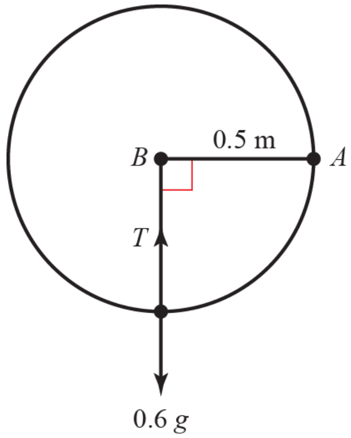
Circular motion 1D

1 a



Let the speed of the particle at the lowest point be v m s^{-1}, and the tension in the rod be TN.

At the lowest point the particle has fallen a distance 0.5 m, so the P.E. lost $= 0.6 \times g \times 0.5$

and the K.E. gained $= \frac{1}{2} \times 0.6 \times v^2$

$$\therefore 0.6 \times g \times 0.5 = \frac{1}{2} \times 0.6 \times v^2$$

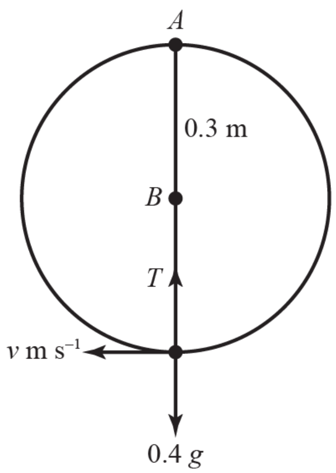
$$v^2 = g,\ v \approx 3.13\,\text{m s}^{-1}$$

b At the lowest point, the force towards the centre of the circle

$$= T - 0.6g = \frac{0.6v^2}{0.5}$$

$$\Rightarrow T = 0.6g + \frac{0.6g}{0.5} = 1.8g \approx 17.6\,\text{N}$$

2 a



Let the speed of the particle at the lowest point be v m s^{-1}, and the tension in the rod be TN.

At the lowest point the particle has fallen a distance 0.6 m, so the P.E. lost $= 0.4 \times g \times 0.6$,

and the K.E. gained $= \frac{1}{2} \times 0.4 \times v^2$

$$\therefore 0.4 \times g \times 0.6 = \frac{1}{2} \times 0.4 \times v^2$$

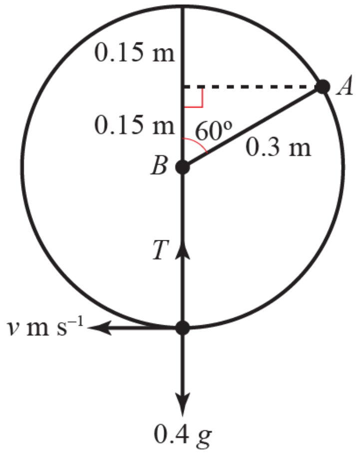
$$v^2 = 2 \times g \times 0.6 = 1.2g,\ v \approx 3.43\,\text{m s}^{-1}$$

b At the lowest point, the force towards the centre of the circle

$$= T - 0.4g = \frac{0.4v^2}{0.3}$$

$$\Rightarrow T = 0.4g + \frac{0.4 \times 1.2g}{0.3} = 2g \approx 19.6\,\text{N}$$

3 a



Let the speed of the particle at the lowest point be v m s^{-1}, and the tension in the rod be TN.

At the lowest point the particle has fallen a distance $0.3\cos 60^\circ + 0.3 = 0.45$ m, so the P.E. lost $= 0.4 \times g \times 0.45$, and the

$$\text{K.E. gained} = \frac{1}{2} \times 0.4 \times v^2$$

$$\therefore\ 0.4 \times g \times 0.45 = \frac{1}{2} \times 0.4 \times v^2$$

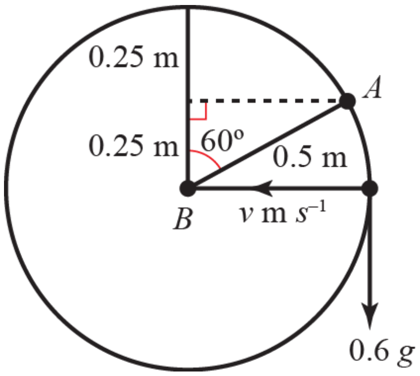
$$v^2 = 2 \times g \times 0.45 = 0.9g,\ v \approx 2.97\,\text{m s}^{-1}$$

b At the lowest point, the force towards the centre of the circle

$$= T - 0.4g = \frac{0.4v^2}{0.3}$$

$$\Rightarrow T = 0.4g + \frac{0.4 \times 0.9g}{0.3} = 1.6g \approx 15.7\,\text{N}$$

