catches the flu or not.



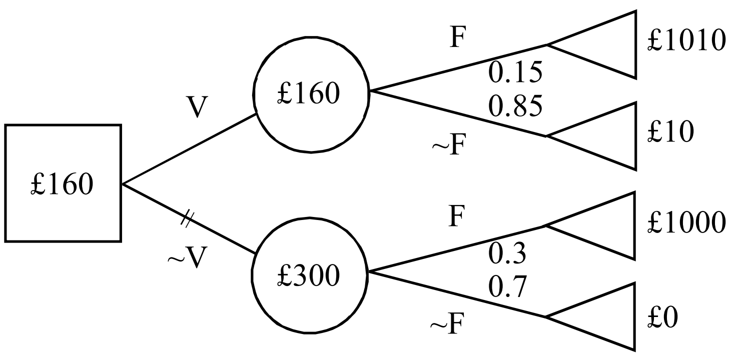
The pay-offs can now be added to the decision tree (the pay-offs in this case are the costs to Stephen. For example, if Stephen has the vaccination and catches the flu, the total cost to him is £10 for the vaccination and an estimated £1000 in lost earnings, which is £1010 in total):



Probabilities are now added to the branches to the right of each chance node.
The probability of Stephen catching the flu if he is vaccinated is 0.5 × 0.3 = 0.15.

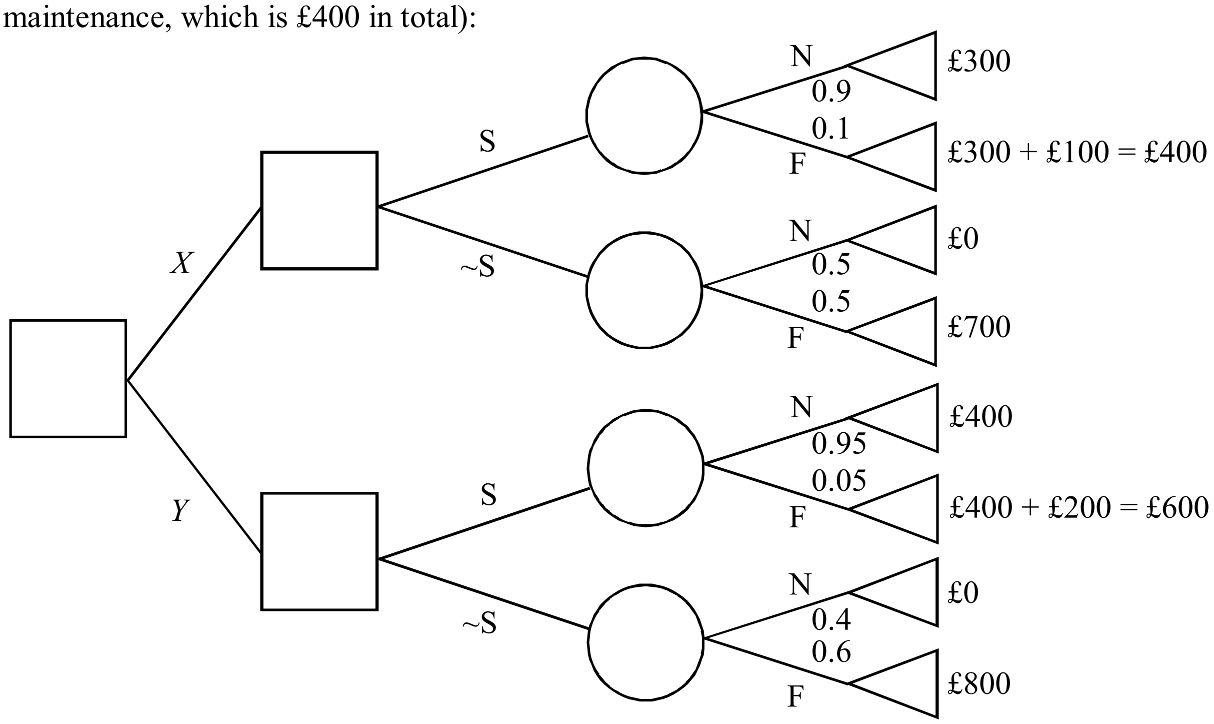


Add EMVs to the chance nodes to complete the decision tree.
Example EMV: the top EMV is (0.15 × £1010) + (0.85 × £10) = £160.

