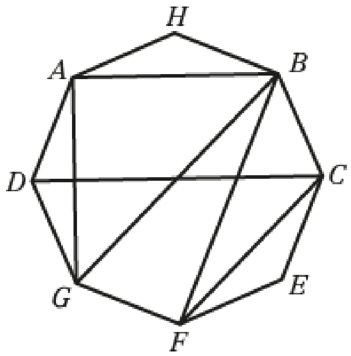
A Level Practice paper

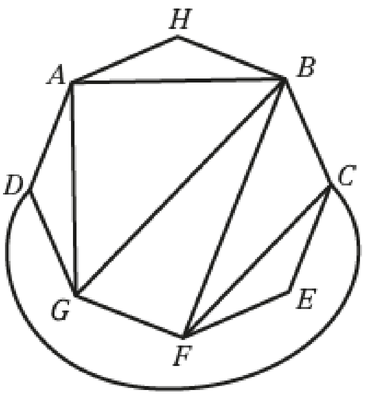
1 a Starting with *AB* means that *H* cannot be included without repeating either *B* or *A*.

b *AHBCEFGDA*

c Draw a polygon matching the Hamiltonian cycle and look at the inside edges one at a time.



AB(I), *AG*, *BF*, *BG*, *CD*, *CF*
AB(I), *AG*(I), *BF*, *BG*, *CD*, *CF*
AB(I), *AG*(I), *BF*(I), *BG*(I), *CD*(O), *CF*
AB(I), *AG*(I), *BF*(I), *BG*(I), *CD*(O), *CF*(I)
The graph is planar.



2 a

42	31	36	18	27	33	41	47	12	24	16
42	36	41	47	[33]	31	18	27	12	24	16
42	47	[41]	36	[33]	31	18	27	24	16	[12]
[47]	42	[41]	[36]	[33]	31	[27]	18	24	16	[12]
[47]	[42]	[41]	[36]	[33]	[31]	[27]	[24]	18	16	[12]
[47]	[42]	[41]	[36]	[33]	[31]	[27]	[24]	18	[16]	[12]

All of the numbers have now been selected as pivots, so the list is in order.

b Using the first fit decreasing algorithm with reels of size 80

$47 \rightarrow 1$

$42 \rightarrow 2$

$41 \rightarrow 3$

$36 \rightarrow 4$

$33 \rightarrow 1$ making it 80

$31 \rightarrow 2$ making it 73

$27 \rightarrow 3$ making it 68

$24 \rightarrow 4$ making it 60

$18 \rightarrow 4$ making it 78

$16 \rightarrow 5$

$12 \rightarrow 5$ making it 28

			18	
33	31	27	24	12
47	42	41	36	16

5 reels are required.

c $0.034 \times \frac{5000 \log 5000}{800 \log 800} = 0.27$ seconds

This is only an estimate because the time taken is only approximately proportional to $n \log n$

3 a A network is semi-Eulerian if it has exactly two odd nodes.

b The odd nodes are A, B, C, D, E, F but we can ignore E and B.
Possible pairings: $AC + DF = 10 + 10 = 20$
$AD + CF = 11 + 9 = 20$
$AF + CD = 2 + 10 = 12$
So we use AF and CD
Minimum length = 72 + 12 = 84 miles

c AF, CD

d This time we can ignore E and C
Possible pairings: $AB + DF = 4 + 10 = 14$
$AD + BF = 11 + 5 = 16$
$AF + BD = 2 + 13 = 15$
So we use AB and DF
The route is extended by $14 - 12 = 2$ miles

4 a Dijkstra's or Floyd's Algorithm

b Arc AC because of lack of symmetry in the table

c 1st iteration (no change)

–	5	∞	∞	∞
5	–	11	16	∞
10	11	–	8	6
∞	16	8	–	10
∞	∞	6	10	–

A	B	C	D	E
A	B	C	D	E
A	B	C	D	E
A	B	C	D	E
A	B	C	D	E

2nd iteration

–	5	16	21	∞
5	–	11	16	∞
10	11	–	8	6
21	16	8	–	10
∞	∞	6	10	–

A	B	B	B	E
A	B	C	D	E
A	B	C	D	E
B	B	C	D	E
A	B	C	D	E

3rd iteration

–	5	16	21	22
5	–	11	16	17
10	11	–	8	6
18	16	8	–	10
16	17	6	10	–

A	B	B	B	C
A	B	C	D	C
A	B	C	D	E
C	B	C	D	E
C	C	C	D	E

4th iteration (no change)

