Data Collection 1A

1 a A census observes or measures every member of a population.

b An advantage is it that it will give a completely accurate result.

A disadvantage could be any one from:

It would be time consuming.
It would be expensive.

2 a The testing process will destroy the harness, so a census would destroy *all* the harnesses, meaning that there would be no harnesses left for climbers to use.

b The claim is misleading. 250 kg is the mean and median load at which the harnesses in the sample break. So we would expect half of the harnesses to break at a load of less than 250 kg.

c Test a larger number of harnesses.

3 a Any one from:

It would be time consuming.
It would be expensive.
It would be difficult to process the data.

b A list of residents.

c Each individual resident.

4 a The testing process would destroy the switches, so a census would destroy *all* the switches, meaning that there would be no switches left to sell.

b The mean is 19 615.4, less than the stated average. One of the switches survived significantly fewer operations, which suggests that the median of 22 921 might be a better average to take, as it is not affected by outliers. The data therefore supports the company's claim.

c Test a larger number of switches.

5 a All the mechanics in the garage.

b Everyone's views will be known.

Data Collection 1B

1 a There are 40 + 60 + 80 = 180 pupils altogether.

Year 1: $40 \times 0.2 = 8$ pupils
Year 2: $60 \times 0.2 = 12$ pupils
Year 3: $80 \times 0.2 = 16$ pupils

b Any one from:

A stratified sample accurately reflects the population structure of the school.
A stratified sample guarantees proportional representation of different year groups in the sample.

2 a Taking every 20th person may introduce bias, as the sampling frame is not random.

b A simple random sample using the alphabetical list as the sampling frame.

3 a No, this is not a systematic sample. Any reason from:

The first person is not selected at random.
The required elements of the sample are not being chosen at regular intervals.

b To improve the reliability of the data collected, the gym could use a larger sample.
To reduce bias, take a simple random sample using the list of members as the sampling frame.

4 a Stratified sampling

b There are 70 + 50 + 85 + 75 = 280 students altogether. All answers should be rounded to the nearest whole number as appropriate.

Year 12 Male: $\frac{70}{280} \times 40 = 10$
Year 13 Male: $\frac{50}{280} \times 40 \approx 7$
Year 12 Female: $\frac{85}{280} \times 40 \approx 12$
Year 13 Female: $\frac{75}{280} \times 40 \approx 11$

5 $k = \frac{480}{30} = 16$

Randomly select a number between 1 and 16. Start with the worker having this clock number.
Then select the workers that have every 16th clock number after this.

6 a Set up a sampling frame. Use any method to select sampling units in which every member of the population has an equal chance of being selected, e.g. lottery sampling. A disadvantage of this method is that it may not reflect the proportion of members at the club who play each sport.

b The sample will have proportional representation of the members who play the different sports.

c There are 120 + 145 + 105 = 370 members altogether. All answers should be rounded to the nearest whole number as appropriate.

Cricket: $\frac{120}{370} \times 30 \approx 10$
Hockey: $\frac{145}{370} \times 30 \approx 12$
Squash: $\frac{105}{370} \times 30 \approx 9$

Data Collection 1C

1 a i Divide the population into groups according to given characteristics. The size of each group determines the proportion of the sample that should have that characteristic. The interviewer should assess which group people fall into, as part of the interview. Once a quota has been filled, no more people in that group are interviewed.

ii Only sample the people who are available at the time the study is carried out, e.g. the first 40 shoppers who are available to be interviewed.

b Quota sampling, as opportunity sampling is unlikely to reflect the characteristics of the whole population.

2 A similarity between quota sampling and stratified random sampling is that the population is divided according to the characteristics of the whole population (into strata for stratified sampling, and groups for quota sampling). A difference between quota sampling and stratified random sampling is that quota sampling is non-random.

3 a Opportunity sampling

b Opportunity sampling is unlikely to reflect the characteristics of the whole population..

c He could survey people at different times of day.
He could survey people in other parts of the town, not just outside the fish and chip shop.

4 a Mean = $\dfrac{4+7+6+8+2}{5} = \dfrac{27}{5}$ = 5.4 hours

b She has used opportunity sampling, which is unlikely to reflect the characteristics of the whole population of the town, and has used a very small sample, which is unlikely to be representative.

c Increase the number of people asked.
Ask people at different times and/or in different locations.

5 a Quota sampling

b Any one from:

No sampling frame is required.
It is quick.
It is easy.
It is inexpensive.
It allows for comparison between male and female boars.

c Mean male weight = $\dfrac{75+80+90+85+82}{5} = \dfrac{412}{5}$ = 82 kg (to the nearest whole number)

Mean female weight = $\dfrac{67+72+75+68+65}{5} = \dfrac{347}{5}$ = 69 kg (to the nearest whole number)

Male range = 90 − 75 = 15
Female range = 75 − 65 = 10

Males are heavier on average, but have a greater spread.

5 d Increase the sample size.
Catch deer at random times during the day.

6 a An example of an opportunity sample would be the first five heights, giving a sample of:

1.8, 1.9, 2.3, 1.7, 2.1

b The second height is 1.9. To take a sample of 5, now choose every 4th height, giving a sample of:

1.9, 2.0, 2.6, 2.3, 2.0

c Mean height for the opportunity sample = $\frac{1.8+1.9+2.3+1.7+2.1}{5} = \frac{9.8}{5} = 1.96$ m (to 2 d.p.)

Mean height for the systematic sample = $\frac{1.9+2.0+2.6+2.3+2.0}{5} = \frac{10.8}{5} = 2.16$ m (to 2 d.p.)

d The systematic sample is likely to be more reliable, because it is random and likely to be more representative. The opportunity sample might just consider all the small values, as it does here.

Data Collection 1D

1 a **Quantitative** as heights are numerical.

b **Qualitative** as colours are not numerical.

c **Quantitative** as time is numerical.

d **Quantitative** as shoe size is numerical.

e **Qualitative** as names are not numerical.

2 a **Discrete –** you cannot have a shoe size of 4.78, for example.

b **Continuous** – you can measure the length of a leaf to any degree of accuracy.

c **Discrete –** you can only have whole numbers of people.

d **Continuous –** you can measure the weight of the sugar to any degree of accuracy.

e **Continuous –** you can measure the time taken to any degree of accuracy.

f **Continuous –** you can measure the lifetime of a battery to any degree of accuracy.

3 a It is descriptive rather than numerical.

b It is discrete because you can only have whole numbers of pupils in a class. It is quantitative as it is numerical.

c Weight can take any value in a given range. Therefore, it is continuous. It is quantitative as it is numerical.

4 a 1.4 and 1.5. There are no gaps, therefore the boundaries are the given boundaries of the class.

b $\frac{1.3+1.4}{2}=\frac{2.7}{2}=1.35$ kg

c $1.3 - 1.2 = 0.1$ kg

Data Collection 1E

1 **a** Leuchars

b Perth

c Any one from: Leeming, Heathrow, Beijing

d Any one from: Leuchars, Hurn, Camborne, Jacksonville, Perth

e Any one from: Beijing, Jacksonville, Perth

2 Daily maximum relative humidity is a continuous variable as it can take any percentage value between 0 and 100.

3 **a** **i** Mean daily total sunshine for Leeming = $\frac{101.4}{10}$ = 10.1 hrs (to 1 d.p.)

ii Mean daily total sunshine for Heathrow = $\frac{76}{10}$ = 7.6 hrs

b **i** Leeming range = 14.6 − 5.1 = 9.5 hrs

ii Heathrow range = 14.4 − 1.6 = 12.8 hrs

c Part (**a**) shows that Leeming has a higher average number of hours of sunshine. Leeming is further north than Heathrow, so the data does not support Supraj's conclusion. (The range calculated in part (**b**) does not affect this.)

4 The rainfall on 2/6/2015, 5/6/2015 and 8/6/2015 was recorded as 'tr', meaning trace, which is between 0 and 0.05 mm of rain. However, we treat the trace amounts as 0 in numerical calculations as anything less than 0.05 mm would be 0.0 to 1 decimal place.

$$\begin{aligned}\text{Mean daily total rainfall in Heathrow} &= \frac{0.6 + 0 \times 3 + 0 \times 5 + 0.8}{10} \\ &= \frac{1.4}{10} \\ &= 0.14 \text{ mm}\end{aligned}$$

5 **a** **i** Using multiple months provides more data.

ii Two days out of each month of 30 or 31 days is a small sample size.

b He is choosing the last day of one month followed by the first day of the next month, which is not random. These consecutive days are likely to be affected by the same weather pattern.

c He could number the days and choose a simple random sample.

6 **a** Perth is in the Southern hemisphere, where it will be winter in August, and Jacksonville is in the Northern hemisphere, where it will be summer. So it is likely to be hotter in Jacksonville.

6 b The lowest temperatures in the UK are at coastal locations – Leuchars and Camborne (14.7 °C and 15.4 °C). The highest temperature is at an inland location – Beijing (26.6 °C). There is some evidence to support this conclusion.

7 1 okta corresponds to 1 eighth of the sky covered by cloud, and so the maximum figure for cloud cover is 8. Brian's answer is more than that and so is incorrect.

8 a Marie should select days at regular intervals from an ordered list. She should put the days into date order. Then she should begin at a random day between 1 and 6 and select every 6th day thereafter until 30 days are selected since $184 \div 30 \approx 6$.

b There may be missing values because the data is not available.

Large data set

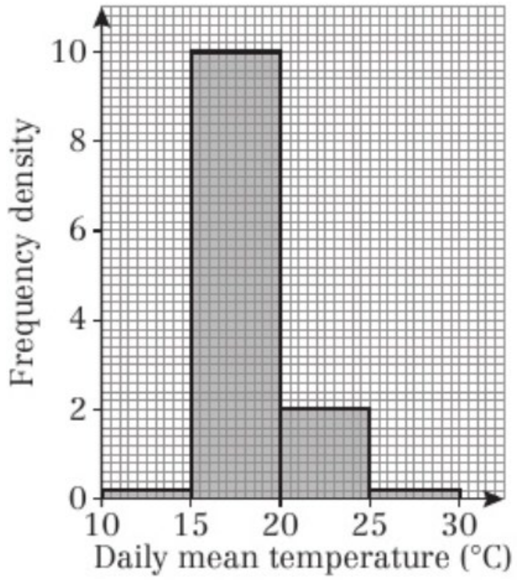
1 a Mean pressure $= \dfrac{31628}{31} = 1020$ hPa (to the nearest whole number)

b When the rainfall for July 2015 is in numerical order, the first seventeen days had zero rainfall, so the median daily rainfall is 0.0 mm.

