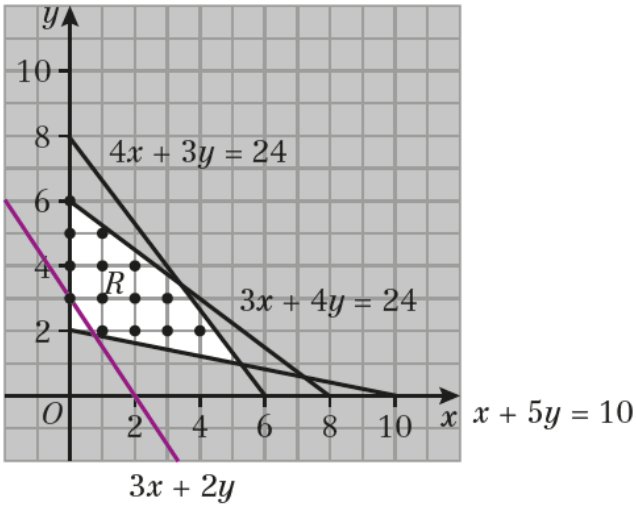
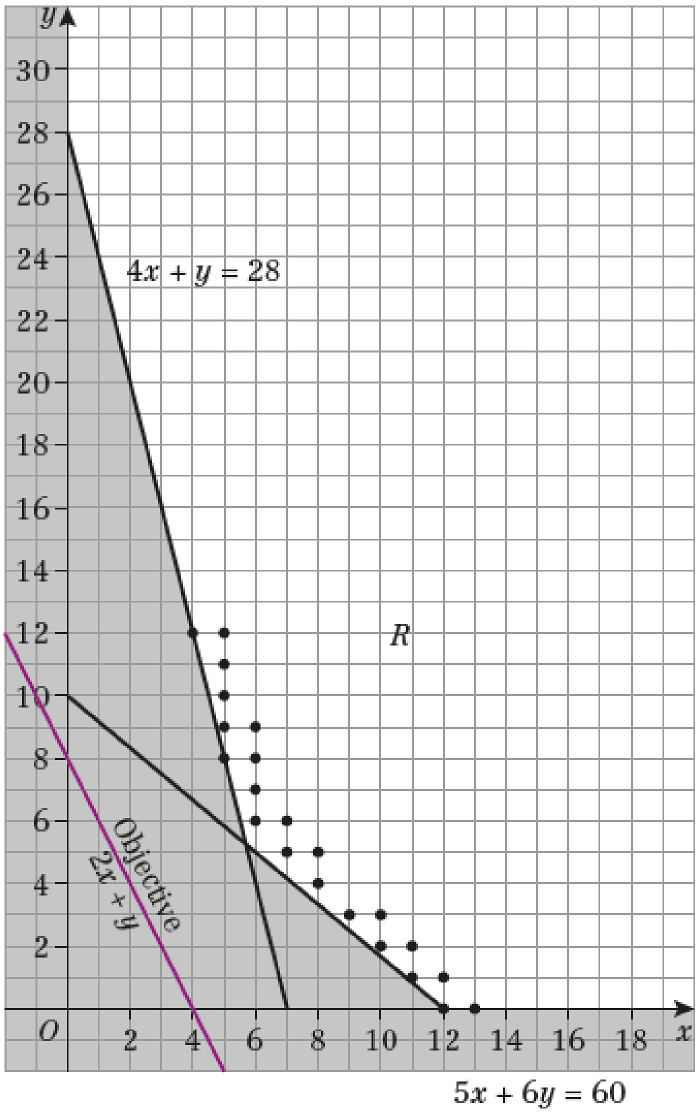
Linear programming 6D