4 a



Let the speed of the particle when the rod is horizontal be v m s^{-1}, and the tension in the rod be TN.

At the horizontal point the particle has fallen a distance $0.5\cos 60^\circ = 0.25$ m, so the P.E. lost $= 0.6 \times g \times 0.25$, and the

$$\text{K.E. gained} = \frac{1}{2} \times 0.6 \times v^2$$

$$\therefore\ 0.6 \times g \times 0.25 = \frac{1}{2} \times 0.6 \times v^2$$

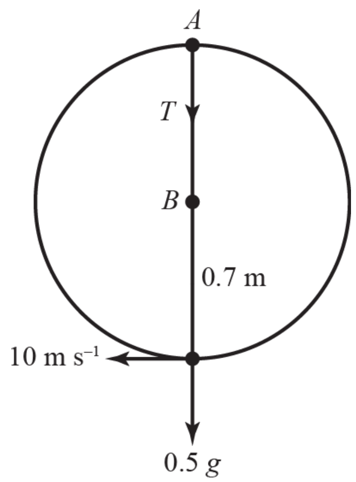
$$v^2 = 2 \times g \times 0.25 = 0.5g,\ v \approx 2.21\,\text{m s}^{-1}$$

b At the horizontal point, the force towards the centre of the circle

$$= T = \frac{0.6v^2}{0.5}$$

$$\Rightarrow T = \frac{0.6 \times 0.5g}{0.5} = 0.6g = 5.88\,\text{N}$$

5 a



Let the speed of the bead at the highest point be v m s^{-1}, and the tension in the wire be TN.

At the highest point the bead has risen a distance 1.4 m, so the

P.E. gained $= 0.5 \times g \times 1.4$, and the

K.E. lost $= \frac{1}{2} \times 0.5 \times 10^2 - \frac{1}{2} \times 0.5 \times v^2$

$\therefore\ 0.5 \times g \times 1.4 = \frac{1}{2} \times 0.5 \times (100 - v^2)$

$100 - v^2 = 2 \times g \times 1.4 = 2.8g,\ v \approx 8.52\,\text{m s}^{-1}$

b The reaction is towards the centre of the circle.

$$R + 0.5g = \frac{0.5(100 - 2.8g)}{0.7}$$

$$R = \frac{0.5(100 - 2.8g)}{0.7} - 0.5g$$

$$= 46.928...$$

$$= 46.9\text{N}$$

6 a When the angle between AB and the vertical is θ the particle has speed v ms^{-1}

P.E. gained $= mgh = 0.5 \times g \times .7(1 - \cos\theta)$

Loss in K.E. $\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = \frac{0.5}{2}\left(u^2 - v^2\right)$

Energy is conserved

$$0.5g \times 0.7(1 - \cos\theta) = \frac{0.5}{2}\left(u^2 - v^2\right)$$

Hence, $v^2 = u^2 - 1.4g\left(1 - \cos\theta\right)$

$$\Rightarrow v = \sqrt{u^2 - 1.4g\left(1 - \cos\theta\right)}$$

b If the particle is to reach the top of the circle then we require $v > 0$ when $\theta = 180^\circ$.