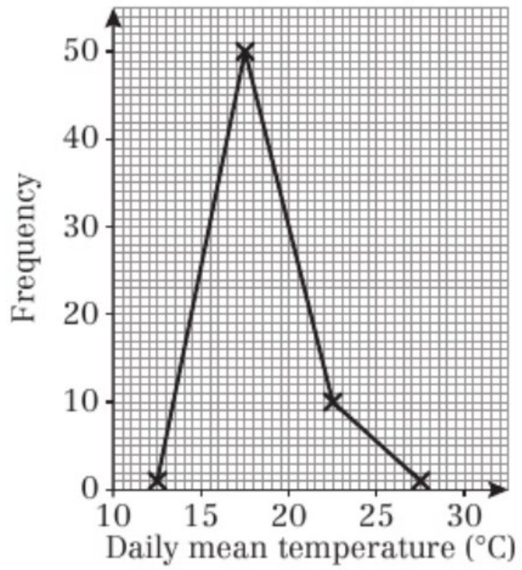
c i

Temperature, t (°C)	Frequency
$10 \leq t < 15$	1
$15 \leq t < 20$	50
$20 \leq t < 25$	10
$25 \leq t < 30$	1

1 c ii



iii



2 a i Give each day a number from 1 to 184. Use a calculator to generate 10 random numbers, for example:

43, 4, 102, 23, 139, 106, 164, 94, 163, 63

This gives the sample of wind speeds: 4, n/a, 4, 12, 3, 8, 4, 8, 5 and 3.

ii Mean daily wind speed excluding the value n/a is $\frac{51}{9}$ = 6 kn (to the nearest whole number)

b i The first ten available values are 5, 8, 7, 11, 9, 7, 10, 7, 5 and 5.

ii Mean daily mean wind speed = $\frac{74}{10}$ = 7 kn (to the nearest whole number)

c The simple random sample is likely to be more representative as the opportunity sample records consecutive days, which are likely to affect one another.

2 d Any two of:

Take a bigger sample, as 10 sample units is likely to be insufficient.
Take a stratified sample, categorising by month.
Take a systematic sample.

Both of the final options guarantee that you will take data spread over a range of months.

e Example: A stratified sample of 30 days.

Month	May	Jun	Jul	Aug	Sep	Oct
Number of days	31	30	31	31	30	31

$\frac{31}{184}\times 30 = 5.05 \approx 5$ days

$\frac{30}{184}\times 30 = 4.89 \approx 5$ days

Therefore, take a simple random sample of 5 days from each month for both Leeming and Leuchars in 1987.

In the sample taken for this example, 4 of the values for Leeming and Leuchars were not available and so the calculations will be evaluated from the remaining 26 sample units.

Mean windspeed for Leeming = $\frac{166}{26}$ = 6 kn (to the nearest whole number)

Mean windspeed for Leuchars = $\frac{183}{26}$ = 7 kn (to the nearest whole number)

Leuchars is a coastal location and has a higher average wind speed than the inland location of Leeming so the calculation supports the statement 'Coastal locations are likely to have higher average windspeeds than inland locations.'

Data Collection, Mixed exercise 1

1 a Mean daily maximum temperature $= \dfrac{14.6 + 8.8 + 7.2 + 7.3 + 10.2}{5} = \dfrac{48}{5} = 9.6°\text{C}$

b The sampling frame is the first 15 days in May 1987. Allocate each date a number from 1 to 15. Use the random number function on a calculator to generate 5 random numbers between 1 and 15 and choose the dates with these numbers.

c For example: The 5 random numbers generated are 12, 8, 13, 6 and 15. These give the temperatures 12.7, 12.1, 8.9, 11.9 and 9.5.

Mean daily max temperature $= \dfrac{55.1}{5} = 11.0°\text{C}$ (to 1 d.p.)

d Mean daily max temperature $= \dfrac{162.2}{15} = 10.8°\text{C}$ (to 1 d.p.)

Using data for the whole period gives a completely accurate result. A sample of one third of the dates will give a less reliable one. This variation is apparent in the answers to a and c, above.

2 a i Advantage: very accurate.
Disadvantages: expensive, time consuming and difficult to process.

ii Advantages: easier data collection, quick and cheap.
Disadvantages: less accurate, less representative and possibly biased.

b Assign unique 3-digit identifiers 000, 001, …, 499 to each member of the population. Use random number tables, a computer or a calculator to generate 3-digit numbers. If these correspond to an identifier then include the corresponding member in the sample, being careful to ignore repeats and numbers greater than 499. Repeat this process until the sample contains 100 members.

3 a i Collection of individual items.

ii List of sampling units, with each unit given an identifying name or number.

b i List of registered owners from the DVLA.

ii List of people visiting a doctor's clinic in Oxford in July 1996.

4 a Advantages:

The results are the most representative of the population since the structure of the sample reflects the structure of the population.
It guarantees proportional representation of groups within a population.

Disadvantages:

You need to know the structure of the population before you can take a stratified sample.
Classification into mutually exclusive strata may be difficult to implement.
The sampling within each strata may suffer from the disadvantages of simple random sampling.

4 **b** Advantages:

Quick
Cheap.
All units have an equal chance of selection.

Disadvantages:

Can introduce bias (e.g. if the sample, by chance, only includes very tall people in an investigation into heights of students).
A sampling frame is needed first.

5 a People on the shop floor are not represented.

b i Get a list of the 300 workers at the factory. $\frac{300}{30} = 10$ so choose one of the first ten workers on the list at random and every subsequent 10th worker on the list, e.g. if person 7 is chosen, then the sample includes workers 7, 17, 27, …, 287 and 297.

ii The sample should contain $\frac{1}{3} \times 30 = 10$ office workers and $\frac{2}{3} \times 30 = 20$ shop floor workers, as those are the corresponding proportions in the whole population. The 10 office workers in the sample should be selected by a simple random sample of the 100 office workers. The 20 shop floor workers should be selected by a simple random sample of the 200 shop floor workers.

iii Decide the categories e.g. age, gender, office/non-office and set a quota for each in proportion to their numbers in the population. Interview workers until quotas are full.

6 a Allocate a number between 1 and 120 (the total number of pupils) to each pupil. Use random number tables, a computer or a calculator to select 15 different whole numbers between 1 and 120.

Pupils corresponding to these numbers become the sample.

b Allocate numbers 1–64 to girls and 65–120 to boys.

Select $\frac{64}{120} \times 15 = 8$ different random numbers between 1 and 64 for girls.

Select the remaining 7 sampling units using random numbers between 65 and 120 for boys.

Include the corresponding boys and girls in the sample.

7 a Stratified sampling.

b This method uses naturally occurring groupings (strata). The results are more likely to represent the views of the whole population since the sample reflects its structure.

8 a Opportunity sampling

b Any one of:

It is easy to carry out.
It is inexpensive.

c The data is continuous, as weight can take any positive value.

8 d Mean weight = $\frac{70+76+82+74+78}{5}=\frac{380}{5}$ = 76 kg

e Mean weight = $\frac{79+86+90+68+75}{5}=\frac{398}{5}$ = 80 kg (to the nearest whole number)

f The second conservationist is likely to have a more reliable estimate as opportunity sampling is unlikely to provide a representative sample of the whole population, as it does not necessarily reflect its structure.

g The second conservationist could select more springboks at each location.

9 a This sample is not entirely random as the dates are selected at regular intervals. It is actually a systematic sample.

b A systematic sample: select the first date at random and then the same date each month. An advantage of a systematic sample is that each month is covered. A disadvantage of a systematic sample is that there may be patterns, and therefore bias, in the sample data.

A simple random sample: select the six days completely at random. An advantage of a simple random sample is that it avoids the likelihood of patterns and unintentional bias. A disadvantage of a simple random sample is that it may not cover the full range of months.

c The data is continuous as temperature can take any value.

d Mean daily maximum temperature = $\frac{59.1}{6}$ = 9.9 °C (to 1 d.p.)

e Any suitable reason e.g. this estimate is unlikely to be reliable as it does not include the winter months / the data is very variable.

Large data set

a $\frac{184}{18}\approx 10$, so take every 10th day, choosing a starting place by getting a calculator to generate a random number, for example, 10. The sample is then:

79, 99, 99, 86, 95, 99, 93, 99, 100, 93, 99, 98, 98, 100, 89, 99, 95, 95

b Any of: easy, quick, more suitable for this relatively large population..

c Mean = $\frac{1715}{18}$ = 95% (to the nearest whole number)

d The sampling frame is not random (it is in date order) so systematic sampling could introduce bias. This bias could be counteracted by using simple random sampling.

Measures of location and spread 2A

1 a 700 g, as this is the most often occurring.

b 500 + 700 + 400 + 300 + 900 + 700 + 700 = 4200

$$\frac{4200}{7} = 600\,\text{g}$$

c 300 400 500 **700** 700 700 900

700 g is the median (the middle value).

d It will increase the mean, as 650 > 600 (the old mean).

The mode will be unchanged.

It will decrease the median. There will now be an *even* number of values, so we take the middle *pair*: 650 and 700. The new median will be half-way between these: 675.

2 a $\frac{256.2}{6} = 42.7$

b It will increase the mean, as the new piece of data (52) is greater than the old mean (42.7).

3 a The mean visibility for May = $\frac{\sum h}{n}$ = $\frac{724000}{31}$ = 23354.8 (to 1 d.p.)

The mean visibility for June = $\frac{\sum h}{n}$ = $\frac{632000}{30}$ = 21066.7 (to 1 d.p.)

b The mean visibility for May and June = $\frac{\sum h}{n}$ = $\frac{724000+632000}{31+30}$ = $\frac{1356000}{61}$ = 22229.5 (to 1 d.p.)

4 a 8 minutes. Everything else occurs only once, but there are two 8's.

b $\frac{102}{10} = 10.2$ minutes

c 5 6 7 8 **8 9** 10 11 12 26

The median is 8.5 minutes.

d The median would be reasonable. The mean is affected by the extreme value of 26.

In this case the mode is close to the median, so would be acceptable; but this would not always be the case.

5 a 2 breakdowns

b The median is the 18.5th value = 1

5 c (8 × 0) + (11 × 1) + (12 × 2) + (3 × 3) + (1 × 4) + (1 × 5) = 53

The mean $= \frac{53}{36} = 1.47$ breakdowns

d The median, since this is the lowest value

6 (5 × 8) + (6 × 57) + (7 × 29) + (8 × 3) + (9 × 1) = 618 petals

8 + 57 + 29 + 3 + 1 = 98 celandines

The mean $= \frac{618}{98} = 6.31$ petals (to 2 d.p.)

7 The mean = $\frac{1 \times 7 + 2 \times p + 3 \times 2}{7 + p + 2}$

$$1.5 = \frac{7 + 2p + 6}{p + 9}$$

$$= \frac{2p + 13}{p + 9}$$

$1.5p + 13.5 = 2p + 13$

$0.5 = 0.5p$

$p = 1$

Measures of location and spread 2B

1 a $351 - 400$

b $$\frac{(200\times 4)+(263\times 8)+(325.5\times 18)+(375.5\times 28)+(450.5\times 7)}{65}$$
$$=\frac{800+2104+5859+10514+3153.5}{65}$$
$$=\frac{22430.5}{65}$$
$$=345.08$$

c There are 65 observations so the median is the 33rd. The 33rd observation will lie in the class 351–400.

2 a $$\frac{(67\times 1)+(72\times 4)+(77\times 6)+(82\times 6)+(87\times 8)+(92\times 4)+(97\times 1)}{30}=\frac{2470}{30}=82.3 \text{ decibels}$$

b The answer is an estimate because you don't know the exact data values.

3 a Modal class = $10 \le t < 12$

b Estimate of the mean = $$\frac{(7\times 3)+(9\times 7)+(11\times 9)+(13\times 7)+(15\times 3)+(17\times 2)}{31}$$
$$=\frac{353}{31}=11.4 \text{ °C (1 d.p.)}$$

4 Store A $$\frac{(20.5\times 5)+(30.5\times 16)+(40.5\times 14)+(50.5\times 22)+(60.5\times 26)+(70.5\times 14)}{97}$$
$$=\frac{4828.5}{97}=50 \text{ years}$$

Store B $$\frac{(20.5\times 4)+(30.5\times 12)+(40.5\times 10)+(50.5\times 28)+(60.5\times 25)+(70.5\times 13)}{92}$$
$$=\frac{4696}{92}=51 \text{ years}$$

Store B employs older workers but not by a great margin.

