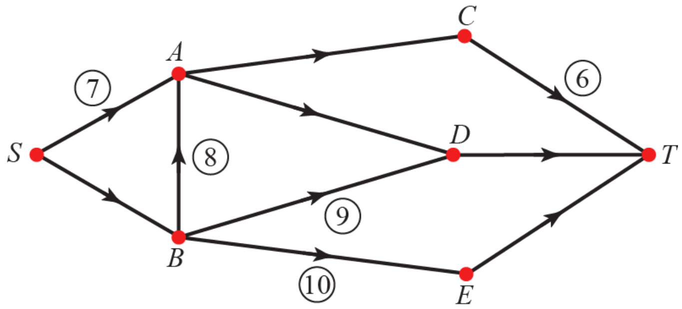
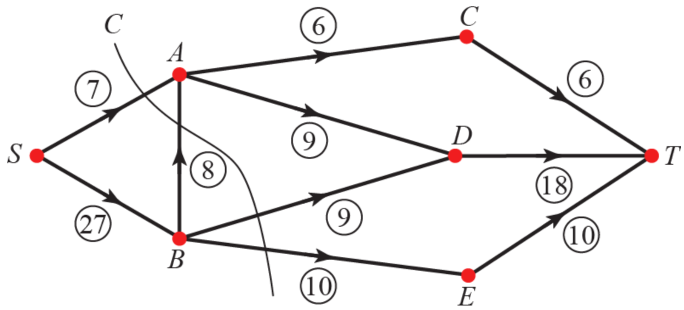
Flow in networks 3E

1 a First, identify saturated arcs

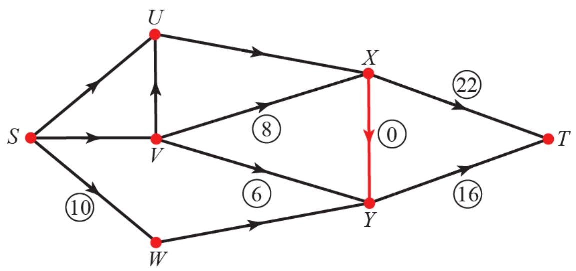


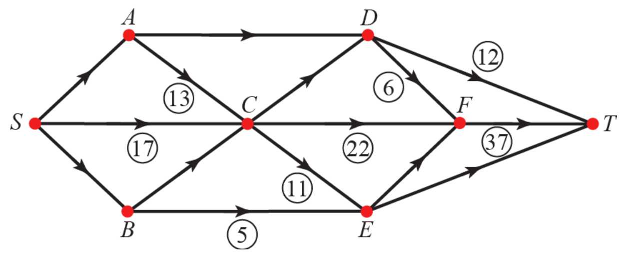
Next, draw a cut passing through *SA*, *AB*, *BD*, *BE*



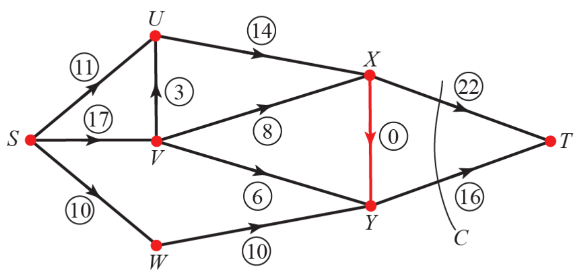
The cut has value 34, which is equal to the flow out of *S* (or into *T*). Hence, by the maximum flow – minimum cut theorem, the flow is maximal.

b First, identify saturated arcs and empty arcs (red)



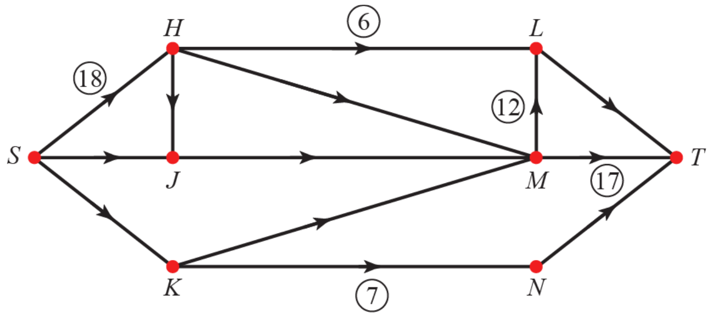


Next, draw a cut passing through *XT* and *YT* (this is only an example, other cuts could be drawn, e.g. *SW*, *VY*, *XY*, *XT*).

