

Circular motion 1B

1 $a = r\omega^2$: $a = 0.16 \times 25 = 4 \text{ ms}^{-2}$

2 $a = \frac{v^2}{r}$: $a = \frac{2.5^2}{0.3} \approx 20.8 \text{ ms}^{-2}$

3 a $a = r\omega^2$: $75 = 3\omega^2$, $\omega^2 = 25$, $\omega = 5 \text{ rad s}^{-1}$

b $a = \frac{v^2}{r}$: $75 = \frac{v^2}{3}$, $v^2 = 3 \times 75 = 225$, $v = 15 \text{ ms}^{-1}$

4 a $a = r\omega^2$: $100 = 0.6\omega^2$, $\omega^2 \approx 166.7$, $\omega \approx 12.9 \text{ rad s}^{-1}$

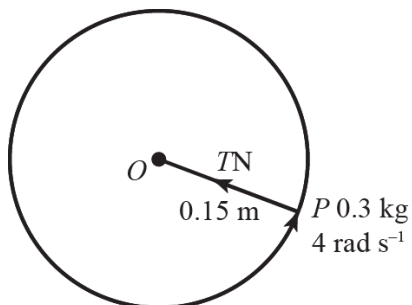
b $a = \frac{v^2}{r}$: $100 = \frac{v^2}{0.6}$, $v^2 = 100 \times 0.6 = 60$, $v = 7.75 \text{ ms}^{-1}$

5 $50 \text{ km h}^{-1} = \frac{50 \times 1000}{3600} \approx 13.89 \text{ ms}^{-1}$

$a = \frac{v^2}{r}$: $a = \frac{13.89^2}{90} \approx 2.14 \text{ ms}^{-2}$

6 $a = r\omega^2$: $6 = 75\omega^2$, $\omega^2 = 0.08$, $\omega \approx 0.283 \text{ rad s}^{-1}$

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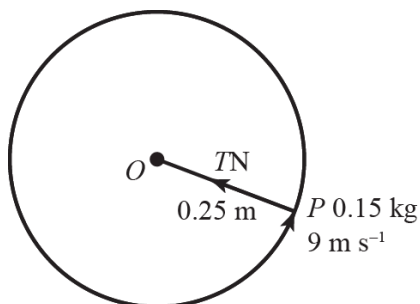


Suppose that the tension in the string is T .

Using $F = ma$

$$T = 0.3 \times 0.15 \times 4^2 = 0.72 \text{ N}$$

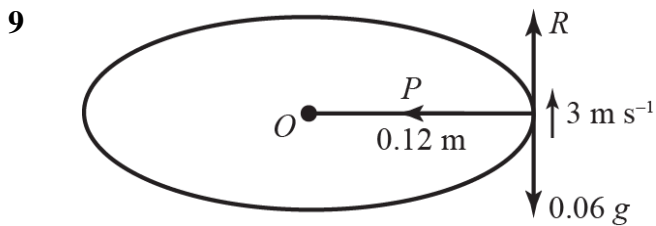
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Suppose that the tension in the string is T .

Using $F = ma$

$$T = \frac{0.15 \times 9^2}{0.25} = 48.6 \text{ N}$$



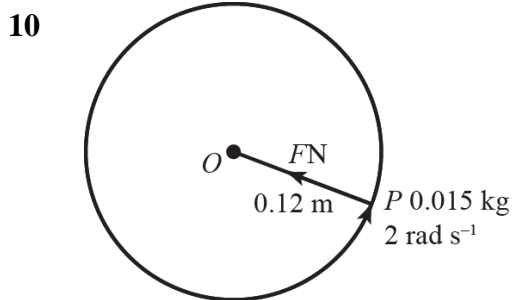
a Suppose the vertical component of the force is R :

$$R(\uparrow)R = 0.06g = 0.588 \text{ N}$$

b Suppose the horizontal component of the force is F :