Measures of location and spread 2C

1 a $Q_2 = \frac{16+1}{2}$th value = 8.5th value

1009, 1013, 1014, 1017, 1017, 1017, 1018, 1019, 1021, 1022, 1024, 1024, 1025, 1027, 1029, 1031

$Q_2 = \frac{1019+1021}{2} = 1020$ hPa

b Q_1 = 4.5th value
Q_1 = 1017 hPa

Q_3 = 12.5th value
Q_3 = 1024.5 hPa

2 $Q_2 = \frac{95+1}{2}$th value = 48th value
$Q_2 = 37$

Q_1 = 24th value
$Q_1 = 37$

Q_3 = 72nd value
$Q_3 = 38$

3 In this case, the number of breakdowns is a discrete variable which in this situation has been treated as a continuous variable. While not ideal, it is the best you can do.

Median value is the 13th value. This is in the first class.
Let m be the median.

$\frac{m-0}{1.5-0} = \frac{13-0}{18-0}$ so $m \approx 1.08$ (3 sf)

4 a Median: $\frac{31}{2}$ = 15.5th value
Using interpolation:

$\frac{Q_2 - 399.5}{449.5-399.5} = \frac{15.5-9}{19-9}$

$Q_2 = 432$ kg

4 b $Q_1 : \frac{31}{4} = 7.75\text{th value, so } Q_1 \text{ is in class } 350-399$

$$\frac{Q_1 - 349.5}{399.5 - 349.5} = \frac{7.75-3}{9-3}$$

$$\frac{Q_1 - 349.5}{50} = \frac{4.75}{6}$$

$Q_1 = 39.58 + 349.5 \ = 389$

c $Q_3 : 3 \times \frac{31}{4} = 23.25\text{th value, so } Q_3 \text{ is in class } 450-499$

$$\frac{Q_3 - 449.5}{499.5 - 449.5} = \frac{23.25-19}{26-19}$$

$$\frac{Q_3 - 449.5}{50} = \frac{4.25}{7}$$

$Q_3 = 30.36 + 449.5 = 480$

d Three-quarters of the cows weigh less than 480 kg.

5 a Estimate for the mean $= \frac{(25\times6)+(35\times10)+(45\times18)+(55\times13)+(65\times2)}{49}$

$= \frac{2155}{49}$

$= 44.0 \text{ minutes } (\text{to 3 s.f.})$

b $65\text{th} : \frac{65}{100} \times 49 = 31.85$

$$\frac{P_{65} - 40}{50-40} = \frac{31.85-16}{34-16}$$

$P_{65} = 48.8$

c $\frac{P_{90} - 50}{60-50} = \frac{44.1-34}{47-34}$

$P_{90} = 57.8$

90th percentile = 57.8 minutes, so more than 10% of customers have to wait longer then 57.8 minutes – not 56 minutes as stated by the firm.

6 a $\frac{P_{80} - 2.5}{3.0 - 2.5} = \frac{80 - 61}{89 - 61}$

$P_{80} = 2.84$ (to 2 d.p.)

80th percentile = 2.84 m, so 80% of condors have a wingspan of less than 2.84 m.

b The 90th percentile is in the $3.0 \leq w$ class. There is no upper boundary for this class, so it is not possible to estimate the 90th percentile.

Measures of location and spread 2D

1 a CF = 4 8 10 17 37 61 71
71 slow worms were measured.

b $Q_1 : \frac{71}{4} = 17.75\text{th value, so } Q_1 \text{ is in class } 185 - 199$

$$\frac{Q_1 - 184.5}{199.5 - 184.5} = \frac{17.75 - 17}{37 - 17}$$

$$Q_1 - 184.5 = 0.5625$$

$$Q_1 = 185.0625$$

$Q_3 : 3 \times \frac{71}{4} = 53.25\text{th value}$
so Q_3 is in class $200 - 214$

$$\frac{Q_3 - 199.5}{214.5 - 199.5} = \frac{53.25 - 37}{61 - 37}$$

$$Q_3 - 199.5 = \frac{243.75}{24}$$

$$Q_3 = 209.656$$

$$\text{IQR} = 209.656 - 185.0625$$
$$= 24.6 \text{ (3 s.f.)}$$

c $$\bar{x} = \frac{(132 \times 4) + (147 \times 4) + (162 \times 2) + (177 \times 7) + (192 \times 20) + (207 \times 24) + (222 \times 10)}{71}$$
$$= \frac{13707}{71}$$
$$= 193.1\text{mm (to 1 d.p.)}$$

d $$\bar{x} + \text{IQR} = 193.1 + 24.6$$
$$= 217.7$$

217.7 is in the class interval 215–229

1 d Using interpolation:

$$\frac{217.7-214.5}{229.5-214.5}=\frac{y-61}{71-61}$$
$$y = 63.13...$$
$$71 - y = 7.87$$

8 slow worms have that length.

2 a 34th: $\frac{34}{100}\times 70 = 23.8$

$$\frac{\text{P}_{34}-1000}{1100-1000}=\frac{23.8-3}{27-3}$$

$$\text{P}_{34} = 1086.7$$

66th: $\frac{66}{100}\times 70 = 46.2$

$$\frac{\text{P}_{66}-1100}{1200-1100}=\frac{46.2-27}{55-27}$$

$$\text{P}_{66} = 1168.6$$

34% to 66% interpercentile range $= \text{P}_{66}-\text{P}_{34} = 1168.6-1086.7 = £81.90$

b $46.2 - 23.8 = 22.4$
So 22 data values

3 a 5th: $\frac{5}{100}\times 60 = 3$

$$\frac{\text{P}_{5}-14.5}{16.5-14.5}=\frac{3-0}{5-0}$$

$$\text{P}_{5} = 15.7$$

95th: $\frac{95}{100}\times 60 = 57$

$$\frac{\text{P}_{95}-20.5}{22.5-20.5}=\frac{57-50}{60-50}$$

$$\text{P}_{95} = 21.9$$

5% to 95% interpercentile range $= 21.9-15.7 = 6.2$

3 b $57 - 3 = 54$
So 54 data values

4 a 9.4, 10.3, 10.3, 10.6, 10.9, 12.1, 12.4, 12.7, 13.2, 14.3

Q_2 = 5.5th value = $\frac{10.9+12.1}{2} = 11.5$
Q_1 = 3rd value = 10.3
Q_3 = 8th value = 12.7

IQR = $12.7 - 10.3 = 2.4$

b On average, the temperature was higher in June than in May as the median is higher. However, the temperature was more variable in May than in June, as the IQR is higher.

c 10th: $\frac{10}{100}\times 31 = 3.1$
90th: $\frac{90}{100}\times 31 = 27.9$

$27.9 - 3.1 = 24.8$

So 24 days

Measures of location and spread 2E

1 a Mean $=\frac{24}{8}=3$

b Variance $=\frac{78}{8}-3^2=0.75$

c Standard deviation $=\sqrt{0.75}=0.866$

2 Standard deviation $=\sqrt{\frac{5905}{10}-\left(\frac{241}{10}\right)^2}=3.11\text{ kg}$

3 a $\sum h=165+170+190+180+175+185+176+184=1425$

Mean $=\frac{1425}{8}=178.125\approx 178$

b Variance $=\frac{254307}{8}-178.125^2=59.9$

c Standard deviation $=\sqrt{59.9}=7.74$

4 $\sum x=50+86=136$

$\sum x^2=310+568=878$

Mean $=\frac{136}{25}=5.44$

Standard deviation $=\sqrt{\frac{878}{25}-\left(\frac{136}{25}\right)^2}=2.35$

5 a Mean $=\frac{869}{85}=10.22$

Standard deviation $=\sqrt{\frac{9039}{85}-\left(\frac{869}{85}\right)^2}=1.35$

b 10.22 + 1.35 = £11.57

$\frac{11.57-11.50}{12.50-11.50}=\frac{s-65}{85-65}$

s = 66.4

85 − 66.4 = 18.6

So 19 students

6 Standard deviation $=\sqrt{\frac{203}{54}-\left(\frac{81}{54}\right)^2}=1.23$

7 Mean $=\frac{805}{50}$ = 16.1 hours

Standard deviation $=\sqrt{\frac{14062.5}{50}-\left(\frac{805}{50}\right)^2}$ = 4.69 hours

One standard deviation below mean = 16.1 − 4.69 = 11.41 hours.

$$\frac{11.41-10}{15-10}=\frac{p-5}{19-5}$$

$p = 8.948$

$50 - p = 41.052$

41 parts tested (82%) lasted longer than one standard deviation below the mean.
According to the manufacturers, this should be 45 parts (90%), so the claim is false.

8 a Mean $=\frac{243}{30}$ = 8.1 kn

Standard deviation $=\sqrt{\frac{2317}{30}-\left(\frac{243}{30}\right)^2}$ = 3.41 kn

b 8.1 + 3.41 = 11.51 kn

$$\frac{11.51-4}{17-4}=\frac{d-0}{30-0}$$

$d = 17.33$

$30 - d = 12.67$

So 12 days

c The windspeeds are equally distributed throughout the range.

Measures of location and spread 2F

1 a 11 + 9 + 5 + 8 + 3 + 7 + 6 = 49

b Mean $=\frac{49}{7}=7$

c $7=\frac{\bar{x}}{10}$ so $\bar{x}=70$

2 a 7 + 10 + 4 + 10 + 5 + 11 + 2 + 3 = 52

b Mean $=\frac{52}{8}=6.5$

c $6.5=\frac{\bar{x}-3}{7}$ so $\bar{x}=48.5$

3 $(1.5\times 200)+65=365$

4 Standard deviation = 2.34

5 a Mean = $\frac{(1\times 3)+(1.1\times 12)+(1.2\times 40)+(1.3\times 10)+(1.4\times 5)}{70}\ \frac{84.2}{70}$

$=\frac{84.2}{70}$

$=1.2$ hours

b $\frac{84.2}{70}=\frac{\bar{x}-1}{20}$ so $\bar{x}=25.1$ hours

c Standard deviation of coded data = $\sqrt{\frac{101.82}{70}-\left(\frac{84.2}{70}\right)^2}$

$=0.0877845...$

Standard deviation = 20 × 0.0877845... = 1.76 hours

6 Standard deviation of coded data $=\sqrt{\frac{176.84}{100}-\left(\frac{131}{100}\right)^2}=0.229$

Standard deviation $=0.229\times 100=22.9$

7 Standard deviation of coded data = $=\sqrt{\frac{147.03}{6}-\left(\frac{16.1}{6}\right)^2}=4.16$

Standard deviation $=\frac{4.16}{0.01}=416$

8 a $t=0.8(m+12)$

8 b Mean of the standardised marks = $\bar{t}$ = 52.8

$\bar{t} = 0.8(\bar{m} + 12)$

$\bar{m} = \dfrac{52.8}{0.8} - 12$

Mean of the original marks = 54

Standard deviation of the standardised marks = $\sqrt{\dfrac{S_{tt}}{n}} = \sqrt{\dfrac{7.3}{28}} = 0.5106...$

Standard deviation of the original marks = $\dfrac{0.5106...}{0.8} = 0.64$

9 Coded mean = 10.15

Mean of the daily mean pressure = 2(10.15 + 500) = 1020.3 hPa

Coded standard deviation = $\sqrt{\dfrac{S_{cc}}{n}} = \sqrt{\dfrac{296.4}{30}} = 3.1432...$

Standard deviation of the daily mean pressure = 2 × 3.1432... = 6.28 hPa

Measures of location and spread, Mixed Exercise 2

1 $(8\times65)+(12\times72)=1384$

$\frac{1384}{20}=69.2$ marks

2 a 10, 12, 9, 2, 2.5, 9.5

b coded mean = $\frac{45}{6}$ = 7.5

c mean = 7.5 × 80 + 7 = 607

3 $18\times1000+720=£18720$

4 a Group A:

(1 × 24.5) + (3 × 34.5) + (6 × 44.5) + (6 × 54.5) + (11 × 64.5) + (10 × 74.5) + (8 × 84.5)

Mean $=\frac{2852.5}{45}=63.39$ marks

Group B:

(1 × 24.5) + (2 × 34.5) + (4 × 44.5) + (13 × 54.5) + (15 × 64.5) + (6 × 74.5) + (3 × 84.5)

Mean $=\frac{2648}{44}=60.18$ marks

b The method used to teach group A is best as the mean mark is higher.

