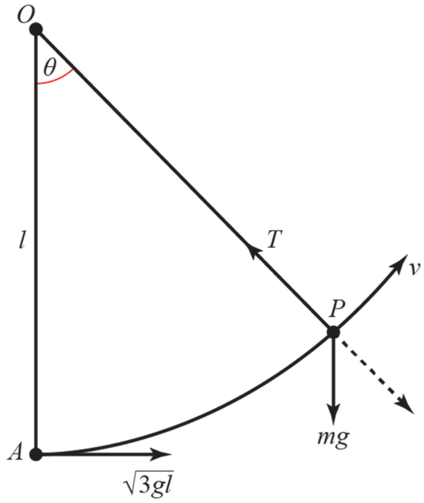
Circular motion 1E

1 a



Conservation of energy.

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mgl(1-\cos\theta)$$
$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mgl(1-\cos\theta)$$
$$v^2 = 3gl - 2gl(1-\cos\theta) = gl(1+2\cos\theta)$$

Resolving towards the centre of the circle:

$$T - mg\cos\theta = \frac{mv^2}{l}$$
$$T = mg\cos\theta + \frac{mgl}{l}(1+2\cos\theta) = mg + 3mg\cos\theta$$

b String slack $\Rightarrow T = 0 \Rightarrow \cos\theta = -\frac{1}{3} \Rightarrow \text{height} = l + \frac{l}{3} = \frac{4l}{3}$

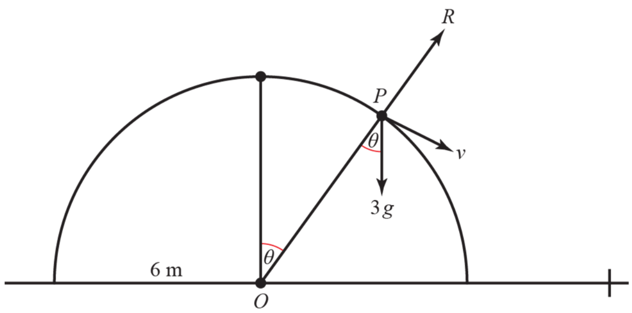
c When the string goes slack, $v^2 = gl\left(1 + 2\times\left(-\frac{1}{3}\right)\right) = \frac{gl}{3}$

So horizontal component of velocity $= \frac{1}{3}\sqrt{\frac{gl}{3}}$

Using energy, if the maximum additional height is h, then

$$mgh + \frac{1}{2}m\times\left(\frac{1}{3}\sqrt{\frac{gl}{3}}\right)^2 = \frac{1}{2}m\left(\frac{gl}{3}\right)$$
$$h = \frac{l}{6} - \frac{l}{6\times 9} = \frac{8l}{54} = \frac{4l}{27},\ \text{height above } A = \frac{4l}{3} + \frac{4l}{27} = \frac{40l}{27}$$

2 a



Conservation of energy from top to P:

$$mg\times 6 = mg\times 6\cos\theta + \frac{1}{2}mv^2$$
$$v^2 = 12g(1-\cos\theta)$$

Resolving towards O:

$$3g\cos\theta - R = \frac{mv^2}{r} = \frac{12\times 3g}{6}(1-\cos\theta)$$
$$9g\cos\theta - 6g = R$$

b $R = 0 \Rightarrow \cos\theta = \frac{2}{3},\ \theta \approx 48^\circ$

2 c $v^2 = 12g \times \frac{1}{3} = 4g$

Speed $\rightarrow v\cos\theta$, $\downarrow v\sin\theta + gt$

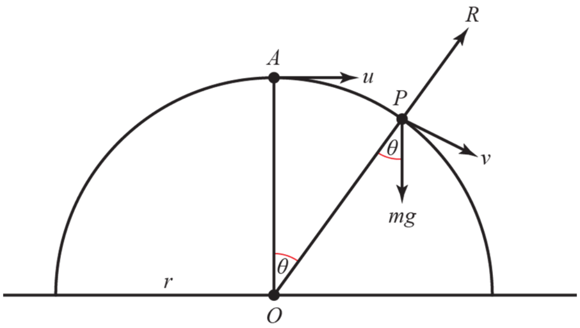
Distance $\rightarrow v\cos\theta t$, $\downarrow v\sin\theta t + \frac{1}{2}gt^2 = 6 \times \frac{2}{3} = 4$