Using $F = ma$,

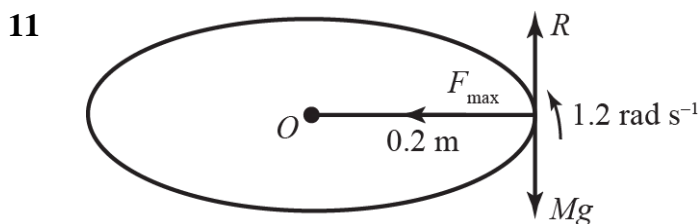
$$F = \frac{0.06 \times 3^2}{0.12} = 4.5 \text{ N}$$



a $v = r\omega$: $v = 0.12 \times 2 = 0.24 \text{ m s}^{-1}$

b Using $F = ma$

$$F = 0.015 \times 0.12 \times 2^2 = 0.0072 \text{ N}$$



Let R be the normal reaction between the particle and the disc, F the frictional force, M the mass of the particle, and μ be the coefficient of friction between the particle and the disc.

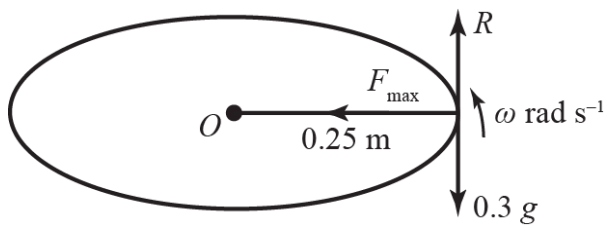
$$R(\uparrow): R = Mg$$

The particle is about to slip, so $F = F_{\max} = \mu R = \mu Mg$.

Using $F = ma$, $\mu Mg = M \times 0.2 \times 1.2^2 = M \times 0.288$,

$$\mu = \frac{0.288}{g} \approx 0.0294$$

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Let R be the normal reaction between the particle and the disc, F the frictional force.

Given $\mu = 0.25$ and $F = F_{max} = 0.25R$.

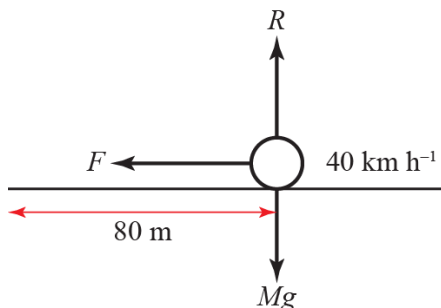
$R(\uparrow)$: $R = 0.3g$, so $F_{max} = 0.25 \times 0.3g$.

Using $F = ma$, $0.25 \times 0.3g = 0.3 \times 0.25 \times \omega^2$,

$$g = \omega^2$$

$$\omega \approx 3.13 \text{ rad s}^{-1}$$

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$$40 \text{ km h}^{-1} = \frac{40 \times 1000}{3600} \approx 11.11 \text{ m s}^{-1}$$

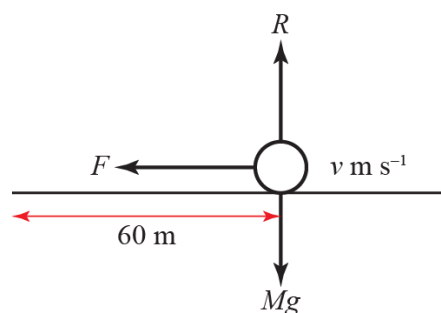
Let the mass of the car be M . Let F be the force due to friction between the car tyres and the road, μ the coefficient of friction, and R the normal reaction between the car and the road.

At maximum speed the car is about to slip, so $F = F_{max}$

$R(\uparrow) R = Mg$, so $F = F_{max} = \mu R = \mu Mg$

$R(\leftrightarrow)$ Using $F = ma$, $\mu Mg = \frac{M \times 11.11^2}{80}$, $\mu = \frac{11.11^2}{80 \times 9.8} \approx 0.157$

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Let the mass of the car be M .