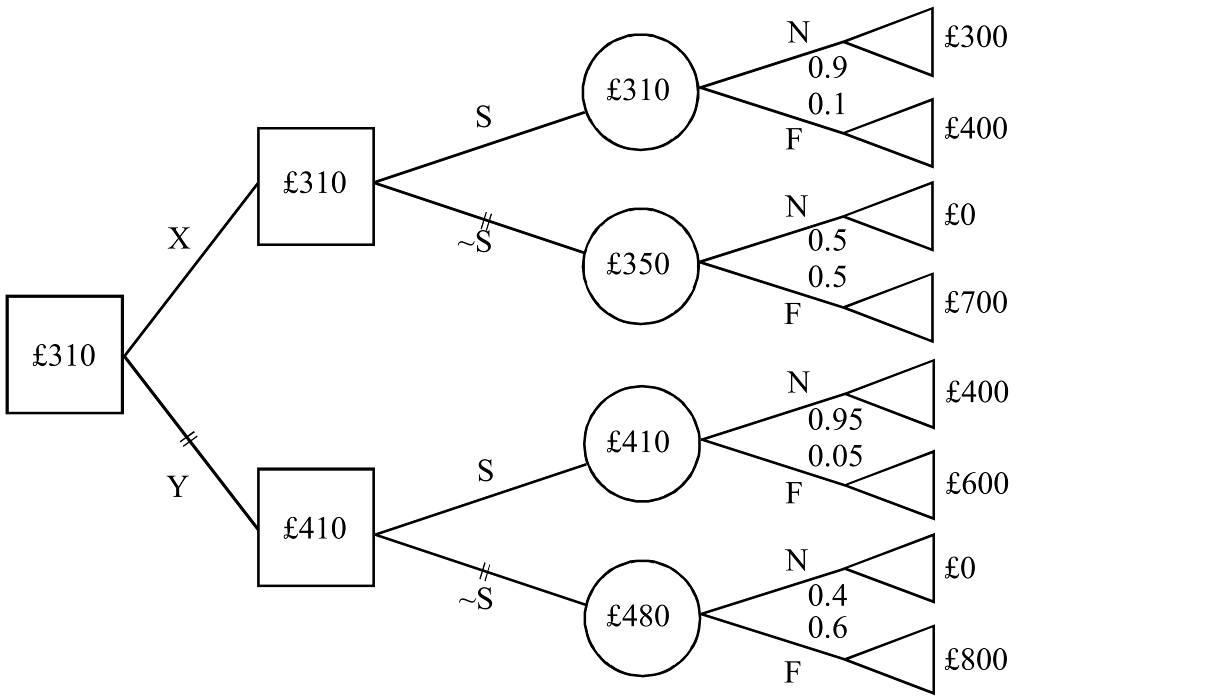


2 b Since £160 < £300 it is more cost-effective for Stephen to have the vaccination.

3 a The structure of the decision tree shows the decisions and possible outcomes using the following notation:
X/*Y*: car *X* and *Y*.
S/~S: have the service or not.
F/N: further costs (F) and no further costs (N).
The pay-offs and probabilities are also shown on the decision tree (the pay-offs in this case are the maintenance costs to Jemima. For example, if Jemima buys car *X*, has it serviced and it requires further maintenance, then the total cost to her is £300 for the service and £100 for further maintenance, which is £400 in total):