$2\sqrt{g}\frac{\sqrt{5}}{3}t + \frac{g}{2}t^2 = 4, 4.9t^2 + \frac{14}{3}t - 4 = 0$

$t \approx 0.545\ldots$

Total horizontal distance from $O = 6\sin\theta + \sqrt{4g}\cos\theta \times t \approx 6.7\,\text{m}$

3 a



Conservation of energy:

$\frac{1}{2}mu^2 + mgr = \frac{1}{2}mv^2 + mgr\cos\theta$

$\frac{rg}{8} + rg = \frac{9rg}{8} = \frac{1}{2}v^2 + rg\cos\theta$

$v^2 = \frac{9rg}{4} - 2rg\cos\theta$

b Resolving towards O: $mg\cos\theta - R = \frac{mv^2}{r} = mg\left(\frac{9}{4} - 2\cos\theta\right)$

$R = 0 \Rightarrow 3mg\cos\theta = mg \times \frac{9}{4},\ \cos\theta = \frac{3}{4}$

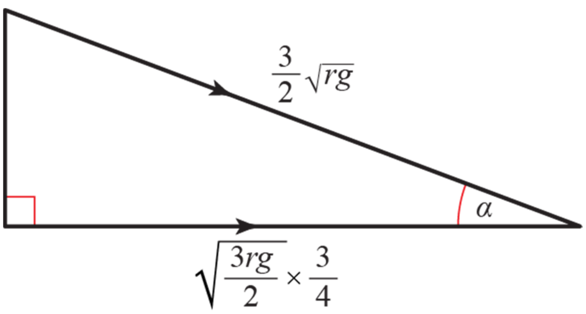
c $v^2 = \frac{9rg}{4} - 2rg \times \frac{3}{4} = \frac{3rg}{4},\ v = \sqrt{\frac{3rg}{4}}$

d Conservation of energy from A to the table:

$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgr$

$v^2 = u^2 + 2gr = \frac{rg}{4} + 2gr = \frac{9rg}{4},\ v = \frac{3}{2}\sqrt{rg}$

e



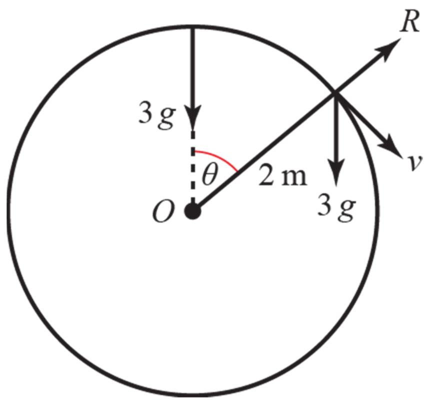
After leaving the hemisphere the horizontal component of the velocity remains constant.

Direction is angle α to the ground,

$\cos\alpha = \frac{\sqrt{\frac{3rg}{4}} \times \frac{3}{4}}{\frac{3}{2} \times \sqrt{rg}} = \frac{\sqrt{3}}{4}$

$\alpha = 64^\circ$

4 a



Conservation of energy:

$$mgr = \frac{1}{2}mv^2 + mgr\cos\theta$$

$$v^2 = 2mgr(1-\cos\theta) = 4mg(1-\cos\theta)$$

Resolving towards O:

$$3g\cos\theta - R = \frac{3v^2}{2} = \frac{3\times 4g(1-\cos\theta)}{2}$$

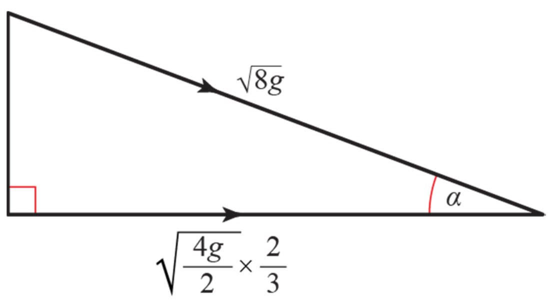
$$R = 0 \Rightarrow 9g\cos\theta = 6g, \quad \cos\theta = \frac{2}{3}$$

$$\theta \approx 48^\circ$$

b Using conservation of energy from the highest point to the ground:

$\frac{1}{2}mv^2 = mgh = mg\times 4,\ v = \sqrt{8g}$ when P hits the ground.

When P leaves the sphere $v^2 = 4mg(1-\cos\theta) = 4mg\times\frac{1}{3},\ v = \sqrt{\frac{4mg}{3}}$

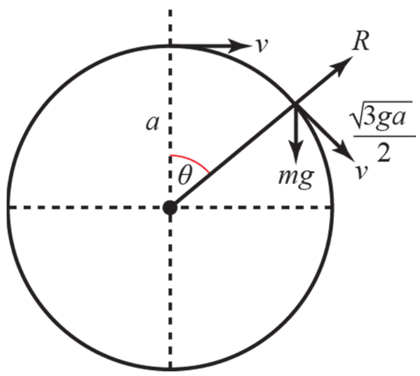


After leaving the hemisphere the horizontal component of the velocity remains constant. Direction is angle α to the ground,

$$\cos\alpha = \frac{\sqrt{\frac{4g}{3}}\times\frac{2}{3}}{\sqrt{8g}} = \frac{2}{3}\times\sqrt{\frac{1}{6}}$$

$$\alpha = 74^\circ$$

5 a



Forces acting along the radius:

$$mg\cos\theta - R = \frac{mv^2}{r} = \frac{m\times 3ga}{4a} = \frac{3mg}{4}$$

$$R = 0 \Rightarrow \cos\theta = \frac{3}{4}$$

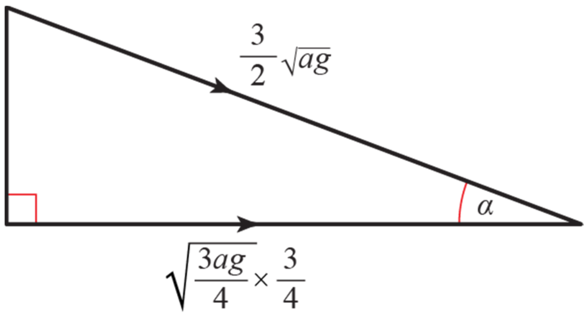
Distance fallen $= a - a\cos\theta = \frac{a}{4}$

b Conservation of energy from the top to the point where the particle leaves the sphere:

$$mg\frac{a}{4} = \frac{1}{2}m\times\frac{3ga}{4} - \frac{1}{2}mv^2,\ \frac{1}{2}v^2 = \frac{3ga}{8} - \frac{ga}{4} = \frac{ga}{8},\ v^2 = \frac{ga}{4},\ v = \sqrt{\frac{ga}{4}}$$

5 c Looking at the energy at the top and level with the centre:

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga = \frac{1}{2}m\frac{ga}{4} + mga, v^2 = \frac{9ga}{4}, v = \frac{3}{2}\sqrt{ga} = \sqrt{\frac{9ga}{4}}$$

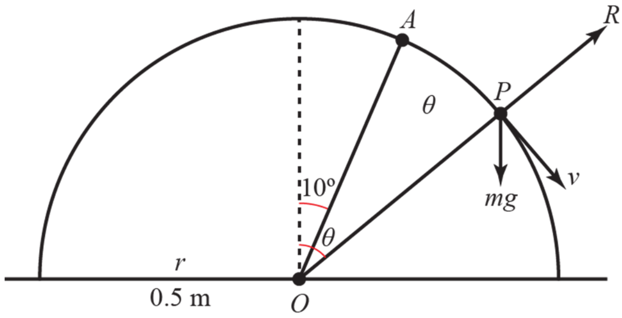


After leaving the hemisphere the horizontal component of the velocity remains constant.