5 a Modal Class is $21-25$ hours

b We need the 40th value. This is in the $21-25$ class.

$$\frac{m-20.5}{25.5-20.5}=\frac{40-30}{75-30}$$

$45m=50+922.5=972.5$
median = 21.6 hours

c mean $=20.6$ hours

d $12\times22.3=267.6$ hours

Total hours for all 92 batteries is $267.6+1645=1912.6$
Mean life for 92 batteries is 20.8 hours.

6 CF = 5 15 30 42 50

6 Data is continuous, so:

$Q_1 : \frac{50}{4} = 12.5\text{th value, so } Q_1 \text{ is in class } 21-40$

$$\frac{Q_1 - 20.5}{40.5 - 20.5} = \frac{12.5 - 5}{15 - 5}$$

$$\frac{Q_1 - 20.5}{20} = \frac{7.5}{10}$$

$Q_1 = 35.5$

$Q_3 : \frac{3 \times 50}{4} = 37.5\text{th value, so } Q_3 \text{ is in class } 61-80$

$$\frac{Q_3 - 60.5}{80.5 - 60.5} = \frac{37.5 - 30}{42 - 30}$$

$Q_3 = 73$

$\text{IQR} = 73 - 35.5 = 37.5$

7 a $30\text{th} : \frac{30}{100} \times 100 = 30$

$P_{30} = 20.5$

b $70\text{th} : \frac{70}{100} \times 100 = 70$

$$\frac{P_{70} - 30.5}{40.5 - 30.5} = \frac{70 - 60}{84 - 60}$$

$P_{66} = 34.7$

c 30% to 70% interpercentile $= 34.7 - 20.5 = 14.2$

8 a $Q_1 : \frac{80}{4} = 20\text{th value, so } Q_1 \text{ is in class } 40-49$

$$\frac{Q_1 - 39.5}{49.5 - 39.5} = \frac{20 - 15}{51 - 15}$$

$$\frac{Q_1 - 39.5}{10} = \frac{5}{36}$$

$Q_1 = 40.9$

8 a Q_3: $\frac{3\times 80}{4} = 60\text{th value, so } Q_3 \text{ is in class } 50-59$

$$\frac{Q_3 - 49.5}{59.5-49.5} = \frac{60-51}{71-51}$$

$Q_3 = 54$

IQR $= 54 - 40.9 = 13.1$

b Variance $= \frac{183040}{80} - \left(\frac{3740}{80}\right)^2 = 102.4375 = 102$

Standard deviation $= \sqrt{102.4375} = 10.1$

9 CF = 5 15 41 49 50

a Data is continuous, so:

Q_1: $\frac{50}{4} = 12.5\text{th value, so } Q_1 \text{ is in class } 95-100$

$$\frac{Q_1 - 95}{100-95} = \frac{12.5-5}{15-5}$$

$Q_1 = 98.75$

b Q_3: $3\times\frac{50}{4} = 37.5\text{th value, so } Q_3 \text{ is in class } 100-105$

$$\frac{Q_3 - 100}{105-100} = \frac{37.5-15}{41-15}$$

$Q_3 = 104.33$

c IQR $= 104.33 - 98.75 = 5.58$

d Standard deviation $= \sqrt{\frac{516112.5}{50} - \left(\frac{5075}{50}\right)^2} = 4.47$

10 a Mean $= \frac{(12\times 12)+(14\times 14)+(16\times 4)}{30} = \frac{202}{15} = 13.5$ (1 d.p)

Standard deviation $= \sqrt{\frac{916}{5} - \left(\frac{202}{15}\right)^2} = 1.36$ (3 s.f.)

10 b $10\text{th}: \frac{10}{100} \times 30 = 3$

$$\frac{P_{10} - 11}{13 - 11} = \frac{3 - 0}{12 - 0}$$

$$P_{10} = 11.5$$

$90\text{th}: \frac{90}{100} \times 30 = 27$

$$\frac{P_{90} - 15}{17 - 15} = \frac{27 - 26}{30 - 26}$$

$$P_{90} = 15.5$$

10% to 90% interpercentile range $= 15.5 - 11.5 = 4$°C

c $13.5 + 1.36 = 14.86$
Using interpolation:

$$\frac{14.86 - 13}{15 - 13} = \frac{d - 12}{26 - 12}$$

$d = 25.02$
$30 - d = 4.98$

So 5 days

11 a Coded mean $= \frac{106}{31} = 3.419...$

Coded standard deviation $= \sqrt{\frac{80.55}{31}} = 1.6119...$

b Mean $= 3.419... \times 2 + 3 = 9.84$ kn

Standard deviation $= 1.6119 \times 2 = 3.22$ kn

12 a Mean $= \frac{316}{20} = 15.8$

Standard deviation $= \sqrt{\frac{5078}{20} - \left(\frac{316}{20}\right)^2} = 2.06$

b It will decrease the mean wing span since $13 < 15.8$

c Coded mean $= \frac{104}{20} = 5.2$
Mean $= 5.2 \times 10 + 5 = 57$ cm

Coded standard deviation $= \sqrt{\frac{1.8}{20}} = 0.3$

Standard deviation $= 0.3 \times 10 = 3$

Challenge

Total = 3.1 × 20 = 62
New total = 62 − 2.3 + 3.2 = 62.9
New mean = $\frac{62.9}{20}$ = 3.145

$\sigma = 1.4$, $\sigma^2 = 1.96$

$1.96 = \frac{\sum x^2}{20} - \left(\frac{62}{20}\right)^2$

$\sum x^2 = 231.4$

New $\sum x^2 = 231.4 - 2.3^2 + 3.2^2 = 236.35$

New standard deviation = $\sqrt{\frac{236.35}{20} - \left(\frac{62.9}{20}\right)^2} = 1.39$

Representations of data 3A

1 IQR = 68−46 = 22

$46 - 1.5 \times 22 = 13$
$68 + 1.5 \times 22 = 101$

a 7 is an outlier as 7 < 13.

b 88 is not an outlier as 13 < 88 < 101.

c 105 is an outlier as 105 > 101.

2 **a** Outliers are < 400 – 180 = 220 or > 580 + 180 = 760. So there are no outliers.

b Outliers are < 260 – 80 = 180 or > 340 + 80 = 420. So 170 g and 440 g are both outliers.

c 760 g

3 **a** Mean = 6.1 kg
Standard deviation = $\sqrt{4.2}$
Mean − 2 × standard deviation = 6.1 − 2 × $\sqrt{4.2}$ = 2.00 (to 3 s.f.)
Mean + 2 × standard deviation = 6.1 + 2 × $\sqrt{4.2}$ = 10.2 (to 3 s.f.)
So 11.5 kg is an outlier.

b The smallest is 2.00 kg.
The largest is 10.2 kg.

4 **a** Mean = $\frac{\sum x}{n} = \frac{92}{9} = 10.2$ (to 3 s.f.)

Standard deviation $= \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$

$= \sqrt{\frac{1428}{9} - \left(\frac{92}{9}\right)^2}$

$= 7.36$ (to 3 s.f.)

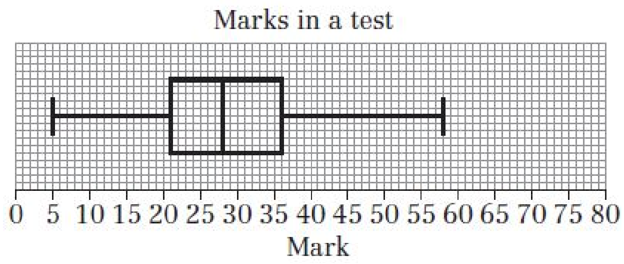
b Mean − 2 × standard deviation = −4.50 (to 3 s.f.)
Mean + 2 × standard deviation = 24.9 (to 3 s.f.)
30 is an outlier, as it is more than 2 standard deviations above the mean (30 > 24.9).

c It could be the age of a parent at the party.

d $\sum x - 30 = 92 - 30 = 62$
Mean = $\frac{62}{8}$ = 7.75
$\sum x^2 - 30^2 = 1428 - 900 = 528$
Standard deviation = $\sqrt{\frac{528}{8} - \left(\frac{62}{8}\right)^2}$ = 2.44 (to 3 s.f.)

Representations of data 3B

1



2 a Upper quartile is 47 marks
Lower quartile is 32 marks

b 38 marks

c $\text{IQR} = 47 - 32 = 15$ marks

d Range $= 76 - 12 = 64$ marks

3 a Male turtles have a higher median mass.

Males have a bigger range (or IQR).

b It is more likely to be a female. Very few male turtles weigh as little as this, but roughly three-quarters of the female turtles weigh even less.

c 500 g despite this data point being an outlier.

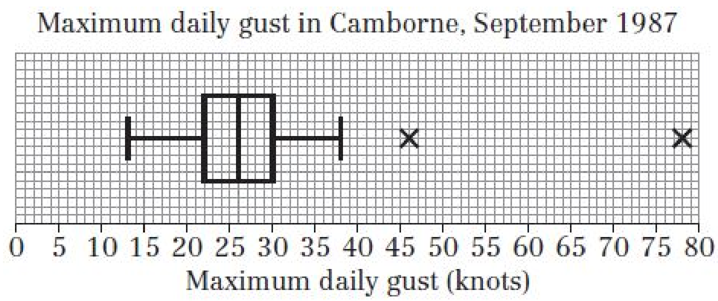
4 a Q_1: $\frac{25}{4}$ = 6.25, so we pick the 7th term: 22

Q_2: $\frac{25+1}{2}$ = 13, so we pick the 13th term: 26

Q_3: $\frac{3}{4}\times 25$ = 18.75, so we pick the 19th term: 30

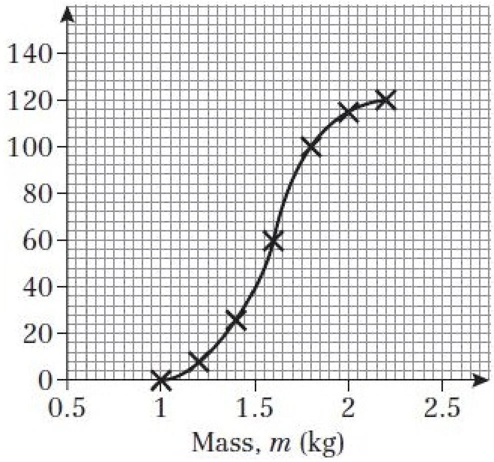
b IQR = 30 − 22 = 8
Outliers are values less than $22 - 1.5 \times 8 = 10$ and values greater than $30 + 1.5 \times 8 = 42$.
46 and 78 are both greater than 42, so 46 and 78 are both outliers.