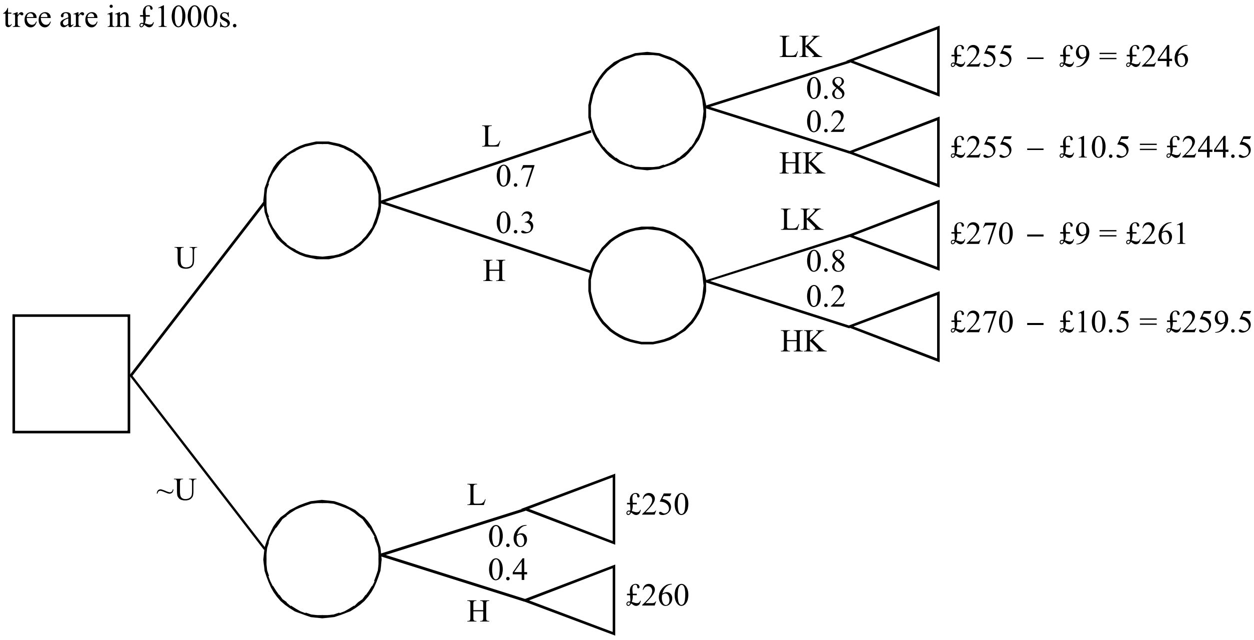


Add EMVs to the chance nodes to complete the decision tree.
Example EMV: the top right EMV is (0.9 × £300) + (0.1 × £400) = £310.

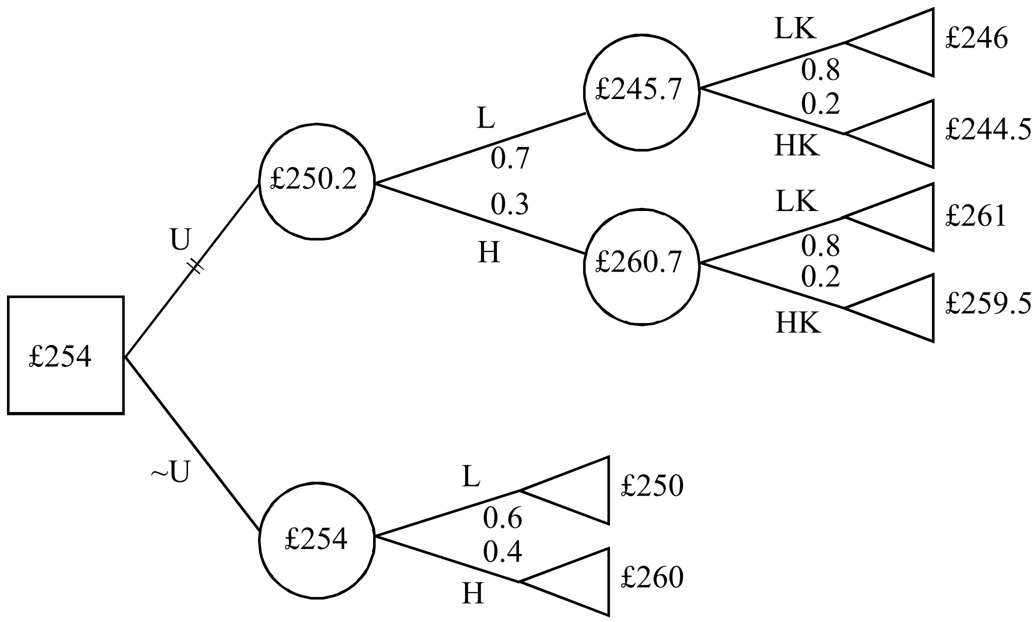


b Since £310 < £410 Jemima's best strategy is to buy car *X* and pay for it to be serviced.