1 a



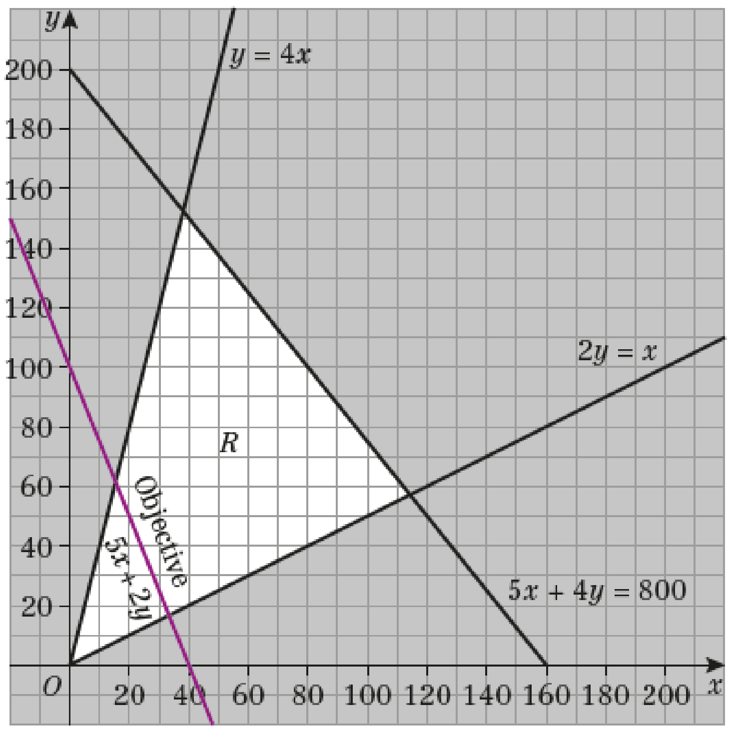
Maximum integer value (5, 1)
$3x+2y=17$

b



Minimum integer value (6, 5)
$2x+y=17$

1 c



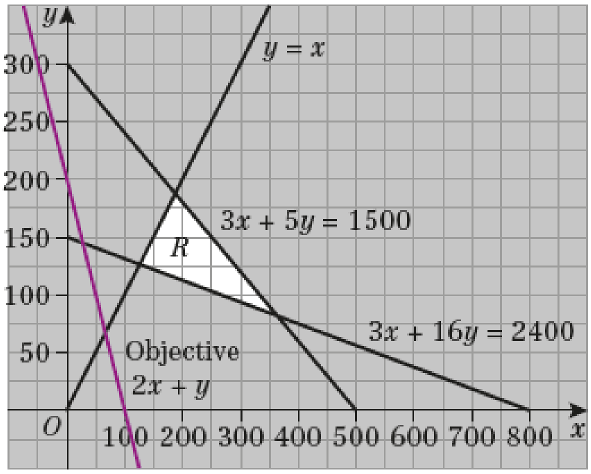
Solving $2y = x$ and $5x + 4y = 800$ simultaneously gives $114\frac{2}{7}, 57\frac{1}{7}$

Test integer values nearby.

Point	$2y \geqslant x$	$5x+4y \leqslant 800$	$5x+2y$
(114, 57)	✓	✓	684
(114, 58)	✓	✗	-
(115, 57)	✗	✗	-
(115, 58)	✓	✗	-

so optimal point is (114, 57) value 684.

1 d



Solving $3x + 16y = 2400$
$3x + 5y = 1500$
simultaneously gives
$\left(363\frac{7}{11}, 81\frac{9}{11}\right)$

Taking integer point

Point	$3x+16y \geqslant 2400$	$3x+5y \leqslant 1500$	$2x+y$
(363, 81)	✓	✗	-
(363, 82)	✓	✓	808
(364, 81)	✗	✓	-
(364, 82)	✓	✗	-

So optimal integer point is (363, 82)
value 808

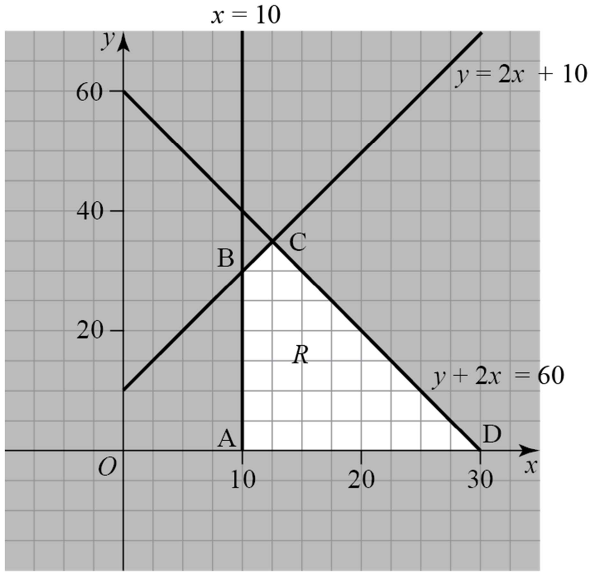
2 Identify vertices as intersections of appropriate lines.