–	5	16	21	22
5	–	11	16	17
10	11	–	8	6
18	16	8	–	10
16	17	6	10	–

A	B	B	B	C
A	B	C	D	C
A	B	C	D	E
C	B	C	D	E
C	C	C	D	E

5th iteration (no change)

–	5	16	21	22
5	–	11	16	17
10	11	–	8	6
18	16	8	–	10
16	17	6	10	–

A	B	B	B	C
A	B	C	D	C
A	B	C	D	E
C	B	C	D	E
C	C	C	D	E

d In final iteration look at row A column E – goes through C
Now look at row A column C – goes through B
So $ABCE$ and length 22

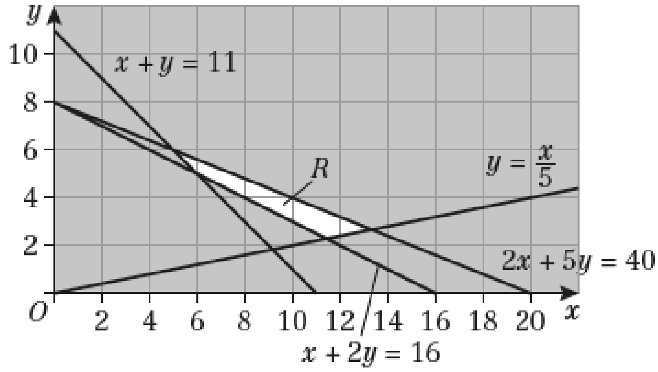
5 a $12x + 30y \leqslant 240$, i.e. $2x + 5y \leqslant 40$

$12x + 30y \leqslant 2(5x + 20y)$, i.e. $x \leqslant 5y$ or $y \geqslant \frac{x}{5}$