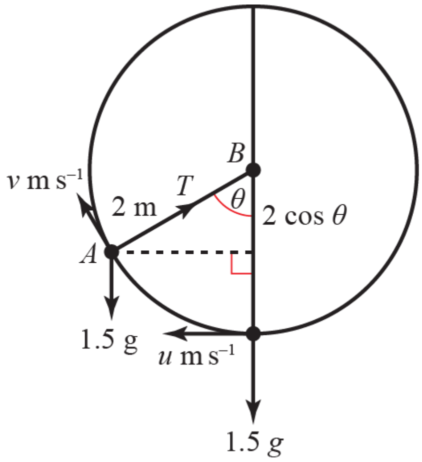
$\Rightarrow u^2 - 1.4g\left(1 - \cos 180^\circ\right) > 0$

$\cos 180^\circ = -1$

But so $u^2 \geqslant 1.4g \times 2$

$\Rightarrow u \geqslant \sqrt{2.8g}$

7 a



Take the lowest point of the circle as the zero level for potential energy.

At the lowest level the particle has

$$\text{K.E.} = \frac{1}{2}\times 1.5\times u^2 = 0.75u^2\text{J} \text{ and P.E.} = 0\text{ J}$$

When the rod is at angle θ to the vertical the particle has

$$\text{K.E.} = \frac{1}{2}\times 1.5\times v^2 = 0.75v^2\text{J and}$$

$$\text{P.E.} = 1.5\times g\times 2(1-\cos\theta)\text{ J}$$

Energy is conserved

$$\therefore\ 0.75u^2 = 0.75v^2 + 3g(1-\cos\theta)$$

Resolving towards the centre of the circle:

$$T - 1.5g\cos\theta = \frac{mv^2}{r} = \frac{1.5v^2}{2}, \text{ so substituting for } v^2$$

gives

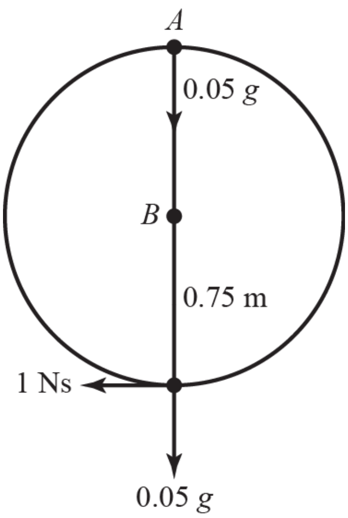
$$T = 1.5g\cos\theta + \frac{3}{4}(u^2 - 4g + 4g\cos\theta)$$

$$= 4.5g\cos\theta + \frac{3u^2}{4} - 3g$$

b If the particle is to reach to top of the circle then we require $T > 0$ when $\theta = 180^\circ$.

$$\Rightarrow -4.5g + \frac{3u^2}{4} - 3g > 0,\ \frac{3u^2}{4} > 7.5g,\ u^2 > 10g,\ u > \sqrt{10g}$$

8 a



Impulse = change in momentum, so if the initial speed of the bead is u m s^{-1} then

$$I = 0.05u,\ u = 20I.$$

Take the lowest point of the circle as the zero level for potential energy.

At the lowest level the particle has

$$\text{K.E.} = \frac{1}{2}\times 0.05\times u^2 = 0.025\times 400I^2 = 10I^2\text{J and}$$

$$\text{P.E.} = 0\text{ J}$$

At the highest level the particle has K.E.= 0 (since we are told that the bead if just reaches the top) and it has risen 1.5 m so it has

$$\text{P.E.} = 0.05\times g\times 1.5 = 0.075g$$

$$\text{Energy is conserved, } \therefore 10I^2 = 0.075g,$$

$$\Rightarrow I^2 = 0.0075g,\ I \approx 0.27$$

8 b Let the lowest point of the circle be the zero level for potential energy.
At the lowest level the particle has

$$\text{K.E.} = \frac{1}{2} \times 0.05 \times u^2 = 0.025 \times 400 I^2 = 10 I^2 J$$

$$\text{P.E.} = 0 \text{ J}$$

When the rod is at angle $\arctan\frac{3}{4}$ to the vertical the particle has K.E. $= 0$ J

as greatest speed is zero,

$$\text{P.E.} = 0.05 \times \text{g} \times 0.75 \left\{1 + \cos\left(\tan^{-1}\frac{3}{4}\right)\right\}$$