$x = 10, y = 0 \Rightarrow A = (10, 0)$

$y = 2x + 10, x = 10 \Rightarrow B = (10, 30)$

$y = 2x + 10, y + 2x = 60 \Rightarrow C = (12.5, 35)$

$y + 2x = 60, y = 0 \Rightarrow D = (30, 0)$



As C is not an integer point so investigate integer points close to it.

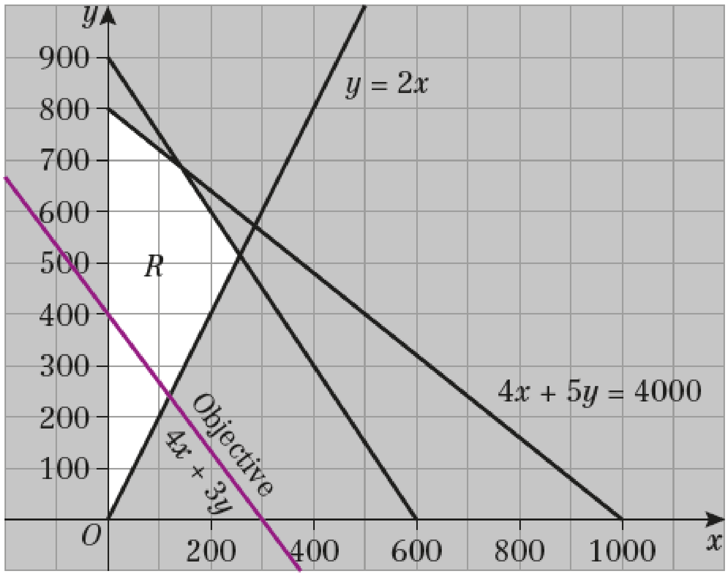
Point	$y \leqslant 2x + 10$?	$y + 2x \leqslant 60$?	**In *R* ?**
$(12, 35)$	$35 > 34$		No
$(13, 35)$	$35 \leqslant 36$	$61 > 60$	No
$(12, 34)$	$34 \leqslant 34$	$58 \leqslant 60$	Yes
$(13, 34)$	$34 \leqslant 36$	$60 \leqslant 60$	Yes

Check values of the objective function at points A, B, D and $C' = (12, 34), C'' = (13, 34)$

Point	**Coordinates**	**Value of** $P = 5x + 3y$
A	$x = 10, y = 0$	50
B	$x = 10, y = 30$	140
C'	$x = 12, y = 34$	162
C''	$x = 13, y = 34$	167
D	$x = 30, y = 0$	150

Maximal value of P we have found is 167 attained at point $(13, 34)$

3 This is the problem formulated in Exercise 6A question 1.



Intersection of $4x+5y=4000$ and $3x+2y=1800$

giving $\left(142\frac{6}{7}, 685\frac{5}{7}\right)$

Testing nearby integer points

Point	$4x+5y \leqslant 4000$	$3x+2y \leqslant 1800$	$80x+60y$
(142, 685)	✓	✓	52460
(142, 686)	✓	✓	52520
(143, 685)	✓	✓	52540
(143, 686)	✗	✗	

so maximum integer solution is 52540 pennies at (143, 685)

4 This is the problem formulated in Exercise 6A question 2.

a Let x and y be the number of displays of type A and B, respectively.