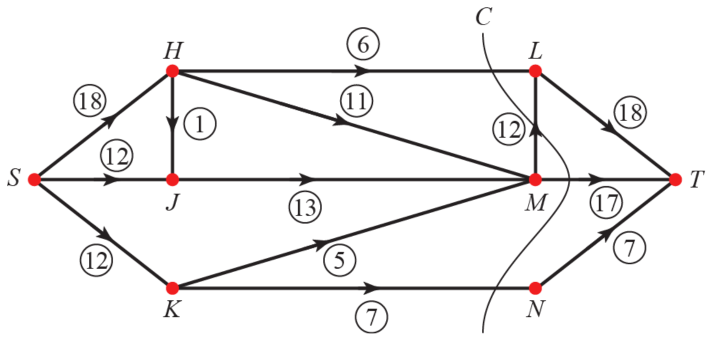


The value of this cut is equal to 38, which is the same as the flow into *T*. Hence, by the maximum flow – minimum cut theorem, the flow is maximal.

1 c First, identify saturated arcs

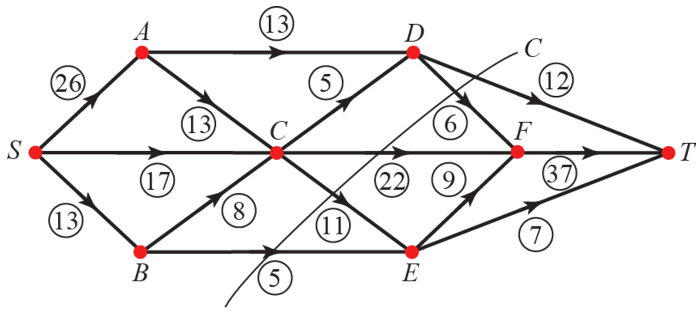


Next, draw a cut passing through *KN*, *MT*, *ML* and *HL*.



The value of the cut is equal to 42, which is the same as the flow into *T*. Hence, by the maximum flow – minimum cut theorem, the flow is maximal.

d First, identify saturated arcs
Next, draw a cut passing through *BE*, *CE*, *CF*, *DF*, and *DT*

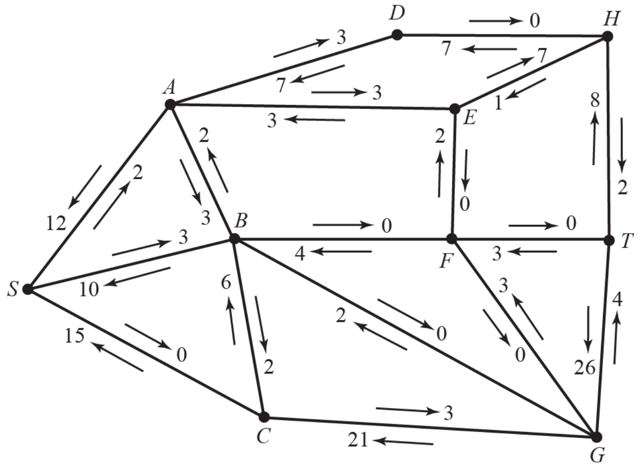


The value of the cut is equal to 56, which is the same as the flow into *T*. Hence, by the maximum flow – minimum cut theorem, the flow is maximal.

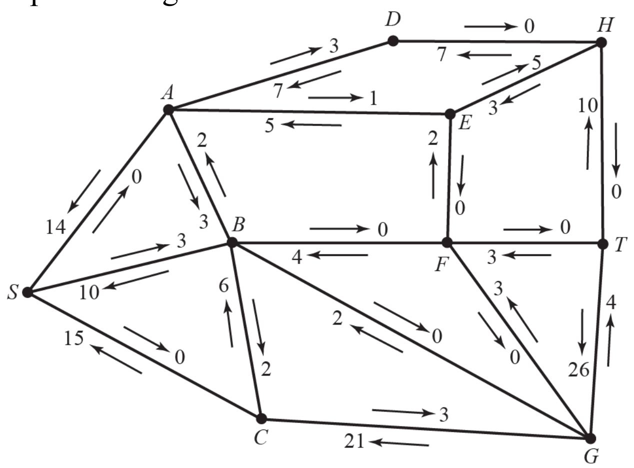
2 a Use flow conservation to work out the values of x and y. Flow into *A* must be equal to flow out of *A*, hence:
$x = 2 + 3 + 7 = 12$
Similarly, for y, the flow into *H* must be equal to the flow out of *H*
$y = 7 + 1 = 8$