$$= 0.0675 \text{ g J}$$

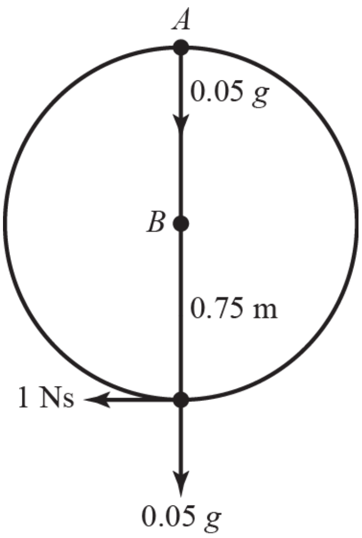
Energy is conserved

$$10 I^2 = 0.0675 \text{ g}$$

$$\Rightarrow I = 0.2571...$$

$$I = 0.26$$

9 a



Impulse = change in momentum, so if the initial speed of the speed is u m s^{-1} then

$l = 0.05u$, $u = 20l$.

Take the lowest point of the circle as the zero level for potential energy.

At the lowest level the particle has

$$\text{K.E.} = \frac{1}{2} \times 0.05 \times u^2 = 0.025 \times 400 l^2 = 10 l^2 \text{J}$$

and P.E. $= 0$ J

If the bead just reaches the top of the circle then this is the point at which the tension in the string becomes zero. If the speed of the bead at this point is v m s^{-1} then

$$0.05g = \frac{mv^2}{r} = \frac{0.05v^2}{0.75}, v^2 = 0.75g$$

The bead has risen 1.5 m so it has

$$\text{P.E.} = 0.05 \times g \times 1.5 = 0.075g$$

Energy is conserved, so

$$10l^2 = 0.075g + \frac{1}{2} \times 0.05 \times 0.75g = 0.09375g, \; l \approx 0.30$$

9 b Let the lowest point of the circle be the zero level for potential energy.
At the lowest level the particle has

$\text{K.E.} = \frac{1}{2} \times 0.05 \times u^2 = 0.025 \times 400 I^2 = 10 I^2 J$

$\text{P.E.} = 0 \text{ J}$

When the bead just reaches the point where AB is at angle $\arctan\frac{3}{4}$ to the vertical the tension in the string becomes zero. If the speed of the bead at this point is $v \text{ ms}^{-1}$ then

$0.05g\cos\left(\tan^{-1}\frac{3}{4}\right) = \frac{mv^2}{r} = \frac{0.05v^2}{0.75}$

$v^2 = 0.75 \times g \times \frac{4}{5} = 0.6 \text{ g}$

The bead has risen $0.75 + 0.6 = 1.35 \text{ m}$

gain in $\text{P.E.} = 0.05 \times g \times 1.35 = 0.0675 \text{ g}$

Energy is conserved

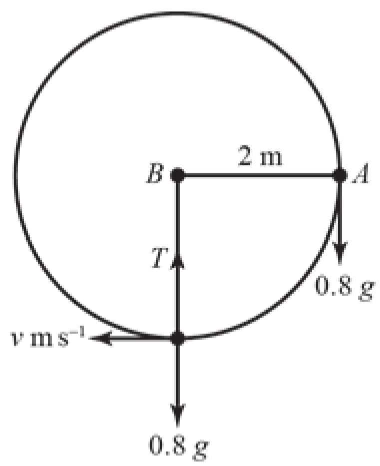
$\Rightarrow 10I^2 = 0.0675g + \frac{1}{2} \times 0.05 \times 0.6g = 0.0825g$

$\Rightarrow I = 0.2843...$

$I = 0.28$

c The particle will continue in a parabolic arc (projectile motion) in the negative x direction, initially increasing in y before decreasing in y.

10 a



Let the speed of the particle at the lowest point be $v \text{ m s}^{-1}$, and the tension in the rod be TN.

Take the starting level as the zero level for potential energy, the particle starts with

$\text{P.E.} = 0$ and $\text{K.E.} = 0$.