c



Representations of data 3C

1 a Cumulative frequencies are: 7, 25, 59, 100, 115, 120

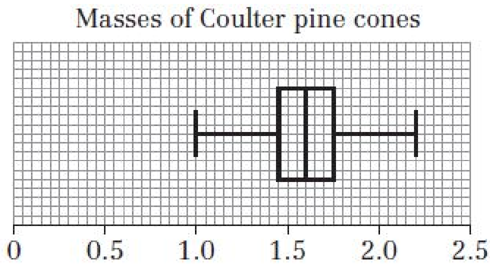


b 1.6 kg

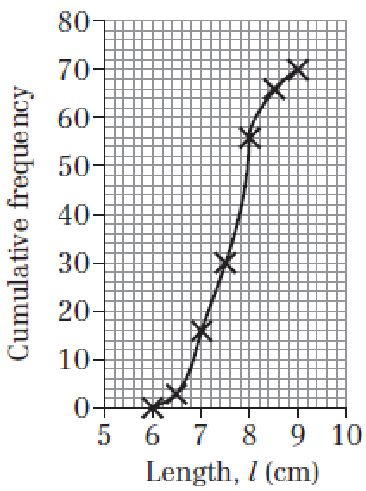
c $Q_1 = 1.43$ kg
$Q_3 = 1.75$ kg
IQR = 1.75 − 1.43 = 0.32 kg

$P_{90} - P_{10} = 1.9 - 1.25 = 0.65$ kg

d



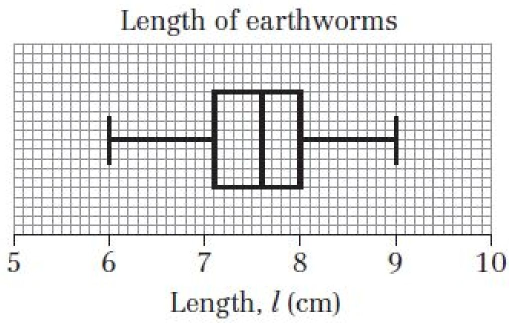
2 a Cumulative frequencies are:
3, 16, 30, 56, 66, 70



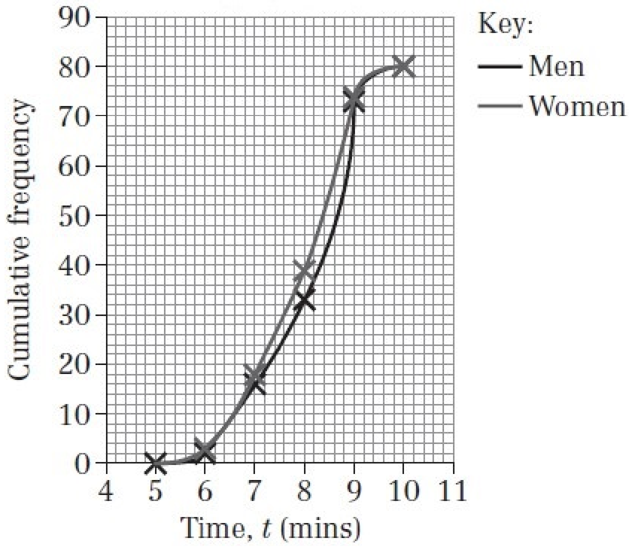
2 b Median = 7.6 cm
Q_1 = 7.1 cm
Q_3 = 8 cm

c i 70 − 62 = 8
ii 24

d



3 a Cumulative frequencies for men are: 2, 16, 33, 73, 80
Cumulative frequencies for women are: 3, 18, 39, 74, 80



b Median for men is 8.2 mins
Median for women is 8 mins

Women have the lower median time.

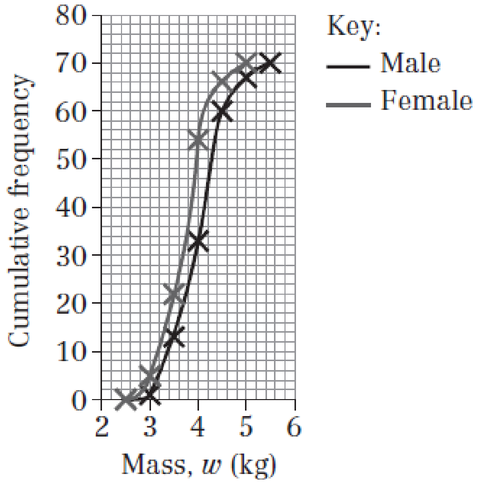
c IQR for men is 8.9 − 7.2 = 1.7 mins
IQR for women is 8.6 − 7.1 = 1.5 mins

Men have the greater spread of times.

d 24 men and 28 women

4 a Cumulative frequencies for males are: 1, 13, 33, 60, 67, 70
Cumulative frequencies for females are: 5, 22, 54, 66, 70, 70

4 a



b IQR for males is 4.4 − 3.6 = 0.8 kg
IQR for females is 4 − 3.4 = 0.6 kg

Males have the greater spread of masses.

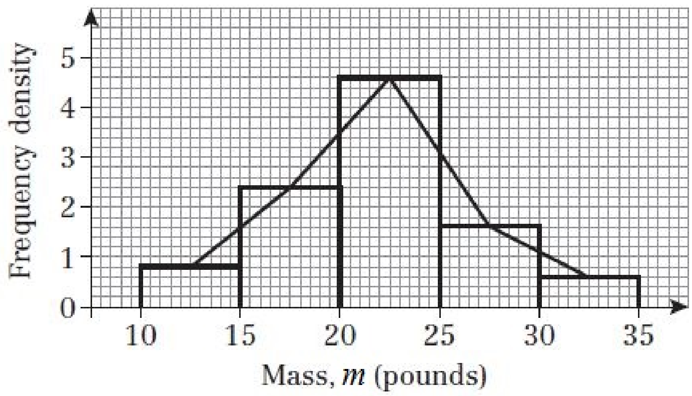
c There are approximately 12 underweight females, and 25 underweight males.

There are fewer female underweight cats.

Representations of data 3D

1 a Class widths are all 5.

Frequency densities are: 0.8, 2.4, 4.6, 1.6, 0.6



2 a Time is a continuous variable.

b Frequency = frequency density × class width
= 5 × 20 = 100

So there were 100 students who took between 40 and 60 seconds.

c 100 students took between 40 and 60 seconds (from part **b**)
6 × 10 = 60 students took between 60 and 70 seconds.
8.6 × 10 = 86 students took between 70 and 80 seconds.

So 100 + 60 + 86 = 246 students took 80 seconds or less.

d 246 students took 80 seconds of less (from part **c**)
14 × 5 = 70 students took between 80 and 85 seconds.
12 × 5 = 60 students took between 85 and 90 seconds.
3 × 30 = 90 students took between 90 and 120 seconds.

Total: 246 + 70 + 60 + 90 = 466
There are 466 students in total.

3 a Distance is a continuous variable.

b Frequency = frequency density × class width
0 m to 20 m: 2 × 20 = 40
20 m to 35 m: 5 × 15 = 75
35 m to 45 m: 10 × 10 = 100
45 m to 60 m: 6 × 15 = 90
60 m to 65 m: 1 × 5 = 5

Total: 40 + 75 + 100 + 90 + 5 = 310

So 310 people entered the competition.

3 c Frequency = frequency density × class width
30 m to 35 m: $5 \times 5 = 25$
35 m to 40 m: $10 \times 5 = 50$

Total: $25 + 50 = 75$

So 75 people threw between 30 and 40 metres.

d From part **b**:
45 m to 60 m: 90 people
60 m to 65 m: 5 people

Total: $90 + 5 = 95$

So 95 people threw between 45 and 60 metres.

e From part **b**, 40 people threw between 0 and 20 metres.
20 m to 25 m: $5 \times 5 = 25$

Total: $40 + 25 = 65$

So 65 people threw between 0 and 25 metres.

4 a The bar for $28 \leqslant m < 32$ has an area of $10 \times 10 = 100$ squares.
If 100 squares represents 32 lambs then
$\frac{100}{4}$ squares represents $\frac{32}{4}$ lambs.
i.e. 25 squares represents 8 lambs.

b The class $24 \leqslant m < 26$ contains $5 \times 20 = 100$ squares.
As above, this represents 32 lambs.

c The class $20 \leqslant m < 24$ contains $10 \times 10 = 100$ squares which represents 32 lambs.

The class $24 \leqslant m < 26$ contains $5 \times 20 = 100$ squares which represents 32 lambs.

The class $26 \leqslant m < 28$ contains $5 \times 40 = 200$ squares which represents 64 lambs.

The class $28 \leqslant m < 32$ contains $10 \times 10 = 100$ squares which represents 32 lambs.

The class $32 \leqslant m < 34$ contains $5 \times 5 = 25$ squares which represents 8 lambs.

So in total we have $32 + 32 + 64 + 32 + 8 = 168$ lambs.

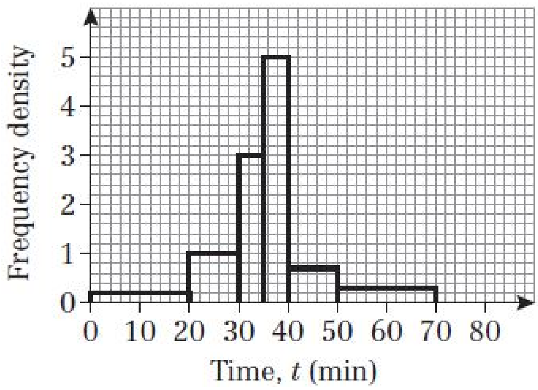
d Class $25 \leqslant m < 26$ is approximately $\frac{1}{2}$ of class $24 \leqslant m < 26$ which equates to 16 lambs.
Class $26 \leqslant m < 28$ represents 64 lambs.
Class $28 \leqslant m < 29$ is approximately $\frac{1}{4}$ of class $28 \leqslant m < 32$ which equates to 8 lambs.

So in total we have $16 + 64 + 8 = 88$ lambs.

5 a i Use extra columns to help, using the frequency densities given in the histogram:

Time, t (min)	Frequency	Class width	Frequency density
$0 \leqslant t < 20$	4	20	0.2
$20 \leqslant t < 30$	$10 \times 1 = 10$	10	1
$30 \leqslant t < 35$	15	5	3
$35 \leqslant t < 40$	25	5	5
$40 \leqslant t < 50$	$10 \times 0.7 = 7$	10	0.7
$50 \leqslant t < 70$	$20 \times 0.3 = 6$	20	0.3

5 a ii



b $\left(\frac{5}{10}\times 10\right)+15+\left(\frac{3}{5}\times 25\right)=35$ passengers.

6 a 12.5 and 14.5 are the class boundaries, as we are dealing with continuous data.

b i The class boundaries for the 15–17 class are 14.5 and 17.5.
This width is 1.5 times the width of the 13–14 class, since 17.5 – 14.5 = 3 = 1.5 x 2.
So the width of the class is $1.5 \times 4 = 6$ cm.

ii The frequency density for the 13–14 class is $\frac{24}{2}=12$.

The frequency density of this class is 6, which is 0.5 times the frequency density above: 12.
So the height of the class is 0.5 x 6 = 3 cm.

7 **a** Width is half of the $8 \leqslant t < 10$ class, which is 0.5 cm.

