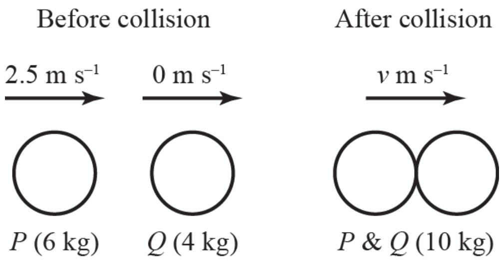
A level Exam-style practice

1 a $e = 0$

b



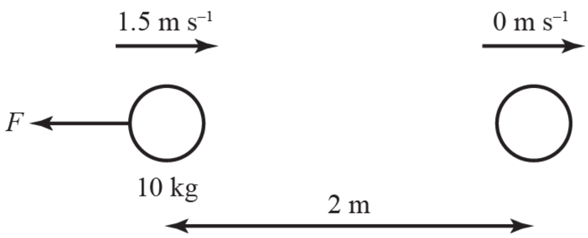
Using conservation of momentum for the system $(\rightarrow)$:

$$6\times2.5+4\times0=10v$$
$$15=10v$$
$$v=\frac{15}{10}=\frac{3}{2}$$
$$v=1.5\ \text{m s}^{-1}$$

c Kinetic energy lost = initial kinetic energy – final kinetic energy

$$=\frac{1}{2}\times6\times2.5^2-\frac{1}{2}\times10\times1.5^2$$
$$=7.5\ \text{J}$$

d



Work done by friction = loss of kinetic energy

$$F\times2=\frac{1}{2}\times10\times1.5^2$$
$$F=5.625\ \text{N} \quad \textbf{(1)}$$

Friction: $F=\mu R$

$(\uparrow)$ R = 10g

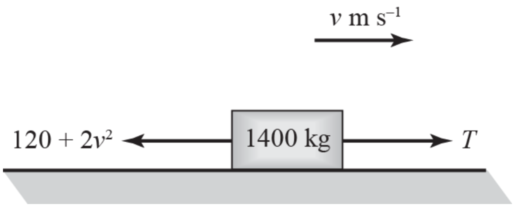
So $F=10g\mu$ **(2)**

From equations **(1)** and **(2)**:

$$10g\mu=5.625$$

So $\mu=\frac{5.625}{10g}=\frac{5.625}{10\times9.8}=0.057$

2 a



Power $= 20$ kW $= 20\,000$ W
Power $= Tv$

So $T = \dfrac{20\,000}{v}$

Using $F = ma$ $(\rightarrow)$

$T - (120 + 2v^2) = 1400a$

$\dfrac{20\,000}{v} - (120 + 2v^2) = 1400a$

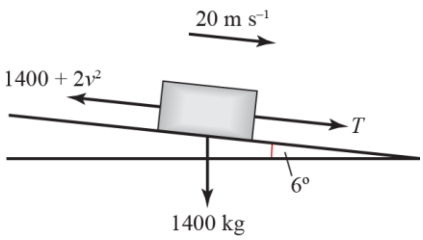
When $v = 16$:

$\dfrac{20\,000}{16} - (120 + 2 \times 16^2) = 1400a$

$618 = 1400a$

So $a = \dfrac{618}{1400} = 0.44 \text{ m s}^{-2}$

b



Power $= 20\,000$ W
Power $= Tv = T \times 20$
So $20\,000 = 20T$
and $T = 1000$ N

The total force down the plane is:

$T + 1400g \sin 6^\circ$

$= 1000 + 1400 \times 9.8 \times \sin 6^\circ$
$= 2434$ N

The resistance to the car's motion is:

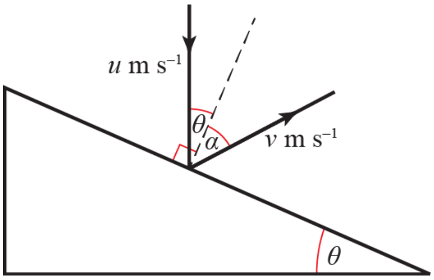
$120 + 2v^2 = 120 + 2 \times 20^2 = 920$ N

The total force down the plane is greater than the resistive force, so there is a net force down the plane.
Therefore the driver will need to brake to maintain his or her original speed.