$5x + 5y \geqslant 55$, i.e. $x + y \geqslant 11$

$6x + 12y \geqslant 96$, i.e. $x + 2y \geqslant 16$

b

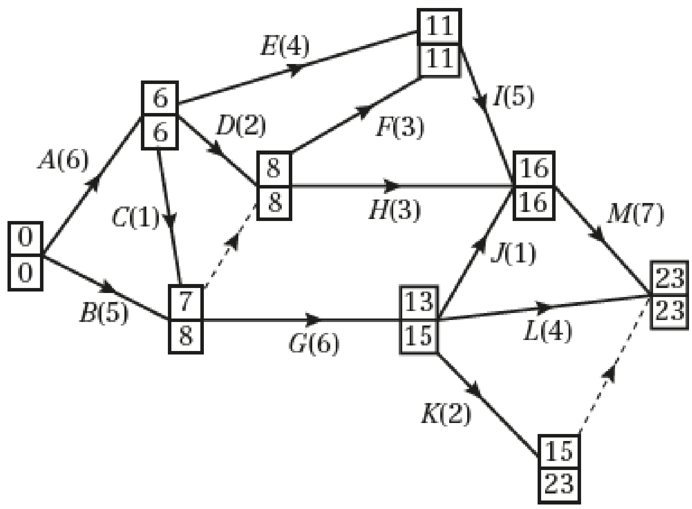


c

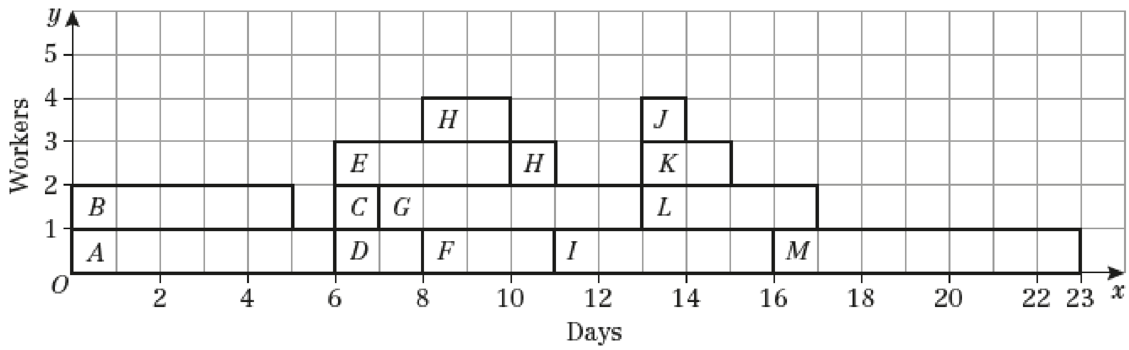
Vertex	(6, 5)	(5, 6)	$\left(\frac{80}{7}, \frac{16}{7}\right)$	$\left(\frac{40}{3}, \frac{8}{3}\right)$
Profit	£153	£166	£139.43	£162.67

So Angie should make 5 mini-packs and 6 mega-packs and the corresponding profit will be £166.

6 a



b



So 4 workers are required

c Delay the start of activity H by 2 days and the start of activity J by 2 days.

7 a The purpose of the first stage is to provide a basic feasible solution as a starting point for the second stage.

b Using slack, surplus and artificial variables we obtain:

$2x + y + z + s_1 = 50$

$x + 3y + z + s_2 = 60$

$x - s_3 + a_1 = 10$

This gives the first 3 lines of the tableau

$P - x - 2y - z = 0$ gives the 4th line

We want to maximise $I = -a_1 = x - s_3 - 10$

$\Rightarrow I - x + s_3 = -10$

which gives the 5th line

b.v.	x	y	z	s_1	s_2	s_3	a_1	Value
s_1	2	1	1	1	0	0	0	50
s_2	1	3	1	0	1	0	0	60
a_1	1	0	0	0	0	−1	1	10
P	−1	−2	−1	0	0	0	0	0
I	−1	0	0	0	0	1	0	−10

c So setting $s_3, y, z = 0$ we obtain the first solution of

$I = 0$ when $a_1 = 0, s_1 = 30, s_2 = 50$

d x now replaces a_1 as a basic variable so we obtain the following table

b.v.	x	y	z	s_1	s_2	s_3	Value
s_1	0	1	1	1	0	2	30
s_2	0	3	1	0	1	1	50
x	1	0	0	0	0	−1	10
P	0	−2	−1	0	0	−1	10

The smallest negative value in the bottom row is −2
So the y-column is the pivot column
The smallest value of θ is then $50 \div 3$
So the pivot is the 3 in the y-column

e The iterations are as follows:

bv	x	y	z	s_1	s_2	s_3	Value
s_1	0	1	1	1	0	2	30
s_2	0	3	1	0	1	1	50
x	1	0	0	0	0	−1	10
P	0	−2	−1	0	0	−1	10

bv	x	y	z	s_1	s_2	s_3	Value	Row operations
s_1	0	1	1	1	0	2	30	
y	0	1	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{50}{3}$	$R_2 \div 3$
x	1	0	0	0	0	−1	10	
P	0	−2	−1	0	0	−1	10	

bv	x	y	z	s_1	s_2	s_3	Value	Row operations
s_1	0	0	$\frac{2}{3}$	1	$-\frac{1}{3}$	$\frac{5}{3}$	$\frac{40}{3}$	$R_1 - R_2$
y	0	1	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{50}{3}$	
x	1	0	0	0	0	−1	10	
P	0	0	$-\frac{1}{3}$	0	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{130}{3}$	$R_4 + 2R_2$

bv	x	y	z	s_1	s_2	s_3	Value	Row operations
z	0	0	1	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{5}{2}$	20	$R_1 \times \frac{3}{2}$
y	0	1	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{50}{3}$	
x	1	0	0	0	0	−1	10	
P	0	0	$-\frac{1}{3}$	0	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{130}{3}$	

bv	x	y	z	s_1	s_2	s_3	Value	Row operations
z	0	0	1	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{5}{2}$	20	
y	0	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	10	$R_2 - \frac{1}{3}R_1$
x	1	0	0	0	0	−1	10	
P	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	50	$R_4 + \frac{1}{3}R_1$

So solution is $P = 50$ when $x = 10$, $y = 10$, $z = 20$