4 a The structure of the decision tree shows the decisions and possible outcomes using the following notation:
U/~U: update the property or not.
L/H: lower pay-off (L) and higher pay-off (H).
LK/HK: lower kitchen cost (LK) and higher kitchen cost (HK).
The pay-offs and probabilities are also shown on the decision tree (the pay-offs in this case are the profit from the sale of the house after home improvements have been paid for. For example, if kitchen improvements are made, the house sells for £255000 and the new kitchen costs £9000, the total profit made from selling the property is £255000 − £9000 = £246000). All pay-offs on the tree are in £1000s.

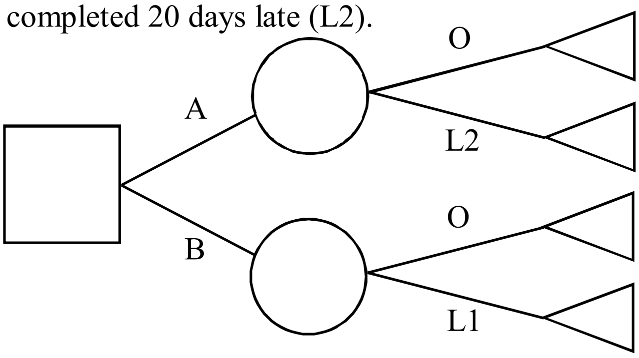


Add EMVs to the chance nodes to complete the decision tree.
Example EMV: the top EMV is (0.8 × £246) + (0.2 × £244.5) = £245.70.

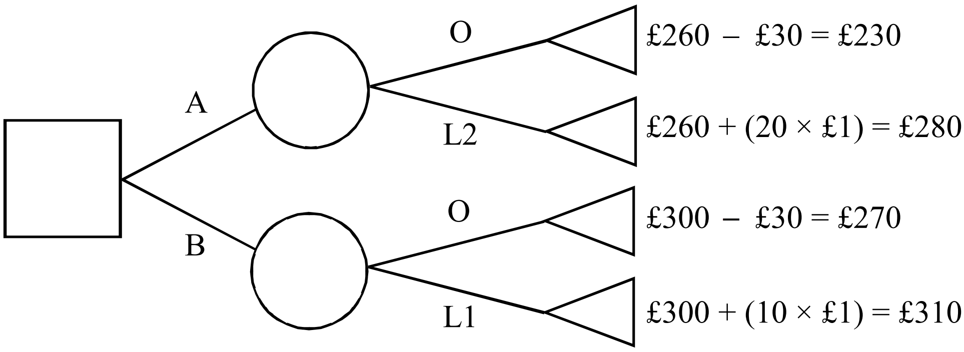


b Since £254 > £250.20 the best strategy in order to maximise the EMV on the sale of the house is not to update the kitchen.

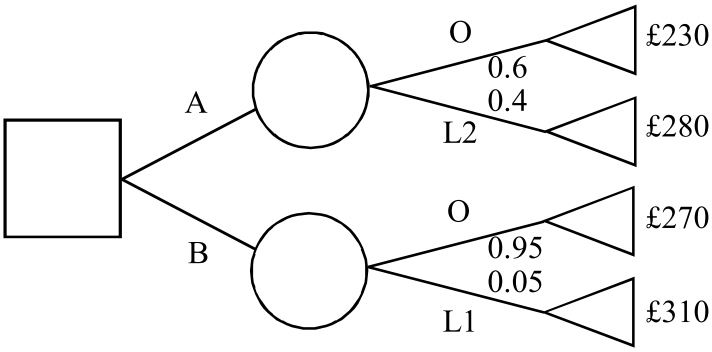
5 a The structure of the decision tree shows the decisions and possible outcomes using the following notation:
A/B: sub-contractors A and B.
O/L1/L2: project completed on time (O), project completed 10 days late (L1) and project completed 20 days late (L2).



