Recurrence relations 7A

1 a $u_n = 1.05u_{n-1}$, $u_0 = 7000$

b Repeating 4 times gives 8508.54 so after 4 years £8508.54

2 a $d_n = 0.78d_{n-1} + 25$, $d_0 = 156$

b Repeating 3 times gives 133.74 so 134 ml

3 Each month 5% is added to the balance so Balance + interest = $b_{n-1} + 0.005b_{n-1} = 1.005b_{n-1}$
£200 is paid off so this amount is reduced by £200.
$k = 1.005$.

4 $P_n = 1.01P_{n-1} + 50\,000$, $P_0 = 12\,500\,000$

5 $u_{n-1} = 5n - 3$, so $u_{n-1} + 5 = 5n + 2 = u_n$

6 $u_{n-1} = 6 \times 2^{n-1} + 1$, so $2u_{n-1} - 1 = 6 \times 2^n + 1 = u_n$

7 a 1, 4, 9, 16

b $u_{n+1} = \sum_{i=1}^{n}(2i-1) + (2(n+1) - 1)$
$= u_n + 2n + 1, n \geqslant 1$

c $u_{n+1} = (n + 1)^2 = n^2 + 2n + 1 = u_n + 2n + 1$

8 a i $2000 \times 1.01^{n-1}$

ii $1800 + 20(n-1) = 1780 + 20n$

b (i)-(ii) = $2000 \times 1.01^{n-1} - 1780 - 20n$

9 a With 1 person there are no handshakes.

b When person n+1 arrives she has to shake hands with all n people already there so $h(n+1) = h(n)+n$

10 a 1, 1, 5, 13, 41, 121

b 1, 1, –1, –3, –1, 5

c 1, 1, 6, 13, 27, 50

11 $B_n = 2B_{n-1} - B_{n-3}$, $n \geqslant 2$; $B_0 = 100$

12 $u_{n-1} = (3 - n)2^n$, $u_{n-2} = (4 - n)2^{n-1}$
$4(u_{n-1} - u_{n-2}) = (3 - n)2^{n+2} - (4 - n)2^{n+1}$
$= (6 - 2n - 4 + n)2^{n+1} = (2 - n)2^{n+1} = u_n$

13 a 10, 10, 10, 10; 20, 10, 10; 10, 20, 10; 10, 10, 20; 20, 20. $J_4 = 5$

b You can jump 10n by jumping 20 less plus a 20 or 10 less plus a 10
so $J_n = J_{n-1} + J_{n-2}$

c Repeating 7 times gives 34.

14 a e.g. Initially there are 4 rabbits so $F_0 = 4$.
$F_1 = 6 \times 4 + 4 = 28$.
Each subsequent year the $F_{n-1} - F_{n-2}$ rabbits just born produce 2 offspring each, and the F_{n-2} older rabbits produce 6 offspring.
So $F_n = 2(F_{n-1} - F_{n-2}) + 6F_{n-2} + F_{n-1} = 3F_{n-1} + 4F_{n-2}$ as required.

b One criticism of this model is that it assumes that no rabbits die.

15 a 2 ways for 1 digit – 1 and 0 so $b_1 = 2$
3 ways for 2 digits – 00, 01 and 10 so $b_2 = 3$

b Strings of length n ending with 0 that do not have consecutive 1s are the strings of length n – 1 with no consecutive 1s with a 0 added at the end, so there are b_{n-1} such strings.
But strings of length n ending with 1 that do not have consecutive 1s must have 0 as their (n – 1)th digit; otherwise they will end with a pair of 1s.
It follows that the strings with length n ending with a 1 that have no consecutive 1s are the strings of length n – 2 with no consecutive 1s with 01 added at the end, so there are b_{n-2} such strings.
We conclude that $b_n = b_{n-1} + b_{n-2}$.

c Repeating gives 2, 3, 5, 8, 13, 21, 34 so $b_7 = 34$