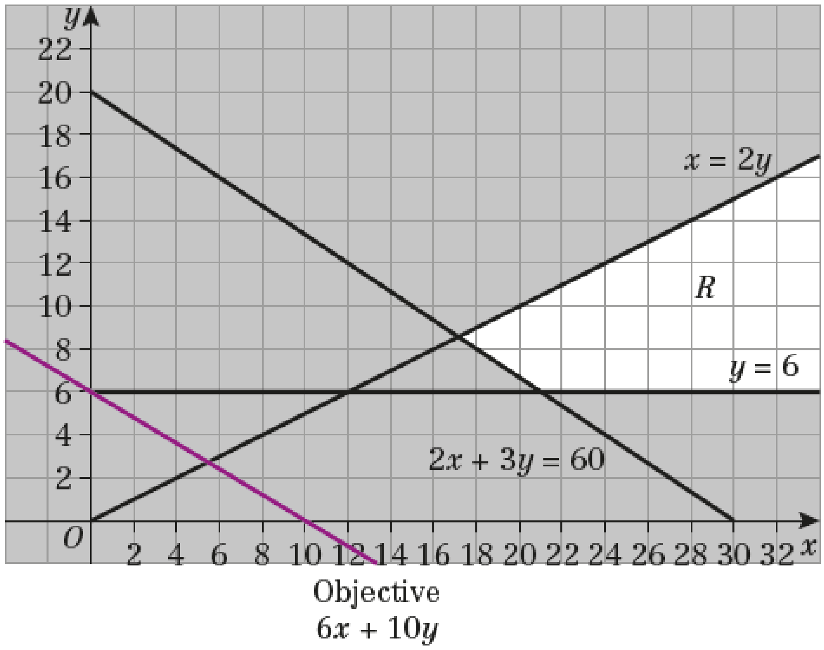
length of display at least 30m $\Rightarrow x+1.5y \geqslant 30 \Rightarrow 2x+3y \geqslant 60$

at least twice as many A as B $\Rightarrow 2y \leqslant x$

at least 6 of type B $\Rightarrow y \geqslant 6$

non-negativity $\Rightarrow x, y \geqslant 0$

4 b



c We aim to minimise cost $C = 6x + 10y$, in pounds.

By objective line method we see that the optimal point lies at the intersection of $y = 6$ and $2x + 3y = 60$, i.e. $x = 21, y = 6$ Optimal value is $6 \times 21 + 10 \times 6 = 186$

Since it is an integer point, it also solves the original problem. The cost is minimized when the client buys 21 displays of type *A* and 6 displays of type *B*. Optimal cost is £186.

5 This is the problem formulated in Exercise 6A question 3.

a Let x and y be the number of games of Cludopoly and Trivscrab, respectively.

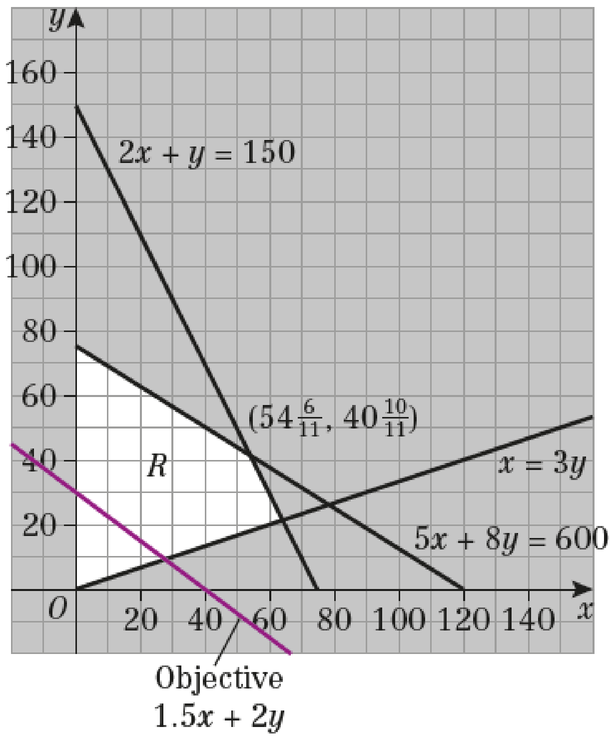
First pieces machine operates max 10h $\Rightarrow \frac{5}{60}x + \frac{8}{60}y \leqslant 10 \Rightarrow 5x + 8y \leqslant 600$

Second pieces machine operates max 10h $\Rightarrow \frac{8}{60}x + \frac{4}{60}y \leqslant 10 \Rightarrow 2x + y \leqslant 150$

At most 3 times as many Cludopoly as Trivago $\Rightarrow 3y \geqslant x$

Non-negativity $\Rightarrow x, y \geqslant 0$

5 b



c We wish to maximise profit $P = 1.5x + 2y$.