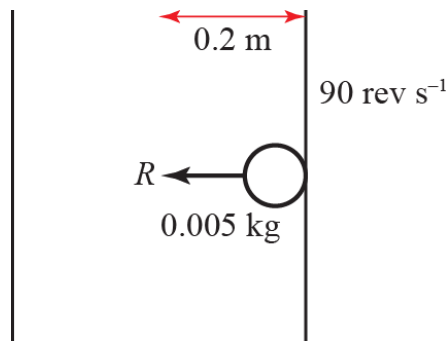
Let F be the force due to friction between the car tyres and the road, and R the normal reaction between the car and the road.

Max speed $\Rightarrow F = F_{max}$

$R(\uparrow) R = ma$, so $F = F_{max} = \mu R = \frac{1}{3} Mg$

$R(\leftrightarrow)$ Using $F = ma$, $\frac{1}{3} Mg = M \times 60 \times \omega^2$, $\omega^2 = \frac{g}{180} \approx 0.0544$, $\omega \approx 0.233 \text{ rad s}^{-1}$

15 a



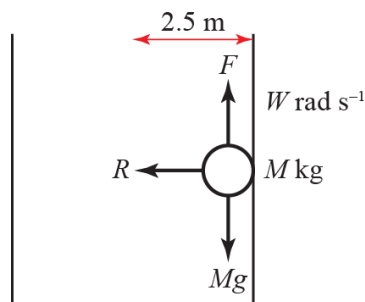
$$90 \text{ rev s}^{-1} = 90 \times 2\pi \text{ rad s}^{-1} \\ = 180\pi \text{ rad s}^{-1}$$

Let the normal reaction between the particle and the cylinder be R .

(\leftrightarrow) Using $F = ma$, $R = 0.005 \times 0.2 \times (180\pi)^2 \approx 320 \text{ N}$

b $F = mg$
 $= 0.005 \times 9.8$
 $R = 319.775$
 $\mu = \frac{F}{R}$
 $= 1.53 \times 10^{-4}$
 $= 0.000153 \text{ (3 s.f.)}$

16 a



Suppose that the person has mass M . Let the normal reaction between the person and the cylinder be R . F is the frictional force between the person and the wall of the cylinder.

Minimum $W \Rightarrow$ the person is about to slip $\Rightarrow F = F_{max} = \frac{2}{3}R$

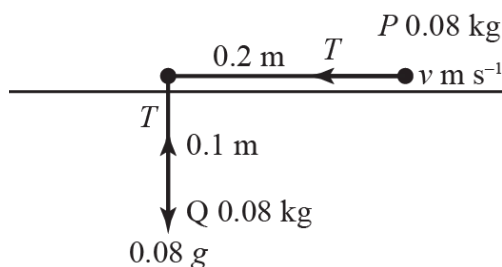
Also $R(\uparrow) \Rightarrow F_{max} = Mg$, so $Mg = \frac{2}{3}R$, $R = \frac{3Mg}{2}$

\leftrightarrow Using $F = ma$,

$$R = \frac{3Mg}{2} = M \times 2.5 \times W^2, W^2 = \frac{3g}{5}, W = \sqrt{\frac{3g}{5}} \approx 2.42 \text{ rad s}^{-1}$$

b No, because it is the minimum possible value for W . If the speed or the coefficient of friction reduced at all, the people would slip down the cylinder.