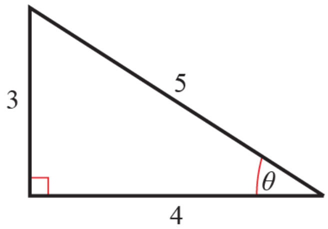
2 c The driver places the car in neutral, so $T = 0$
The maximum speed of the car will occur when its acceleration is $0\ \text{m s}^{-2}$
i.e. when the resultant force parallel to the plane is zero.
Therefore $120 + 2v^2 = 1400g\sin 6^\circ$

So $v = \sqrt{\dfrac{1400g\sin 6^\circ - 120}{2}}$ and $v = 25.6\ \text{m s}^{-1}$

3



Since $\tan\theta = \dfrac{3}{4}$, you have that $\sin\theta = \dfrac{3}{5}$ and $\cos\theta = \dfrac{4}{5}$ from the right-angled triangle:



Suppose the ball hits the plane with speed $u\ \text{m s}^{-1}$ and rebounds with speed $v\ \text{m s}^{-1}$

Then down the plane $(\searrow)$: $u\sin\theta = v\sin\alpha$ **(1)**

Newton's law of restitution parallel to the plane $(\nearrow)$: $v\cos\alpha = eu\cos\theta$ **(2)**

Squaring equation **(1)** gives $u^2\sin^2\theta = v^2\sin^2\alpha$
Squaring equation **(2)** gives $v^2\cos^2\alpha = e^2u^2\cos^2\theta$

Adding these equations gives:

$$v^2\sin^2\alpha + v^2\cos^2\alpha = u^2\sin^2\theta + e^2u^2\cos^2\theta$$
$$v^2\left(\sin^2\alpha + \cos^2\alpha\right) = u^2\sin^2\theta + e^2u^2\cos^2\theta$$
$$v^2 = u^2\sin^2\theta + e^2u^2\cos^2\theta$$
$$v^2 = \left(\frac{3}{5}\right)^2 u^2 + \left(\frac{4}{5}\right)^2 e^2u^2$$
$$v^2 = \frac{9u^2}{25} + \frac{16e^2u^2}{25} \qquad \textbf{(3)}$$

Since the ball loses half its kinetic energy upon impact, you have

$$\frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{1}{2}mu^2\right) = \frac{1}{4}mu^2$$

So $v^2 = \dfrac{u^2}{2}$ **(4)**

3 continued

Solving equations **(3)** and **(4)** simultaneously, you obtain:

$$\frac{u^2}{2} = \frac{9u^2}{25} + \frac{16e^2u^2}{25}$$

$$\frac{1}{2} = \frac{9}{25} + \frac{16e^2}{25}$$

$$\frac{1}{2} - \frac{9}{25} = \frac{16e^2}{25}$$

$$\frac{7}{50} = \frac{16e^2}{25}$$

So $e^2 = \frac{7}{32}$

and $e = \sqrt{\frac{7}{32}} = 0.468$

4 a Impulse on the football = change in momentum of the football

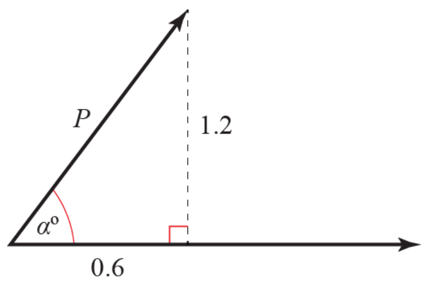
$$\mathbf{P} = m\mathbf{v} - m\mathbf{u}$$

$$\mathbf{P} = 0.2\begin{pmatrix} 8 \\ 4 \end{pmatrix} - 0.2\begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{8}{5} \\ \frac{4}{5} \end{pmatrix} - \begin{pmatrix} 1 \\ \frac{-2}{5} \end{pmatrix} = \begin{pmatrix} \frac{3}{5} \\ \frac{6}{5} \end{pmatrix}$$

$$|\mathbf{P}| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{6}{5}\right)^2} = \sqrt{\frac{45}{25}} = \frac{3\sqrt{5}}{5} = 1.34 \text{ N s}$$

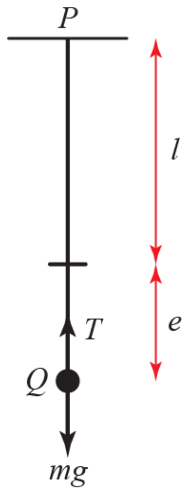
b Let α be the angle between $\mathbf{P}$ and $\mathbf{i}$



$$\tan\alpha = \left(\frac{1.2}{0.6}\right)$$

So $\alpha = 63.4^\circ$

5 a



Consider the ball hanging in vertical equilibrium.