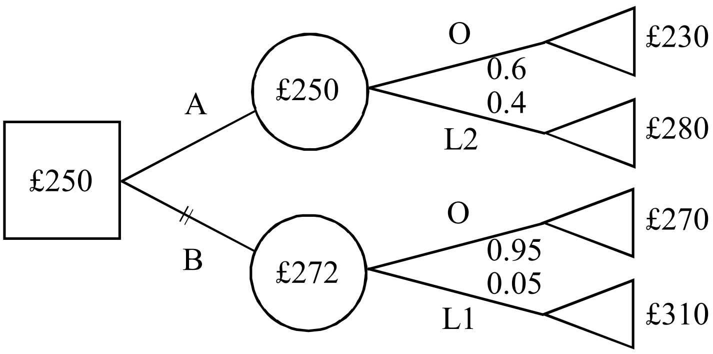
The pay-offs can now be added to the decision tree (the pay-offs in this case are the costs to Me. For example, if I go for contractor A and they deliver the project late by 20 days then this will cost me £260000 + (20 × £1000) = £280000). All pay-offs on the tree are in £1000s.



Probabilities are now added to the branches to the right of each chance node:



Add EMVs to the chance nodes to complete the decision tree.
Example EMV: the top EMV is (0.6 × £230) + (0.4 × £280) = £250.



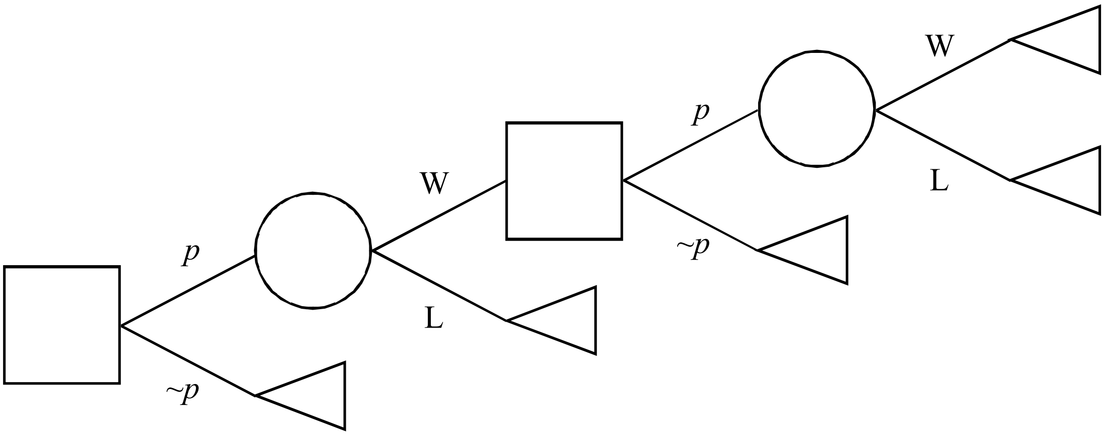
b Since £250 < £272 the contract should be awarded to contractor A.

6 **a** The structure of the decision tree shows the decisions and possible outcomes using the following notation:

p/~p: play the game or not.

W: a score of seven when throwing two dice and summing their totals (a win).

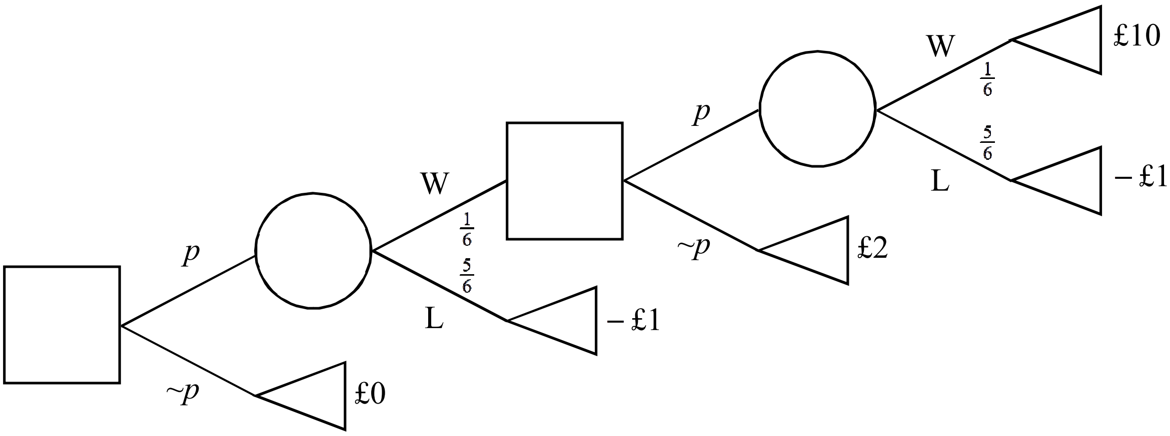
L: a score of other than seven when throwing two dice and summing their totals (a loss).