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Let the tension in the string be $T \text{ N}$, and the speed of P be $v \text{ m s}^{-1}$
 Q is in equilibrium, so
 $R(\uparrow) \text{ at } Q \Rightarrow T = 0.08 \text{ g}$

For P , Using $F = ma$,

$$T = 0.08 \text{ g} = \frac{0.08v^2}{0.2}, v^2 = 0.2 \times g \approx 1.96, v = 1.4 \text{ m s}^{-1}$$

- 18 a** Let the frictional force between the car tyre and the road be F , and the coefficient of friction be μ . The normal reaction between the car and the road is R .

$$R(\uparrow): R = Mg$$

$$R(\leftarrow): F = \frac{mv^2}{r}$$

The car does not slip at this speed,

$$F < \mu R$$

$$\mu > \frac{Mv^2}{Mg r}$$

$$\mu > \frac{v^2}{g r}$$

- b** Model assumes that the tyres all experience the same friction.

- 19** If the extension in the spring is x m, then the radius of the circle is $(0.3 + x)$.

The tension in the string is given by $T = \frac{\lambda x}{a} = \frac{10x}{0.3}$

Using $F = ma$,

$$\frac{10x}{0.3} = 0.25 \times (0.3 + x) \times 3^2$$

$$10x = \frac{9}{4} \times \frac{3}{10} (0.3 + x)$$

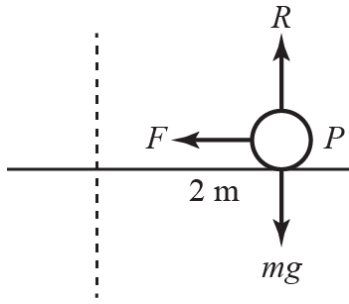
$$400x = 8.1 + 27x$$

$$373x = 8.1$$

$$x = 0.02171\dots$$

$$\begin{aligned} \text{Radius} &= 0.3 + 0.0217 \\ &= 0.322 \text{ m (3 s.f.)} \end{aligned}$$

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F is force due to friction,

R is the normal reaction.

$$R(\updownarrow): R = mg$$

$$R(\leftrightarrow): F = mr\omega^2$$

If P is not to slip then

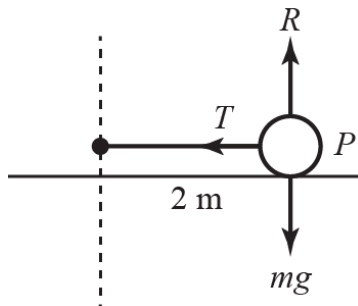
$$0.3 \times 4g \geq 4 \times 2 \times \omega^2$$

$$\therefore \omega^2 \leq \frac{147}{100}$$

T is the tension in the elastic string:

$$T = \frac{\lambda x}{l} = \frac{12 \times (2 - 1.5)}{1.5}$$

$$= 4 \text{ N}$$



The limits for ω^2 depends on whether the friction is acting with the tension or against it:

$$R(\leftrightarrow): 0.3 \times 4 \times g + 4 \geq 4 \times 2 \times \omega^2$$

$$\omega^2 = \frac{197}{100}$$

$$R(\leftrightarrow): -0.3 \times 4 \times g + 4 \leq 4 \times 2 \times \omega^2$$

$$\omega^2 \geq \frac{97}{100}$$

$$\frac{\sqrt{97}}{10} \leq \omega \leq \frac{\sqrt{197}}{10}$$

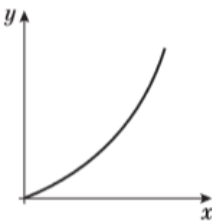
Challenge

a

$$x = pt$$

$$t = \frac{x}{p}$$

$$y = q \left(\frac{x}{p} \right)^2$$

$$= \frac{q}{p^2} x^2$$


b $\frac{dx}{dt} = p; \frac{dy}{dt} = 2qt$

Acceleration is $2q$ in the positive y -direction. Speed at the origin is p .

c Solve for y ,

$$x^2 + (y - R)^2 = R^2$$

$$(y - R)^2 = R^2 - x^2$$

$$y = R \pm \sqrt{R^2 - x^2}$$

For lower half of a circle

$$y = R - \sqrt{R^2 - x^2}$$

d $R = \frac{p^2}{2q}$

e $2q$

f The acceleration of P and Q are equal.