b The initial flow is the flow out of *S* (or the flow into *T*).
Thus:
Flow $= 15 + 10 + x = 25 + 12 = 37$
So the initial flow is 37

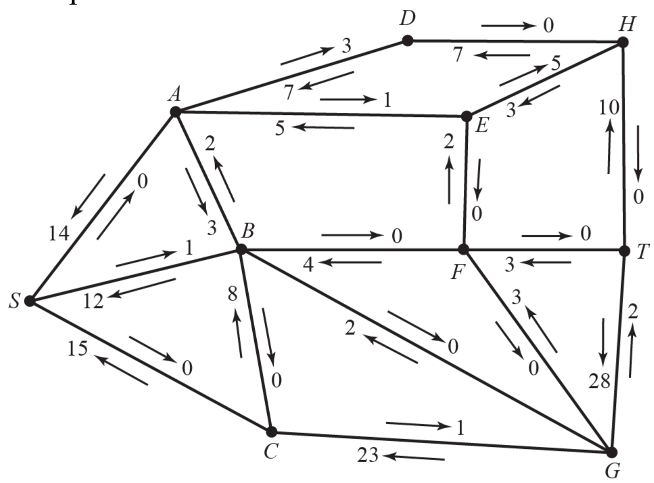
2 c The initial diagram looks as follows

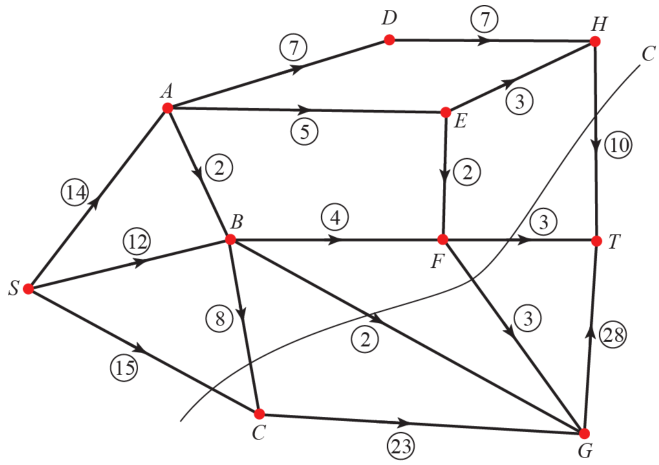


From *S* we could go to either *B* or *A*, since *SC* is already saturated. From *B* the only unsaturated route leads us back to *A*, so let's follow *SA*. There is a spare capacity of 2 along *SAEHT*.
Updated diagram:



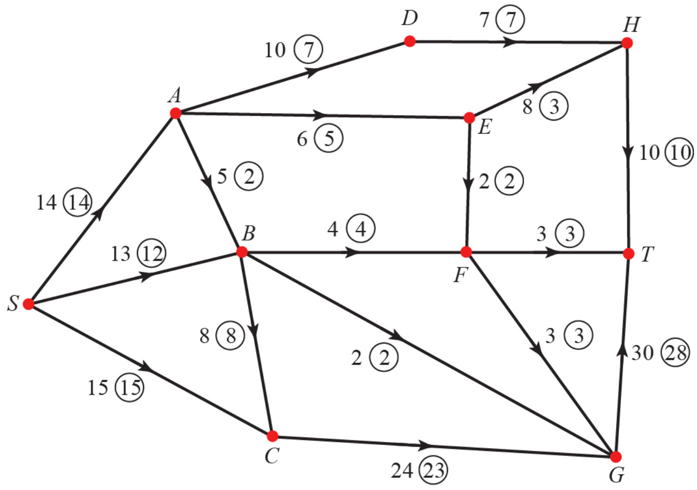
Now we can only follow *SB*. We see that there is a spare capacity of 2 along SBCGT.
Updated network:



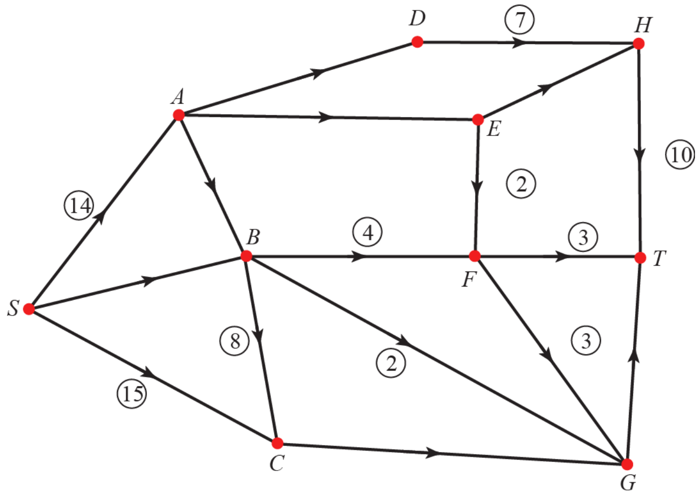


2 c (continued)

Further inspection shows there are no other routes with spare capacities. Hence, the final network looks as follows



d First, identify saturated arcs



Next, draw a cut through *SC*, *BC*, *BG*, *FG*, *FT* and *HT* (this is only an example, other cuts could be drawn).
The value of this cut is equal to 41, which is the same as the flow into *T*. Hence, by the maximum flow – minimum cut theorem, the flow is maximal.

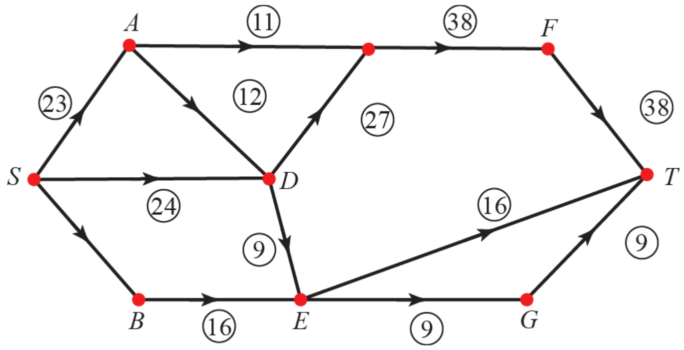
3 a The saturated arcs are *SA*, *AD*, *DE*, *ET* and *EG*.

b Initial flow is equal to
Flow = $SA + SD + SB = 23 + 18 + 16 = 57$

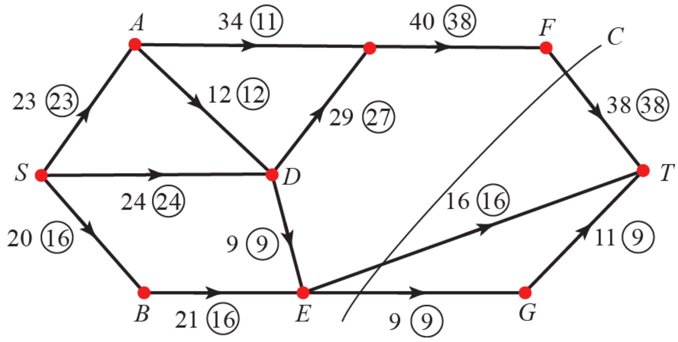
c The capacity of an arc is equal to the capacity flowing out of that cut.
Hence:
$C_1 = AC + AD + SD + ET + EG$
$= 34 + 12 + 24 + 16 + 9 = 95$
Note that *DE* flows into the cut.
$C_2 = CF + ET + GT = 40 + 16 + 11 = 67$

3 d By inspection, we find that there is spare capacity of 6 along *SDCFT*.

e First, identify all saturated arcs



Next, draw a cut through *EG, ET* and *FT*.



The value of the cut is equal to 63, which is the same as the flow into *T*. Hence, by maximum flow – minimum cut theorem, the flow is maximal.