Direction is α, $\cos\alpha = \dfrac{\sqrt{\frac{3ag}{4}} \times \frac{3}{4}}{\frac{3}{2} \times \sqrt{ag}} = \dfrac{\sqrt{3}}{4}$

$\alpha = 64^\circ$ to the horizontal

6 a



Conservation of energy:

$$\frac{1}{2}mv^2 + mg\frac{1}{2}\cos\theta = mg\frac{1}{2}\cos 10^\circ$$

$$v^2 = g(\cos 10^\circ - \cos\theta)$$

Forces acting towards O:

$$mg\cos\theta - R = \frac{mv^2}{0.5} = 2mv^2$$

$$R = 0 \Rightarrow g\cos\theta = 2v^2 = 2g(\cos 10^\circ - \cos\theta) \Rightarrow 3g\cos\theta = 2g\cos 10^\circ$$

$$\cos\theta = \frac{2}{3}\cos 10^\circ,\ \theta \approx 49^\circ$$

b The particle will fall through a parabolic arc (projectile motion) towards the surface in the positive x direction.

7 a Total height lost

$= 5(1 - \cos 70^\circ) + 7(1 - \cos 40^\circ) + 0.5$

$= 5.427...$

$= 5.4$ m

Conservation of energy:

$\frac{1}{2} \times 2 \times v^2 = 2 \times g \times 5.427...$

$\Rightarrow v = 10.3\ \text{ms}^{-1}$

b At R: $\frac{1}{2} \times 2 \times v^2 = 2g(12 - 5\cos 70^\circ - 7\cos 40^\circ)$

$\Rightarrow v^2 = 96.58$

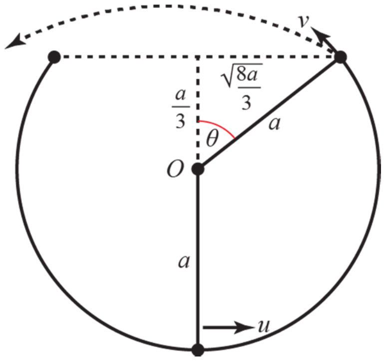
R(↗) towards B:

$$mg\cos\theta - R = \frac{mv^2}{7}$$

$$R = 2g\cos 40^\circ - \frac{2v^2}{7} = -12.6 < 0$$

This is impossible, so the particle must have lost contact with the chute before this point.

7 c In reality, energy is lost due to friction between the laundry bag and the chute.

8 a 

K.E. + P.E. at lowest point $= \frac{1}{2}mu^2$

K.E. + P.E. at rim $= \frac{1}{2}mv^2 + mg \times \frac{4a}{3}$

$$\Rightarrow u^2 = v^2 + \frac{8ga}{3}$$

After the particle leaves the bowl:

The vertical speed when the particle returns to the level of the rim of the bowl is $v\sin\theta$ downwards, so using $v = u + at, -v\sin\theta = v\sin\theta - gt,\ t = \frac{2v\sin\theta}{g}$

The horizontal distance covered in this time is $v\cos\theta \times \frac{2v\sin\theta}{g}$

The width of the top of the bowl $= 2 \times \frac{\sqrt{8}}{3}a = \frac{4\sqrt{2}a}{3}$

$\Rightarrow 2\frac{v^2}{g}\sin\theta\cos\theta > \frac{4\sqrt{2}a}{3},\ v^2 \times \frac{\sqrt{8}}{3} \times \frac{1}{3} > \frac{2\sqrt{2}ag}{3},\ v^2 > 3ag$

$\Rightarrow u^2 > 3ag + \frac{8ga}{3} = \frac{17ga}{3}$

so minimum value of u is $\sqrt{\frac{17ag}{3}}$

b Energy would be lost due to the frictional force acting on the marble, requiring a larger initial speed for the marble to leave the bowl.