The pay-offs (amounts won/lost) can now be added to the decision tree.

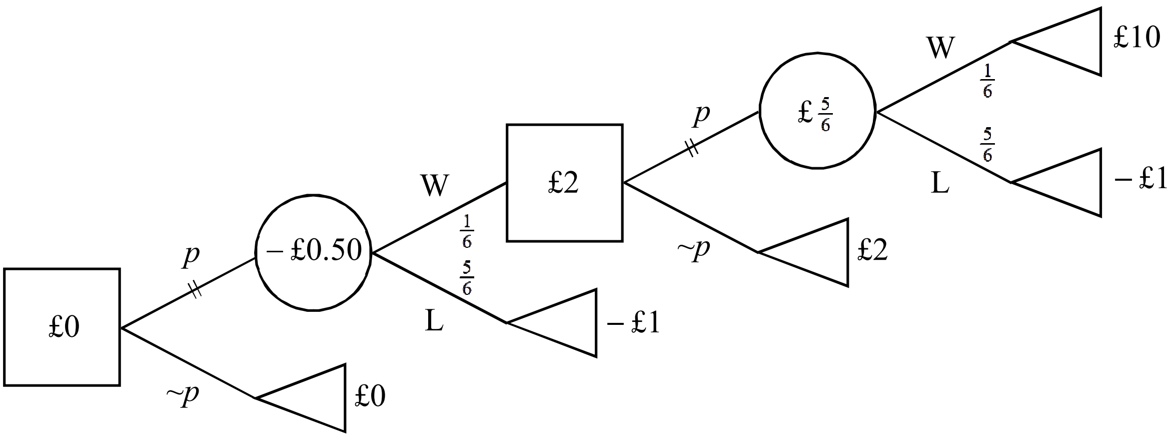
Probabilities are also now added to the branches to the right of each chance node:

There are 36 outcomes in total, 6 of which result in a score of 7 (1, 6), (2, 5), (3, 4), (4, 3), (5, 2) and (6, 1). Therefore, the probability of winning (i.e. scoring a total of 7) = $\frac{6}{36} = \frac{1}{6}$.



Add EMVs to the chance nodes to complete the decision tree.

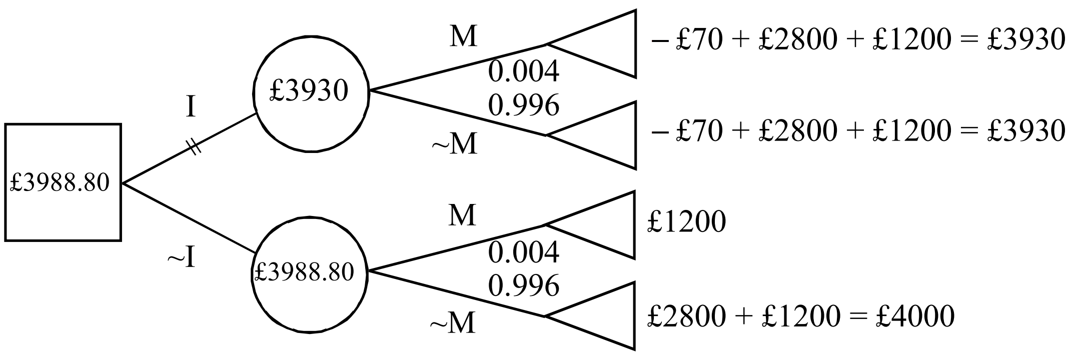
Example EMV: the top right EMV is $\left(\frac{1}{6} \times £10\right) + \left(\frac{5}{6} \times -£1\right) = -£1$.