Height is double the frequency density, so must be $\frac{7}{1} \times 2 = 14$ cm.

b Mean $= \frac{\sum fx}{\sum f} = \frac{645}{62} = 10.4$°C (to 3 s.f.) where x is taken as the midpoint of each class.

Standard deviation $= \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} = \sqrt{113.89 - \left(\frac{645}{62}\right)^2} = 2.4$ (to 2 s.f.)

c Q_1 is the $\frac{31}{4} = 7.75^{\text{th}}$ piece of data.

$\frac{Q_1 - 8}{10 - 8} = \frac{7.75 - 4}{12 - 4}$ using linear interpolation on the $8 \leq t < 10$ class.

$Q_1 = 8.94$°C

d Mean + standard deviation = 12.8°C

$\frac{12.8 - 12}{15 - 12} = \frac{d - 25}{30 - 25}$

$d = 26.33$ days

So 4.7 days

Representations of data 3E

1 Median for motorway A = 76 mph
Median for motorway B = 72 mph

IQR for motorway A = 80 − 75 = 5 mph
IQR for motorway B = 75 − 65 = 10 mph

The median speed is greater on motorway A than on motorway B. The spread of speeds for motorway B is greater than the spread of speeds for motorway A.

2 Mean for class 2B = $\frac{\sum x}{n} = \frac{650}{20}$ = 32.5 minutes

Mean for class 2F = $\frac{\sum x}{n} = \frac{598}{22}$ = 27.2 minutes (to 3 s.f.)

Standard deviation for class 2B = $\sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$

$$= \sqrt{\frac{22000}{20} - \left(\frac{650}{20}\right)^2}$$

$$= 6.61 \text{ (to 3 s.f.)}$$

Standard deviation for class 2F = $\sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$

$$= \sqrt{\frac{19100}{22} - \left(\frac{598}{22}\right)^2}$$

$$= 11.4 \text{ (to 3 s.f.)}$$

The mean time for Class 2B is higher than the mean time for Class 2F, showing that Class 2F are generally faster at completing the puzzle. The standard deviation for Class 2F is bigger than for Class 2B, showing that the times are more spread out.

3 Boys median = 163 cm
Girls median = 158 cm

Boys IQR = 171 − 154.5 = 16.5 cm
Girls IQR = 164.5 − 149.5 = 15 cm

The median height for boys is higher than the median height for girls, showing that boys are gnerally taller. Comparing interquartile ranges,,the spread of heights for boys is greater than the spread of heights for girls.

4 a Leuchars: 89, 91, 94, 98, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100

Q_1 = 4th value = 98

Median = 8th value = 100

Q_3 = 12th value = 100

4 a Camborne: 81, 90, 91, 92, 95, 96, 98, 98, 99, 99, 99, 100, 100, 100, 100

Q_1 = 4th value = 92

Median = 8th value = 98

Q_3 = 12th value = 100

b The median for Leuchars is higher than for Camborne, showing that the humidity in Leuchars is higher. Comparing interquartile ranges, the spread of humidity in Camborne is greater than the spread of humidity in Leuchars.

Large data set

1 a Hurn 1987 mean $= \dfrac{\sum x}{n}$

$$= \frac{1103}{167}$$
$$= 6.60 \text{ kn (to 3 s.f.)}$$

Hurn 2015 mean $= \dfrac{\sum x}{n}$

$$= \frac{1418}{184}$$
$$= 7.71 \text{ kn (to 3 s.f.)}$$

b Hurn 1987 mode = 4 kn
Hurn 2015 mode = 7 kn

c Hurn 1987 standard deviation $= \sqrt{\dfrac{\sum x^2}{n} - \left(\dfrac{\sum x}{n}\right)^2}$

$$= \sqrt{\frac{8793}{167} - \left(\frac{1103}{167}\right)^2}$$
$$= 3.00 \text{ kn (to 3 s.f.)}$$

Hurn 2015 standard deviation $= \sqrt{\dfrac{\sum x^2}{n} - \left(\dfrac{\sum x}{n}\right)^2}$

$$= \sqrt{\frac{12410}{184} - \left(\frac{1418}{184}\right)^2}$$
$$= 2.84 \text{ kn (to 3 s.f.)}$$

2 The mean and modal windspeeds were higher in 2015 than in 1987. The spread of the speeds was greater in 1987 than in 2015 due to a higher standard deviation.

Representations of data, Mixed Exercise 3

1 a Q_1: $\frac{31}{4} = 7.75$ so we pick the 8th value: 178

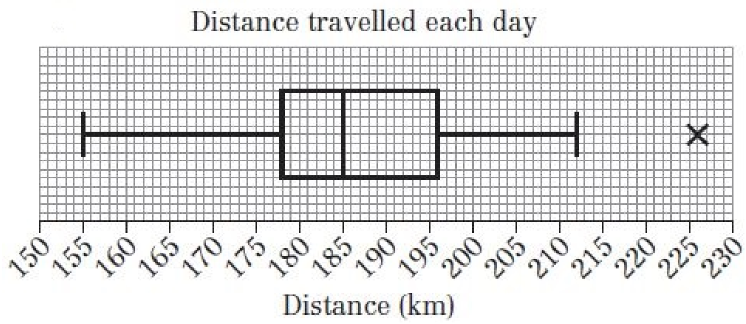
Q_2: $\frac{31+1}{2} = 16$ so we pick the 16th value: 185

Q_3: $\frac{3}{4} \times 31 = 23.25$ so we pick the 24th value: 196

b $Q_1 - 1.5(Q_3 - Q_1) = 178 - 1.5(196 - 178) = 151$
$Q_3 + 1.5(Q_3 - Q_1) = 196 + 1.5(196 - 178) = 223$

So 226 km is an outlier.

c



2 a 45 minutes

b 60 minutes

c This is an outlier that does not fit the pattern.

d The Irt club had the highest median, so overall they had the slowest runners.

The IQR ranges were about the same, with the Irt club slightly more spread out.

e With the exception of the outlier, the Esk Club runners were faster in every respect. Their minimum, Q_1, Q_2, Q_3 and maximum times were all lower than the corresponding times for the Irt Club runners.

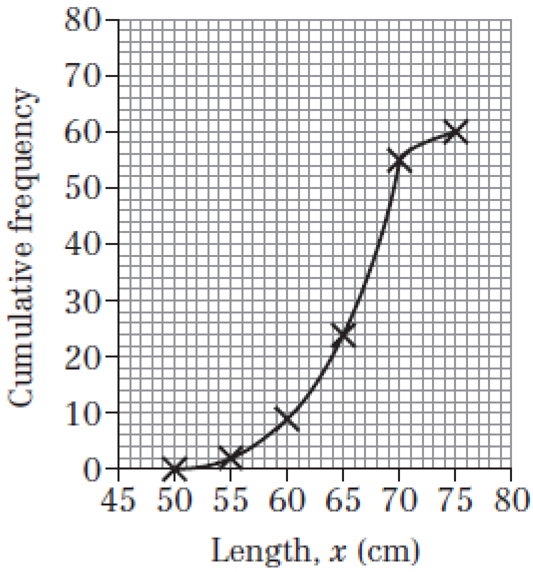
f Advantages, any one from:

It helps us to see the spread of the data easily.
The plot is clear and easy to understand.
It uses the range and the median values.
It is easy to compare the stratified data.

Disadvantages, any one from:

Original data is not clearly shown in the box plot.
Mean and mode cannot be identified using the box plot.
It can be easily misinterpreted.
If large outliers are present, the box plot is more likely to give an incorrect representation.

3 a Cumulative frequencies are: 2, 9, 24, 55, 60



b Median = 66 cm

c IQR = 69 − 62.5 = 6.5 cm

d Median = 65.5 cm

IQR = 70 − 63.25 = 6.75

The median length of the honey badgers is slightly higher than the median length for the European badgers, showing that honey badgers are only a little longer on average than European badgers. Comparing interquartiles ranges, the spread of lengths of European badgers is slightly greater than the spread of lengths of honey badgers.

e The cumulative frequency diagram shows us how the lengths of the badgers is spread and enables us to estimate the median and quartiles, but tells us little about individual data points.

4 a The areas of the bars is proportional to the frequency represented.

$2k(1+1.5+5.5+4.5)+4k(1)=58$

$29k=58$ so $k=2$

Number of girls who took longer than 56 seconds $=2\left((4.5\times 2)+(1\times 4)\right)=26$ girls

b Number of girls who took between 52 and 55 seconds $=2\left((1.5\times 2)+(\tfrac{1}{2}\times 2\times 5.5)\right)=17$ girls

5 $1.5\times 5.7\times k=2565$, so $k = 300$

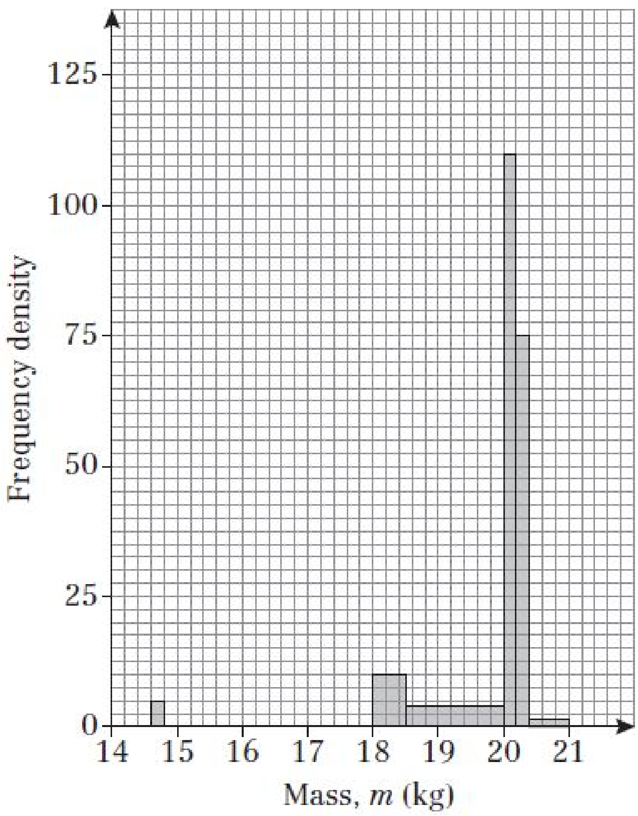
a width $=1\times 1.5=1.5\,\text{cm}$

height $=\dfrac{\text{frequency}}{k\times\text{width}}=\dfrac{1170}{300\times 1.5}=2.6\,\text{cm}$

5 b width $= 5 \times 1.5 = 7.5\,\text{cm}$

height $= \dfrac{630}{300 \times 7.5} = 0.28\,\text{cm}$

6 a



b Mean $= \dfrac{\sum fy}{n} = \dfrac{988.85}{50} = 19.777$ kg

Standard deviation $= \sqrt{\dfrac{\sum fy^2}{n} - \mu^2} = \sqrt{\dfrac{19602.84}{50} - 19.777^2} = \sqrt{0.927} = 0.963$ (to 3 s.f.)

c Median $= 20.0 + \dfrac{13.5}{22} \times 0.2 = 20.123$ (to 3 d.p.)