By objective line method we find the optimal non-integer point as an intersection

$$5x + 8y = 600, 2x + y = 150 \Rightarrow x = 54\frac{6}{11}, y = 40\frac{10}{11}$$

Investigate neighbouring integer points

Points	$5x+8y \leqslant 600$?	$2x+y \leqslant 150$?	$1.5x+2y$
(54, 40)	$590 \leqslant 600$	$148 \leqslant 150$	161
(54, 41)	$598 \leqslant 600$	$149 \leqslant 150$	163
(55, 40)	$599 \leqslant 600$	$150 \leqslant 150$	162.5
(55, 41)	$603 > 600$	–	–

Maximal value of profit we have found is £163 achieved by producing 54 games of Cludopoly and 41 games of Trivscrab.

6 a Let x and y be the number of bookcases of type 1 and 2, respectively.

We want to optimise shelving $S = 40x + 60y$ subject to the following constraints:

budget constraint $\Rightarrow 150x + 250y \leqslant 3000 \Rightarrow 3x + 5y \leqslant 60$

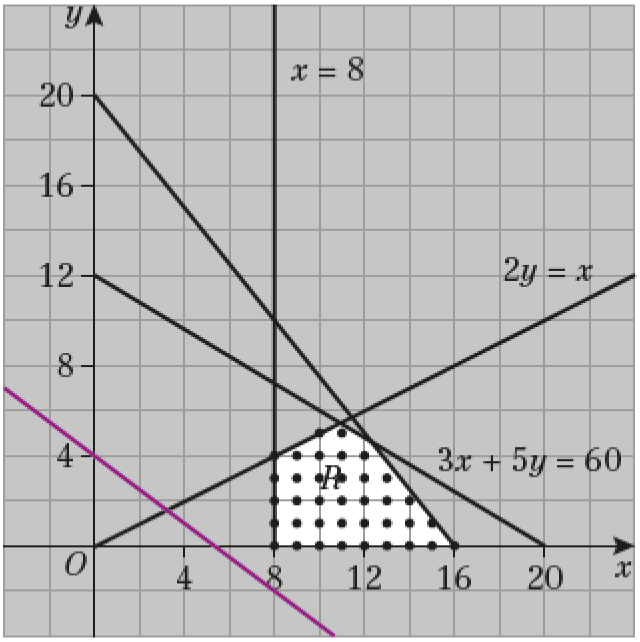
space constraint $\Rightarrow 15x + 12y \leqslant 240 \Rightarrow 5x + 4y \leqslant 80$

at least 8 type 1 $\Rightarrow x \geqslant 8$

at most $\frac{1}{3}$ of type 2 $\Rightarrow y \leqslant \frac{1}{3}(x + y) \Rightarrow 2y \leqslant x$

non-negativity $x, y \geqslant 0$

6 b Represent all inequalities on a diagram and find feasible region *R*.



Integer points are fairly sparse, so we can use Method 1 from Example 11 (p. 153), i.e. identify the optimal point directly by the ruler method.
Optimal integer solution is $(11,5)$ and at this point $S = 740$.
Thus, the maximal amount of shelving is 740m and to achieve that the librarian should buy 11 bookcases of type 1 and 5 bookcases of type 2.

Using Method 1 from Example 13 shows you that the optimal integer solution is (11, 5) giving 740 m of shelving.
Using Method 2 gives you the following solution:
Intersection of $3x + 5y = 60$
$5x + 4y = 80$ giving $\left(12\frac{4}{13}, 4\frac{8}{13}\right)$

Points	$3x+5y \leqslant 60$	$5x+4y \leqslant 80$	$40x+60y$
(12, 4)	✓	✓	720
(12, 5)	✗	✓	
(13, 4)	✓	✗	
(13, 5)	✗	✗	

Maximum value is 720 at (12, 4).
In this instance, the solution produced by Method 2 is actually incorrect, but it requires a very particular set of circumstances to create this discrepancy. It is generally safe to assume that a solution found using Method 2 will be correct, but do check your graph to see whether there could be an alternative optimal integer solution.