6 b Since £0 > – £0.50 Beth's best strategy is to not play the game. If she chooses to play and wins on the first throw, then she should keep the winnings and not take the second throw.

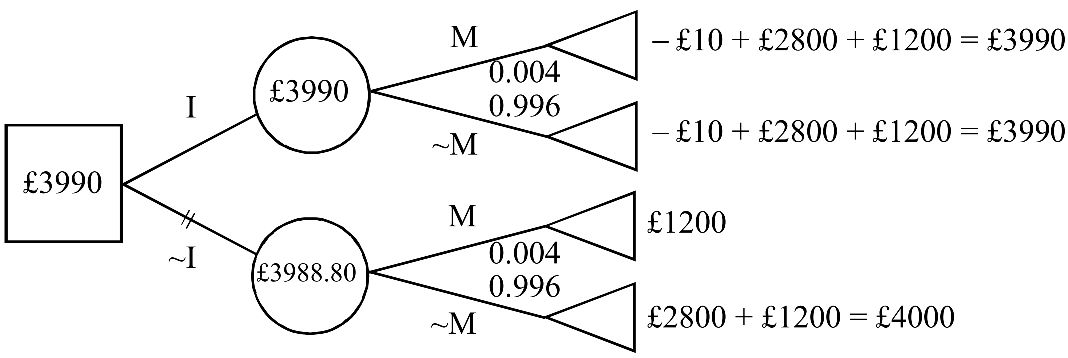
7 a The structure of the decision tree shows the decisions and possible outcomes using the following notation:
I/~I: insure or not.
M/~M: miss holiday or not.

The pay-offs in this case are the amounts of budget that remain. For example, if the couple pays for the insurance and misses the holiday they pay £70 for the insurance, but they receive back the £2800 paid for the holiday and keep the remaining holiday budget of £1200.
Adding EMVs to the chance nodes completes the decision tree.
Example EMV: the top EMV is (0.004 × £3930) + (0.996 × £3930) = £3930.



Since £3988.80 > £3930 it is better not to take out the insurance cover.

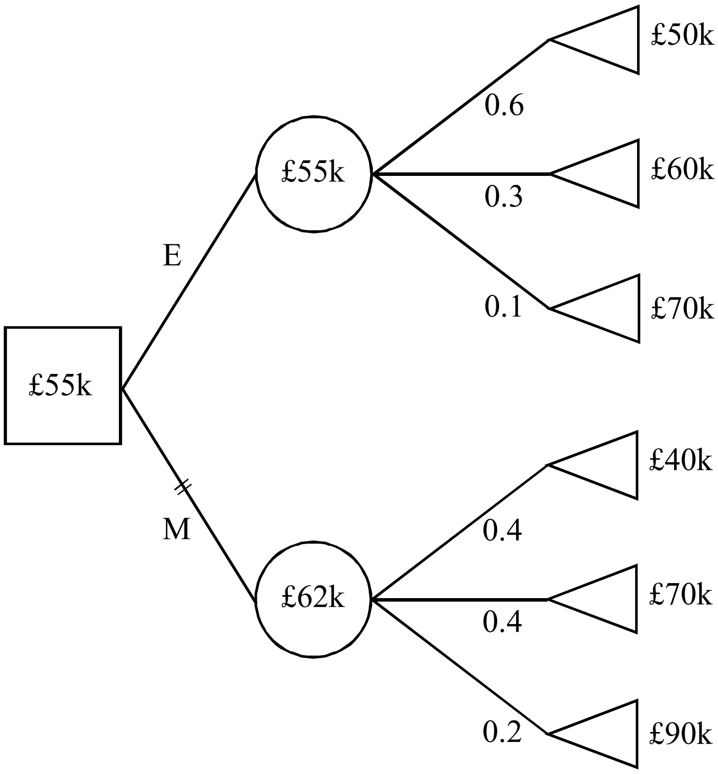
b If the insurance cover costs £10 instead of £70, the decision tree becomes:



Since £3990 > £3988.80 it is now better to take out the insurance cover.