7 a $\dfrac{312}{14} = 22.286$ (to 3 d.p.)

b Median is the $\dfrac{14+1}{2} = 7.5$th piece of data: 20

Q_1 is the $\dfrac{14}{4} = 3.5$th piece of data, so we choose the 4th: 13

Q_3 is the $\dfrac{3 \times 14}{4} = 10.5$th piece of data, so we choose the 11th: 31

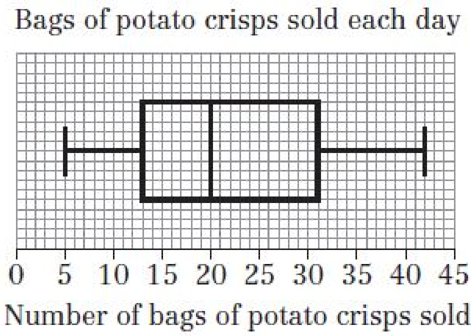
7 c $\text{IQR} = 31 - 13 = 18$ so $1.5 \times \text{IQR} = 27$

$13 - 27 = -14$

$31 + 27 = 58$

So there are no outliers.

d



8 a The maximum gust is a continuous variable and the data is given in a grouped frequency table.

b $10 \leqslant g < 15$ bar:

Width = 2.5, class width = 5, so width = class width × 0.5
Area = 2.5 × 1.8 = 4.5 cm^2, frequency = 3, so area = frequency × 1.5

$18 \leqslant g < 20$ bar:

Width = class width × 0.5 (from above) = 2 × 0.5 = 1 cm
Area = frequency × 1.5 (from above) = 9 × 1.5 = 13.5 cm^2
Height $= \dfrac{\text{Area}}{\text{Width}} = 13.5$ cm

c Mean $= \dfrac{\sum fx}{\sum f} = \dfrac{1334.5}{57} = 23.4$ kn (to 3 s.f.)

Standard deviation $= \sqrt{\dfrac{\sum fx^2}{\sum f} - \left(\dfrac{\sum fx}{\sum f}\right)^2} = \sqrt{\dfrac{34299.25}{57} - \left(\dfrac{1334.5}{57}\right)^2} = 7.32$ (to 3 s.f.)

d Using exact figures throughout:

lower boundary = mean – standard deviation = 16.1 (to 3 s.f.)
higher boundary = mean + standard deviation = 30.7 (to 3 s.f.)

$\dfrac{\text{lower boundary} - 15}{18 - 15} = \dfrac{g - 3}{12 - 3} \Rightarrow g = 6.27$ (to 3 s.f.)

$\dfrac{30.72 - 30}{50 - 30} = \dfrac{h - 50}{57 - 50}$

$h - g = 44.0$ (to 3 s.f.)

9 **a** Mean for 1987 = $\frac{\sum x}{n} = \frac{356.1}{30}$ = 11.9 °C (to 1 d.p.)

Mean for 2015 = $\frac{\sum x}{n} = \frac{364.1}{30}$ = 12.1 °C (to 1 d.p.)

b Standard deviation for 1987 = $\sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} = \sqrt{\frac{4408.9}{30} - \left(\frac{356.1}{30}\right)^2}$ = 2.46 °C (to 3 s.f.)

The mean temperature was slightly higher in 2015 than in 1987. The standard deviation of temperatures was higher in 1987 than in 2015, so the temperatures were more spread out.

c In all calculations, exact figures are used.
critical temperature = mean − 1.02 = 11.08 °C

Let a = the number of the day with this temperature.

$$\frac{\text{critical temperature} - 10.1}{14.1 - 10.1} = \frac{a - 0}{30 - 0} \text{ so } a = 7.35$$

So there are 7 days where the temperature ⩽ 11.08 °C.

critical temperature = mean + 1.02 = 13.12 °C

Let b = the number of the day with this temperature.

$$\frac{\text{critical temperature} - 10.1}{14.1 - 10.1} = \frac{b - 0}{30 - 0} \text{ so } b = 22.65$$

So there are 22 days where the temperature ≤ 13.12 °C.

30 − (b − a) = 14.7 (to 3 s.f.)

So there are 15 'abnormal' days.

Assumption: the temperatures are equally distributed throughout the range.

Challenge

Length (mins)	Frequency	Area of bar	Class width	Bar height
70–89	4	$4k$	20	x
90–99	17	$17k$	10	$x + 3$
100–109	20	$20k$	10	
110–139	9	$9k$	30	
140–179	2	$2k$	40	

Area of 70–89 bar = $20x = 4k$, so $x = \frac{k}{5}$

Area of 90–99 bar = $10(x + 3) = 10x + 30 = 17k$

Using substitution

$10 \times \frac{k}{5} + 30 = 17k$, so $2k + 30 = 17k$, i.e. $k = 2$

$x = \frac{k}{5} = 0.4$

Area of 110–139 class = $9k = 9 \times 2 = 18$ cm^2

Height = $\frac{\text{Area}}{\text{Class width}} = \frac{18}{30} = 0.6$ cm

Correlation 4A

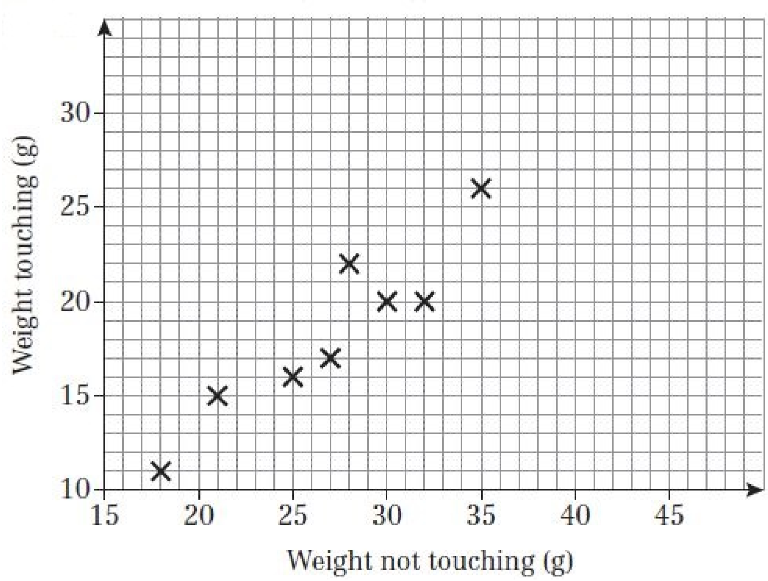
1 a Positive correlation.

b The longer the treatment the greater the loss of weight.

2 a There is no correlation.

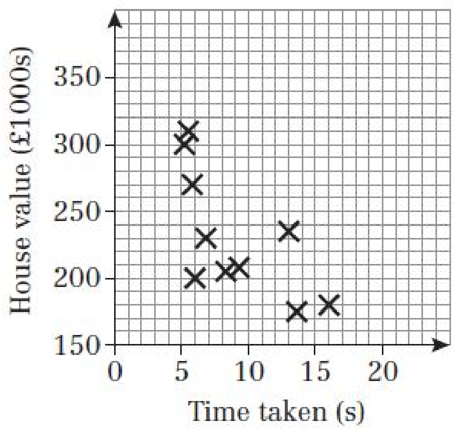
b The scatter graph does not support the statement that hotter cities have less rainfall.

3 a



b It shows positive correlation. Each student's tendency to guess lower or higher than the mean was the same in both tests.

4 a



b There is weak negative correlation

c For example, there may be a third variable that influences both house value and internet connection, such as the distance from built up areas.

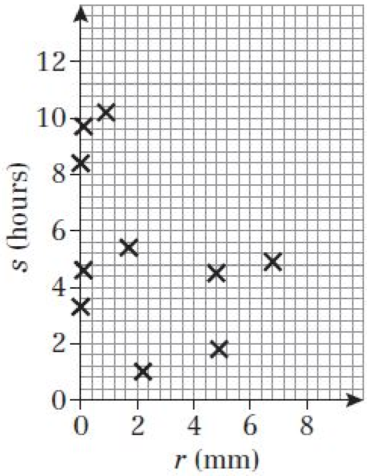
5 a $Q_3 + 1.5(Q_3 - Q_1) = 4.85 + 1.5(4.85 - 0.1) = 11.975$

As $21.7 > 11.975$, r is an outlier.

5 b i The outlier is part of the distribution of the data.
It is unlikely to be an anomaly.

ii 21.7 is an outlier so may not be representative of the typical rainfall

c

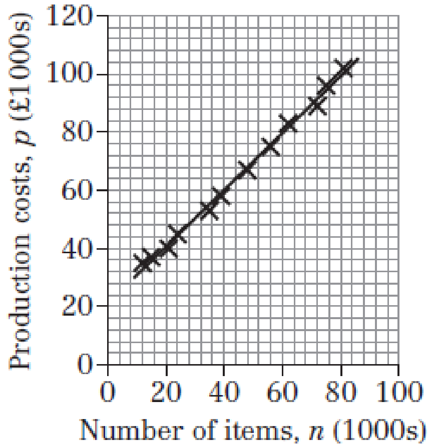


d There is weak negative correlation

e There could be a causal relationship, as days with more rainfall will have more clouds and therefore less sunshine.

Correlation 4B

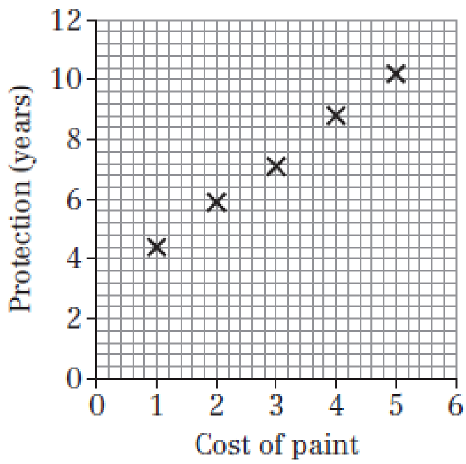
1 a and **b**



c If the number of items produced is zero, the production costs will be approximately £21 000. If the number of items produced increases by 1000, the production costs increase by approximately £980.

d The prediction for 74 000 is within the range of the data (interpolation) so is more likely to be reliable The prediction for 95 000 is outside the range of the data (extrapolation) so is less likely to be reliable.

2 a

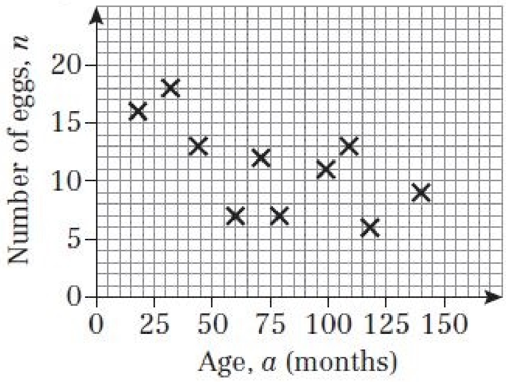


b There are two key problems with Helen's statement:

First, 10 coats of paint is very far outside our range of given data, and we cannot assume that this linear relationship continues as we extrapolate, so using the regression line is not necessarily valid.

Second, even if we accept the extrapolation as valid, a gradient of 1.45 means that, for every extra coat of paint, the protection will increase by 1.45 years. Therefore, if 10 coats of paint are applied, the protection will be 14.5 years longer than if no paint were applied. Helen has, however, forgotten to include the constant 2.93 years, which is the weather resistance if no paint were applied. After 10 coats of paint the protection will last approximately 2.93 + 14.5 = 17.43 years.

3 a



b The scatter diagram shows weak negative correlation, therefore the gradient in the regression equation, given as 0.063, should be negative.

4 This is not a reasonable statement as there are unlikely to be any houses with no bedrooms, so she is extrapolating outside of the range of data, where the linear relationship is unlikely to continue.