At the lowest level,

$\text{P.E.} = -0.8 \times g \times 2 = -1.6g$

$\text{K.E.} = \frac{1}{2} \times m \times v^2 = \frac{1}{2} \times 0.8v^2 = 0.4v^2$

Conservation of energy

$\Rightarrow -1.6g + 0.4v^2 = 0, \quad v^2 = 4g$

$v \approx 6.26 \text{ m s}^{-1}$

Force towards the centre of the circle $= T - 0.8g = \frac{mv^2}{r} = \frac{0.8 \times 4g}{2} = 1.6g$

$T = 2.4g \approx 23.5 \text{ N}$

10 b Let tension in the rod be T N and the speed of the particle at the point when

$\theta = \tan^{-1}\frac{3}{4}$ be $v\text{ ms}^{-1}$

Take the starting level as the zero level for potential energy, the parcel starts with

P.E. $= -0.8 \times g \times 1.6 = -1.28g$

K.E. $= \frac{1}{2} \times m \times v^2 = \frac{1}{2} \times 0.8v^2 = 0.4v^2$

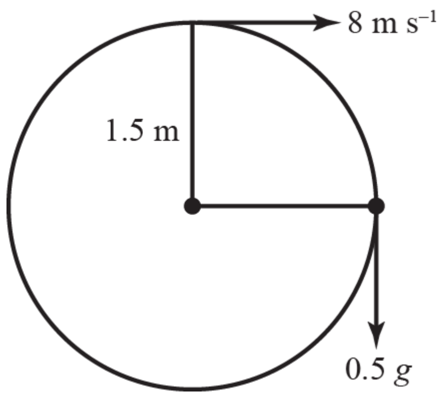
Conservation of energy

$$\Rightarrow -1.28g + 0.4v^2 = 0$$
$$v^2 = 3.2g$$
$$v = 5.6\text{ ms}^{-1}$$

Force towards the centre of the circle,

$$T - 0.8g\cos\theta = \frac{mv^2}{r}$$
$$= \frac{0.8 \times 3.2g}{2} = 1.28g$$
$$T = 1.28g + 0.8g \times \frac{4}{5}$$
$$= 1.8816g$$
$$= 18.8\text{N } (3\text{ s.f.})$$

11 a



Let the speed of the particle when the string is horizontal be v m s^{-1}

Take the lowest point as the zero level for potential energy, the particle starts with

P.E. $= 0.5 \times g \times 3$ and K.E. $= \frac{1}{2} \times 0.5 \times 8^2$

When the string is horizontal,

P.E. $= 0.5 \times g \times 1.5$ and K.E. $= \frac{1}{2} \times 0.5 \times v^2$

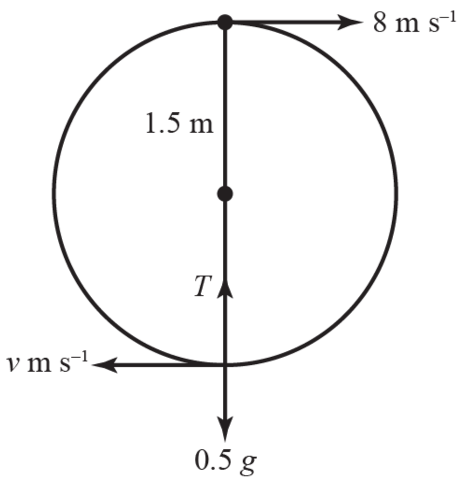
Energy is conserved

$$\Rightarrow 1.5g + 16 = 0.75g + \frac{v^2}{4}$$
$$v^2 = 4(0.75g + 16),\ v \approx 9.66\text{ ms}^{-1}$$

b The only force with a vertical component is the weight.

Acceleration $= g$ m s^{-2}

11 c



Let the speed of the particle at the lowest point be v m s^{-1}, and the tension in the rod be TN.

