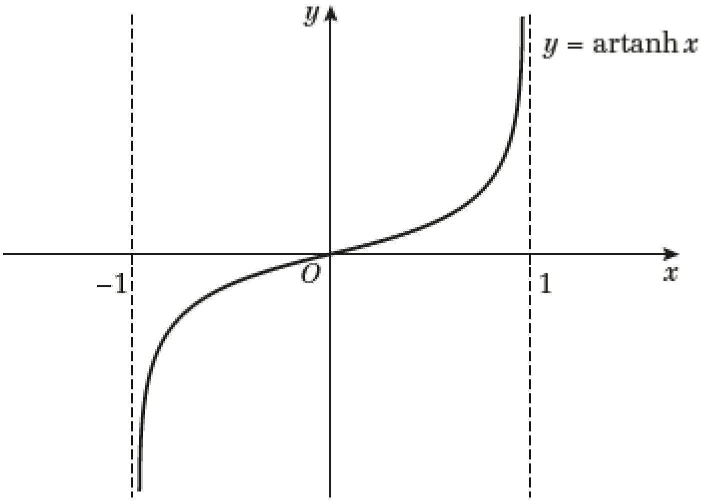
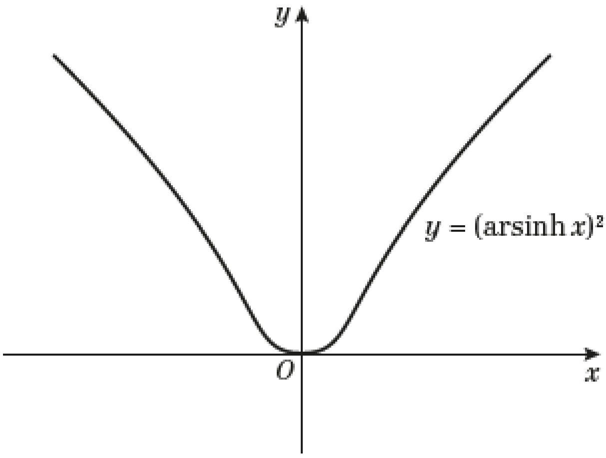
Hyperbolic Functions 6B

1



$y = \text{artanh}x,\ |x| < 1.$

2



$y = (\text{arsinh}\, x)^2$

3 $y = \text{artanh}\, x$

$$x = \tanh\, y = \frac{e^{2y}-1}{e^{2y}+1}$$

$$x\left(e^{2y}+1\right) = e^{2y}-1$$

$$1+x = e^{2y}(1-x)$$

$$e^{2y} = \frac{1+x}{1-x}$$

$$2y = \ln\left(\frac{1+x}{1-x}\right)$$

$$y = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

$$\text{artanh}\, x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right),$$

$$|x| < 1$$

For $|x| \geq 1$, $\ln\left(\frac{1+x}{1-x}\right)$ is not defined, since $\frac{1+x}{1-x} \leq 0$.

4 a $\text{arsinh}\, 2 = \ln(2+\sqrt{2^2+1})$

$= \ln(2+\sqrt{5})$

b $\text{arcosh}\, 3 = \ln(3+\sqrt{3^2-1})$

$= \ln(3+\sqrt{8})$

$= \ln(3+2\sqrt{2})$

c $\text{artanh}\left(\frac{1}{2}\right) = \frac{1}{2}\ln\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right)$

$= \frac{1}{2}\ln 3$

5 a $\text{arsinh}\sqrt{2} = \ln(\sqrt{2}+\sqrt{2+1})$

$= \ln(\sqrt{2}+\sqrt{3})$

b $\text{arcosh}\sqrt{5} = \ln(\sqrt{5}+\sqrt{5-1})$

$= \ln(2+\sqrt{5})$

c $\text{artanh}\, 0.1 = \frac{1}{2}\ln\left(\frac{1+0.1}{1-0.1}\right)$

$= \frac{1}{2}\ln\left(\frac{11}{9}\right)$

6 **a** $\text{arsinh}(-3) = \ln(-3 + \sqrt{(-3)^2 + 1})$

$= \ln(-3 + \sqrt{10})$

b $\text{arcosh}\left(\frac{3}{2}\right) = \ln\left(\frac{3}{2} + \sqrt{\left(\frac{3}{2}\right)^2 - 1}\right)$

$= \ln\left(\frac{3}{2} + \sqrt{\frac{5}{4}}\right)$

$= \ln\left(\frac{3}{2} + \frac{\sqrt{5}}{2}\right)$

$= \ln\left(\frac{3 + \sqrt{5}}{2}\right)$

c $\text{artanh}\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{2}\ln\left(\frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}\right)$

$= \frac{1}{2}\ln\left(\frac{\sqrt{3} + 1}{\sqrt{3} - 1}\right)$

$= \frac{1}{2}\ln\left(\frac{\sqrt{3} + 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}\right)$

$= \frac{1}{2}\ln\left(\frac{4 + 2\sqrt{3}}{2}\right)$

$= \frac{1}{2}\ln(2 + \sqrt{3})$

7 $\text{artanh}\, x + \text{artanh}\, y$

$$= \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) + \frac{1}{2}\ln\left(\frac{1+y}{1-y}\right)$$

Use $\ln a + \ln b = \ln(ab)$.

$$= \frac{1}{2}\ln\left(\frac{1+x}{1-x} \times \frac{1+y}{1-y}\right)$$

$$= \frac{1}{2}\ln\left(\frac{1+x+y+xy}{1-x-y+xy}\right)$$

Use $\frac{1}{2}\ln a = \ln a^{\frac{1}{2}}$.

$$= \ln\sqrt{\left(\frac{1+x+y+xy}{1-x-y+xy}\right)}$$

So $\frac{1+x+y+xy}{1-x-y+xy} = 3$

$$1+x+y+xy = 3-3x-3y+3xy$$

$$1+x-3+3x = -3y+3xy-y-xy$$

$$2xy-4y = 4x-2$$

$$y(x-2) = 2x-1$$

$$y = \frac{2x-1}{x-2}$$