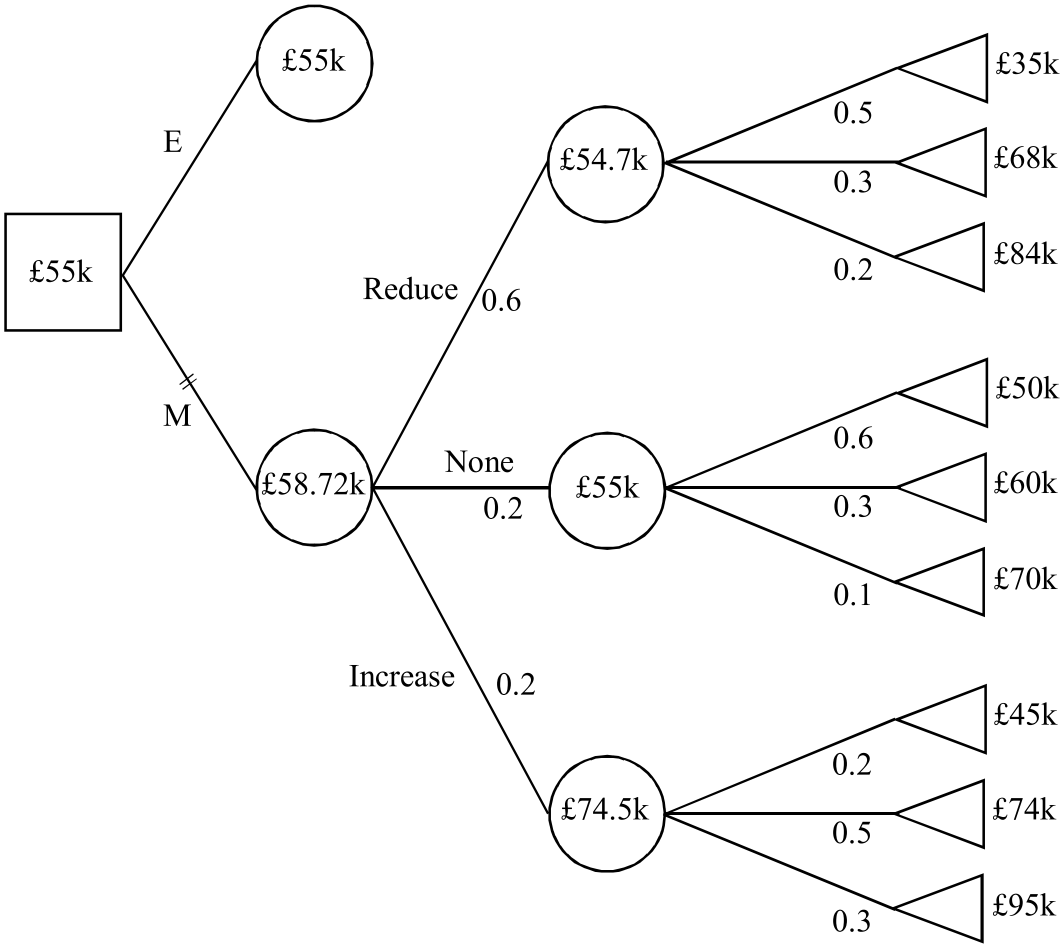
c The insurance cover would have to be ⩽ £4000 – £3988.80 = £11.20 to make it worthwhile in terms of EMV.

8 a The structure of the decision tree shows the decisions and possible outcomes using the following notation:
E/M: extend or move.
The pay-offs in this case are the costs of extending or moving.
Adding EMVs to the chance nodes completes the decision tree.
Example EMV: the top EMV is (0.6 × £50k) + (0.3 × £60k) + (0.1 × £70k) = £55k.



Since £55k < £62k it is cheaper to extend than move.

8 b The revised decision tree is as follows:



The expected costs of moving have reduced but since £55k < £58.72k it is still cheaper to extend than move.