Take the lowest point as the zero level for potential energy, the particle starts with

$$\text{P.E.} = 0.5 \times g \times 3 \text{ and K.E.} = \frac{1}{2} \times 0.5 \times 8^2$$

When the string is vertical,

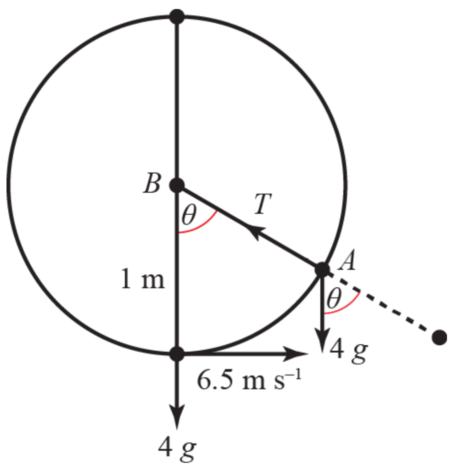
$$\text{P.E.} = 0 \text{ and K.E.} = \frac{1}{2} \times 0.5 \times v^2$$

Energy is conserved

$$\Rightarrow 1.5g + 16 = \frac{v^2}{4}, \quad v \approx 11.1 \text{ m s}^{-1}$$

$$T - 0.5g = \frac{0.5v^2}{1.5}, \quad T \approx 45.8 \text{ N}$$

12



At the lowest point let P.E.= 0 J.

The K.E. at the lowest point is

$$\frac{1}{2} \times 4 \times 6.5^2 = 84.5 \text{ J.}$$

When AB is at angle θ to the vertical, the tension in the rod is T, and the particle has speed v m s^{-1}

Particle has risen $(1 - \cos\theta)$, so

$$\text{P.E.} = 4 \times g \times (1 - \cos\theta) \text{ J and}$$

$$\text{K.E.} = \frac{1}{2} \times 4 \times v^2 = 2v^2$$

Energy is conserved $\Rightarrow 84.5 = 2v^2 + 4g(1 - \cos\theta)$,

$$v^2 = 42.25 - 2g(1 - \cos\theta)$$

Force towards the centre of the circle

$$= T - 4g\cos\theta = \frac{mv^2}{r} = \frac{4(42.25 - 2g(1 - \cos\theta))}{1}$$

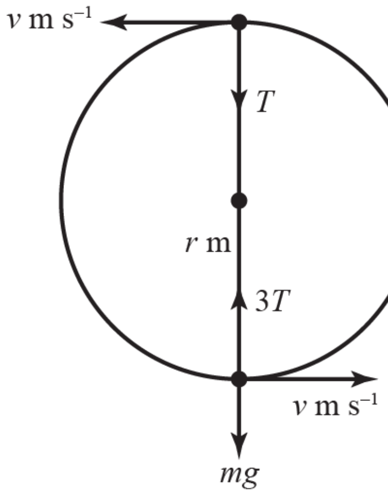
$$\therefore T = 4g\cos\theta + 169 - 8g(1 - \cos\theta) = 169 + 12g\cos\theta - 8g = 0 \text{ when}$$

$$\cos\theta = \frac{8g - 169}{12g} = -0.77\ldots \text{ giving } \theta \approx 140.4^\circ$$

i.e. $\theta \approx 39.6^\circ$ to the upward vertical and

$$v^2 = 42.25 - 2g(1 - \cos\theta) = 7.5\ldots, v \approx 2.74 \text{ m s}^{-1}$$

13



Let the speed at the lowest point be u m s^{-1}, and the speed at the highest point be v m s^{-1}

The gain in P.E. in moving from the lowest point to the highest is $2mgr$.

The loss in K.E. is $\frac{1}{2}mu^2 - \frac{1}{2}mv^2$

Energy is conserved

$\therefore\ 2mgr = \frac{1}{2}mu^2 - \frac{1}{2}mv^2,\ v^2 = u^2 - 4gr$

At the lowest point $3T - mg = \frac{mu^2}{r}$

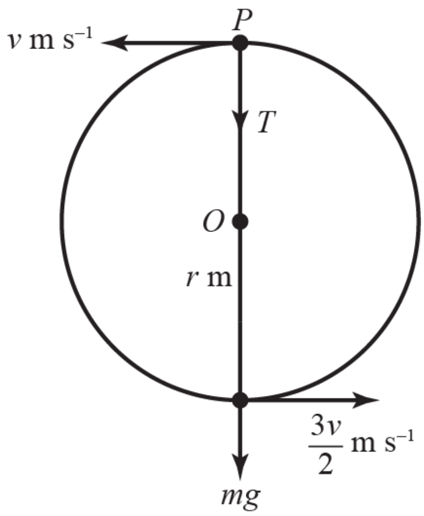
At the highest point $T + mg = \frac{mv^2}{r}$