$(\uparrow) T = mg$

Hooke's Law gives $T = \frac{\lambda e}{l}$

So $mg = \frac{\lambda e}{l}$

So $e = \frac{mgl}{\lambda} = \frac{0.25 \times 9.8 \times 1.2}{15} = 0.196$ m

So $PQ = l + e = 1.2 + 0.196 = 1.4$ m

b When the string has length 1.9 m, its extension is $1.9 - 1.2 = 0.7$ m

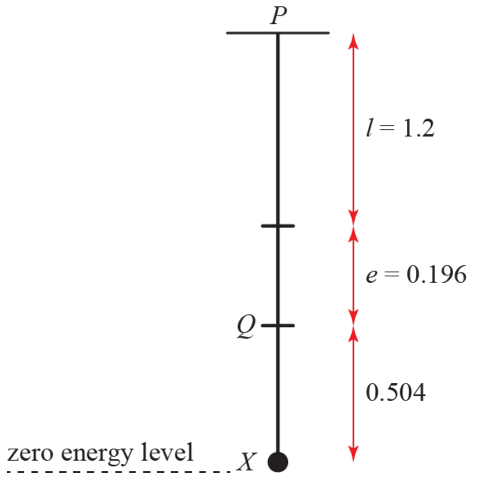
So its elastic potential energy is $\frac{15 \times 0.7^2}{2 \times 1.2} = 3.0625\,\text{J}$

When the string is in equilibrium, its extension is 0.196 m

So its elastic potential energy is $\frac{15 \times 0.196^2}{2 \times 1.2} = 0.2401\,\text{J}$

Therefore the work done in stretching the string to a length of 1.9 m is $3.0625 - 0.2401 = 2.8\,\text{J}$

5 c



$PX = 1.9$, so $QX = PX - PQ = 1.9 - (1.2 + 0.196) = 0.504$ m

Let v be the velocity of the ball as it passes through Q.

Using the conservation of energy:

EPE + PE + KE at Q = EPE at X

$$\frac{15 \times 0.196^2}{2 \times 1.2} + 0.25 \times 9.8 \times 0.504 + \frac{1}{2} \times 0.25 \times v^2 = \frac{15 \times 0.7^2}{2 \times 1.2}$$

$$0.2401 + 1.2348 + 0.125v^2 = 3.0625$$

$$0.125v^2 = 1.5876$$

So $v = \sqrt{\frac{1.5876}{0.125}} = 3.6 \text{ m s}^{-1}$

d Let h be the distance travelled by the ball above point X

After travelling through a distance h, the string will be slack and its velocity will be zero.

Using the work-energy principle:

Potential energy gained = elastic potential energy lost

$$0.25 \times 9.8 \times h = \frac{15 \times 0.7^2}{2 \times 1.2}$$

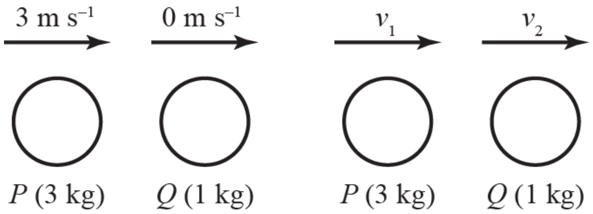
$$2.45h = 3.0625$$

So $h = \frac{3.0625}{2.45} = 1.25$

This is a distance of $1.9 - 1.25 = 0.65$ m from the ceiling.

Hence, the ball will not hit the ceiling.

6 a i



Using conservation of momentum for the system $(\rightarrow)$:

$3 \times 3 = 3v_1 + 1v_2$

$9 = 3v_1 + 1v_2$ **(1)**

Consider the final kinetic energy of Q: $\frac{1}{2} \times 1 \times v_2^{\,2} = 3.645$

$v_2^{\,2} = 7.29$

$v_2 = \sqrt{7.29} = 2.7 \text{ m s}^{-1}$

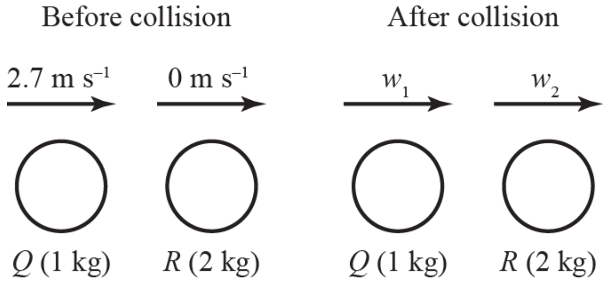
Substituting v_2 into equation (**1**) gives $9 = 3v_1 + 2.7$

$v_1 = \frac{9 - 2.7}{3} = 2.1 \text{ m s}^{-1}$

ii Newton's law of restitution:

$e = \frac{\text{separation speed}}{\text{approach speed}} = \frac{2.7 - 2.1}{3 - 0} = 0.2$

6 b Consider the collision between Q and R:



Using conservation of momentum for the system $(\rightarrow)$:

$2.7 = w_1 + 2w_2$ **(2)**

Newton's law of restitution:

$0.2 = \frac{w_2 - w_1}{2.7}$

$0.54 = w_2 - w_1$ **(3)**

Adding equations **(2)** and **(3)** gives:

$3.24 = 3w_2$

So $w_2 = 1.08 \text{ m s}^{-1}$

Substituting w_2 into equation **(3)** gives:

$0.54 = 1.08 - w_1$

So $w_1 = 1.08 - 0.54 = 0.54 \text{ m s}^{-1}$

So the kinetic energy lost in this collision is given by

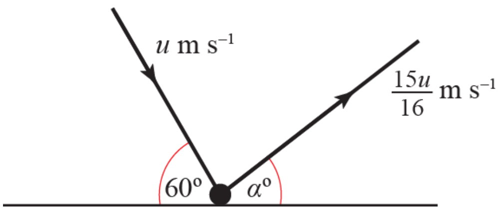
$\frac{1}{2}\times 1\times 2.7^2 - \left(\frac{1}{2}\times 1\times 0.54^2 + \frac{1}{2}\times 2\times 1.08^2\right)$

$= 2.33 \text{ J}$

c P moves with speed 2.1 m s^{-1} and Q moves with speed 0.54 m s^{-1}

Since P and Q are moving in the same direction, they will collide again.

7 a



i $(\rightarrow)$ $u\cos 60^\circ = \frac{15u}{16}\cos\alpha$

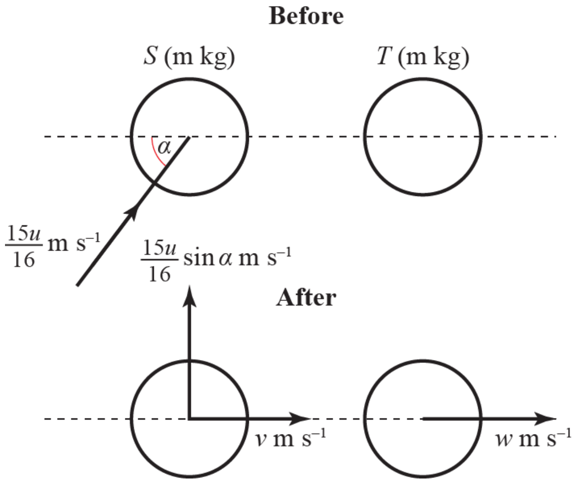
$\cos\alpha = \frac{15\cos 60}{15} = \frac{8}{15}$

$\alpha = 57.8^\circ$

ii Newton's law of restitution

gives: $e = \frac{\text{final speed (vertically)}}{\text{approach speed (vertically)}} = \frac{\frac{15u}{16}\sin\alpha}{u\sin 60^\circ} = \frac{15}{16}\left(\frac{\sin 57.8^\circ}{\sin 60^\circ}\right) = 0.916$

7 b



Conservation of momentum parallel to the line of centres:

$$m\frac{15u}{16}\cos\alpha = mv + mw$$

$$\frac{15u\cos\alpha}{16} = v + w \qquad \textbf{(1)}$$

Newton's law of restitution:

$$\frac{3}{4} = \frac{w - v}{\frac{15u}{16}\cos\alpha}$$

$$\frac{45}{64}u\cos\alpha = w - v \qquad \textbf{(2)}$$

Adding equations **(1)** and **(2)** gives:

$$\frac{15}{16}u\cos\alpha + \frac{45}{64}u\cos\alpha = 2w$$

$$\frac{105}{64}u\cos\alpha = 2w$$

Substituting $\cos\alpha = \frac{8}{15}$ leads to

$$\frac{105}{64}u\left(\frac{8}{15}\right) = 2w$$

$$w = \frac{7u}{16} = 0.4375u$$

Substitute $w = \frac{7u}{16}$ in equation **(1):**

$$\frac{15u}{16}\cos\alpha = v + \frac{7u}{16}$$

$$v = \frac{15u}{16}\left(\frac{8}{15}\right) - \frac{7u}{16}$$

$$v = \frac{u}{16} = 0.0625u$$

7 b continued

So the speed of S is given by $\sqrt{\left(\frac{15u}{16}\sin 57.769^\circ\right)^2 + \left(\frac{u}{16}\right)^2} = 0.795u$

$$\tan\theta = \frac{\frac{15u}{16}\sin 57.769^\circ}{v} = \frac{\frac{15u}{16}\sin 57.769^\circ}{0.0625u} = \frac{\frac{15}{16}\sin 57.769^\circ}{0.0625}$$

So $\theta = 85.5^\circ$

Therefore S has velocity $0.795u \text{ m s}^{-1}$ at 85.5° to the line of centres.

T has velocity $0.4375u \text{ m s}^{-1}$ along the line of centres.