Substituting for T and v^2 in the first of these two equations:

$$3\left(\frac{m(u^2-4gr)}{r} - mg\right) - mg = \frac{mu^2}{r},\ 3\frac{(u^2-4gr)}{r} - 4g = \frac{u^2}{r}$$

$$\frac{2u^2}{r} = 16g,\ u^2 = 8gr,\ u = \sqrt{8gr}$$

14 a



Let the speed at the lowest point be $\frac{3v}{2}$ m s^{-1}, and the speed at the highest point be v m s^{-1}

The gain in P.E. in moving from the lowest point to the highest is $2mgr$.

The loss in K.E. is $\frac{1}{2}m\left(\frac{3v}{2}\right)^2 - \frac{1}{2}mv^2$

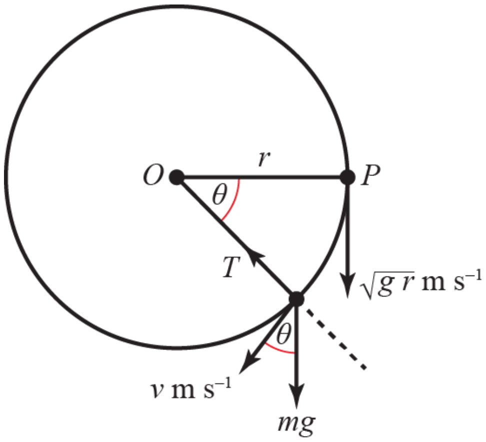
Energy is conserved

$$\therefore\ 2mgr = \frac{1}{2}m \times \frac{9v^2}{4} - \frac{1}{2}mv^2 = \frac{5}{8}mv^2$$

$$v^2 = \frac{16gr}{5},\ v = \sqrt{\frac{16gr}{5}}$$

b At the highest point, $T + mg = \frac{mv^2}{r} = \frac{m\frac{16gr}{5}}{r} = \frac{16mg}{5},\ T = \frac{11mg}{5}$

15 a



With OP horizontal, the particle has

$$\text{P.E.} = 0 \text{ and K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}mgr.$$

When OP is θ° below the horizontal, the tension in the string is T and the speed of the particle is v.

The particle has

$$\text{P.E.} = -mgr\sin\theta \text{ and K.E.} = \frac{1}{2}mv^2$$

Energy is conserved

$$\therefore \frac{1}{2}mgr = -mgr\sin\theta + \frac{1}{2}mv^2$$

$$v^2 = gr(1+2\sin\theta)$$

Resolving towards O:

$$T - mg\sin\theta = \frac{mv^2}{r} = \frac{mgr(1+2\sin\theta)}{r}$$

$$T = mg(1+3\sin\theta)\text{N}$$

b When $T = 2mg$ N, $2mg = mg(1+3\sin\theta)$, $\sin\theta = \frac{1}{3}, \theta \approx 19.5^\circ$

16 a At point S: $\text{G.P.E.} = 0.4 \times g \times 3.8 = 1.52g$

$\text{K.E.} = 0$

At point P: $\text{G.P.E.} = 0.4 \times g \times 4\sin\theta$

$= 1.6g\sin\theta$

$$\text{K.E.} = \frac{1}{2} \times 0.4 \times v^2 = 0.2v^2$$

By conservation of energy:

$1.52g = 1.6g\sin\theta + 0.2v^2$

$\Rightarrow 0.2v^2 = 1.52g - 1.6g\sin\theta$

$\Rightarrow v^2 = 7.6g - 8g\sin\theta$

$\Rightarrow v = \sqrt{7.6g - 8g\sin\theta}$

b Vertical height above O, 3.8 m.

c In reality there will be frictional forces acting on the handle so the height will be less than 3.8 m.