5 a If the humidity increases by 1%, the decrease in visibility is approximately 106 m.

b High humidity (> 95%) can give rise to misty and foggy conditions, which in turn will cause lower visibility. Hence there is likely to be a causal relationship.

c i The prediction for 100% is outside the range of the data (extrapolation) so is less likely to be accurate.

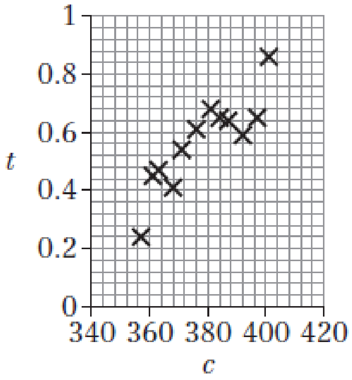
ii This particular regression equation should only be used to predict a value for v given h.

d The data provided is only useful for analysing the first two weeks of September. Random values throughout September should be used and analysis made of the whole month. The sample size could also be increased across multiple months as data between May and October is available.

Correlation, Mixed Exercise 4

1 The data shows that the number of serious road accidents in a week strongly correlates with the number of fast food restaurants. However, it does not show whether the relationship is causal. Both variables could correlate with a third variable, e.g. the number of roads coming into a town.

2 a



b There is strong positive correlation.

c As mean CO_2 concentration in the atmosphere increased, mean temperatures also increased.

3 a There is strong positive correlation.

b If the number of items increases by 1, the time taken increases by approximately 2.64 minutes.

4 The answer is likely to be unreliable as it involves extrapolation. 3500 is well outside the limits of the data set used.

The economist used the regression line of y on x. Estimating energy consumption (x) from Gross National Product (y) would need the regression line of x on y.

5 a $15.2 + 2 \times 11.4 = 38$

As $50 > 38$, $t = 50$ °C is an outlier.

b The outlier should be omitted, as it is very unlikely that the average temperature was 50 °C in a climate where people need to buy gloves, and so this data point is likely an anomaly.

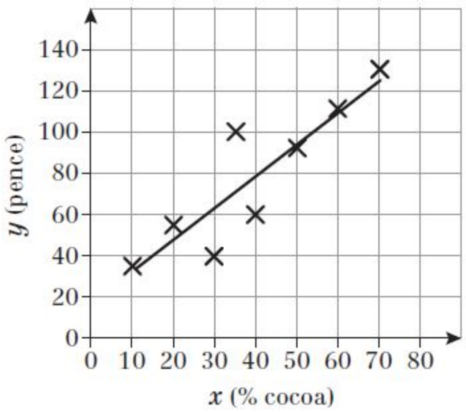
c If the temperature increases by approximately 1 °C, the number of pairs of gloves sold each month decreases by 5.2.

6 a 44 cm is the length of the spring with no mass attached. If a mass of 1 g is attached, the spring will increase in length by approximately 0.2 cm.

b i $m = 150$ is outside the range of the data (extrapolation) so is less likely to be accurate.

ii This particular regression equation should only be used to predict a value of s given m. To predict a value of m given s, you should use the regression equation of m on s.

7 a and **b**

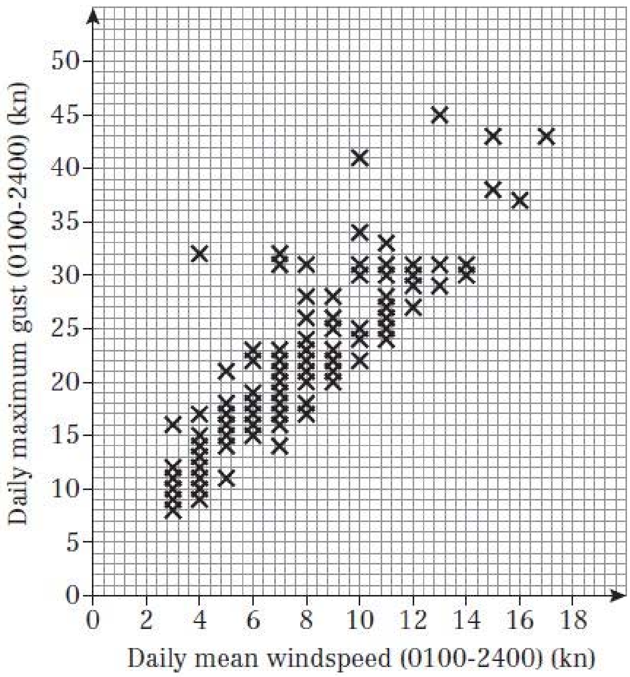


c Brand D is overpriced, since it's price is much more than you would expect (the data point is far above the regression line)..

d The regression equation should be used to predict a value for y given x, i.e. the price given the percentage of cocoa solids. So the student's method is a valid one.

Large data set

1 a



b There is moderate positive correlation

c The relationship is causal, as the maximum gust is related to the mean windspeed. A higher mean windspeed is likely to result in a higher maximum gust.

d i $g = 4.97 + 2.15 \times 0.5$
$= 6.045$

d **ii** $g = 4.97 + 2.15 \times 5$
$= 15.72$ kn

iii $g = 4.97 + 2.15 \times 12$
$= 30.77$ kn

iv $g = 4.97 + 2.15 \times 40$
$= 90.97$ kn

e Parts **ii** and **iii** are within the range of the data (interpolation), so are more likely to be accurate. Parts **i** and **iv** are outside the range of the data (extrapolation), so are less likely to be accurate.

f Using the 'SLOPE' and 'INTERCEPT' function in a spreadsheet:

$w = 0.05 + 0.35\,g$

$w = 0.05 + 0.35 \times 30$
$= 10.55$ kn

2 **a** Using the 'SLOPE' and 'INTERCEPT' function in a spreadsheet:

$s = 14.6 – 1.7c$ (values taken to 2 d.p.)

The missing total sunshine data values are approximately:

2.7, 7.8, 6.1, 11.2, 2.7. 7.8, 11.2, 14.6, 9.5, 6.1. 2.7, 4.4, 7.8, 2.7, 4.4, 4.4, 1.0

b The relationship is likely to be causal: greater cloud cover is likely to result in less sunshine.

Probability 5A

1

	Coin 1		
Coin 2		**H**	**T**
	H	HH	TH
	T	HT	TT

P(same) = $\frac{2}{4}=\frac{1}{2}$

2 a

		Second roll					
		1	2	3	4	5	6
First roll	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

b i $P(X = 24) = \frac{2}{36}=\frac{1}{18}$

ii $P(X < 5) = \frac{8}{36}=\frac{2}{9}$

iii $P(X \text{ is even}) = \frac{27}{36}=\frac{3}{4}$

3 a $P(m \geqslant 54) = \frac{33+21+2}{140}=\frac{56}{140}=\frac{2}{5}$

b $P(48 \leqslant m < 57) = \frac{25+42+33}{140}=\frac{100}{140}=\frac{5}{7}$

c Let B = the number of Bullmastiffs with mass less than 53 kg.

Using interpolation:

$$\frac{53-51}{54-51}=\frac{B-(17+25)}{42}$$

So $B = 70$

3 c $\text{P}(m < 53) = \frac{70}{140} = 0.5$, so half of the Bullmastiffs are estimated to have a mass less than 53 kg.

This probability is lower than the probability of 0.54 for Rottweilers, and so it is less likely.

The assumption made is that the frequency is uniformly distributed throughout the class.

4 a $\text{P(female)} = \frac{14+15+32+27+26}{240} = \frac{114}{240} = \frac{19}{40}$

b $\text{P}(l < 80) = \frac{4+14+20+15+24+32}{240} = \frac{109}{240}$

c $\text{P(male and } 75 \leqslant l < 85) = \frac{24+47}{240} = \frac{71}{240}$

d Using interpolation for males:

The number of male juvenile koalas is approximately $4 + \frac{72-70}{75-70} \times 20 = 4 + 8 = 12$.

The number of female juvenile koalas is approximately $14 + \frac{72-70}{75-70} \times 15 = 14 + 6 = 20$.

So $\text{P(juvenile)} = \frac{12+20}{240} = \frac{32}{240} = \frac{2}{15}$

The assumption is that the distribution of lengths of koalas between 70 and 75 cm is uniform.

5 a $\text{P}(m > 5) = \frac{(1\times 24)+(2\times 4)}{70} = \frac{32}{70} = \frac{16}{35}$

b Start with the probability that the cat has a mass *greater* than 6.5.

$\text{P}(m > 6.5) = \frac{\frac{3}{4}\times(2\times 4)}{70} = \frac{6}{70} = \frac{3}{35}$

So $\text{P}(m < 6.5) = 1 - \frac{3}{35} = \frac{32}{35}$

The fact that we have ignored the case 6.5 is not a problem in this estimate. We are assuming that the class is continuous when we interpolate, and that the probability of being exactly equal to any individual value is negligible.

Challenge

	A			
B	×	**2**	**7**	**5**
	4	8	28	20
	$\boldsymbol{x}$	$2x$	$7x$	$5x$

If x is even, all the products are even, so P(Y is even) = 1

But $P(Y \geqslant 20) = 1$ is impossible, as the product of 2 and 4 is only 8, so x cannot be even.

If x is odd, there are four even values of Y: 8, 28, 20 and $2x$.

But P(Y is even) = $P(Y \geqslant 20)$, so there must also be four values where $Y \geqslant 20$.

Two of them are in the top row: 28 and 20, leaving two in the bottom row.

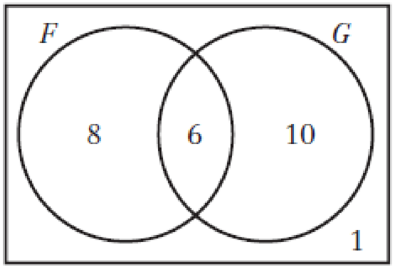
Given that exactly two of these three values are greater than or equal to 20:

$2x < 20$ and $5x \geqslant 20$, i.e. $x < 10$ and $x \geqslant 4$.

Hence $4 \leqslant x < 10$ and x is odd so the possible values of x are 5, 7 and 9.

Probability 5B

1 a



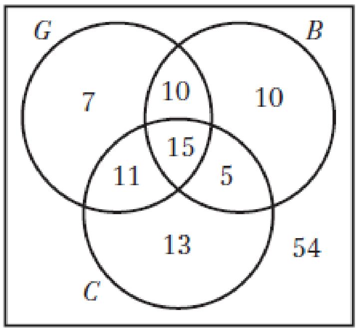
b i $\text{P}(F) = \frac{14}{25}$

ii $\text{P}(F \cap G) = \frac{6}{25}$

iii $\text{P(French but not German)} = \frac{8}{25}$

iv $\text{P(Neither French nor German)} = \frac{1}{25}$

2 a



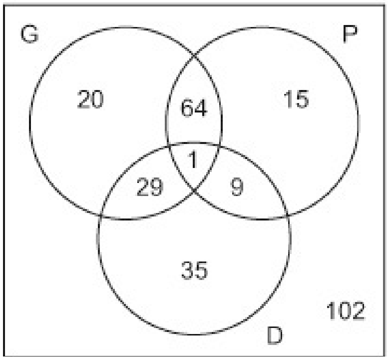
b i $\text{P(All three)} = \frac{15}{125} = \frac{3}{25}$

ii $\text{P(Beer but not cheesecake and not garlic bread)} = \frac{10}{125} = \frac{2}{25}$

iii $\text{P(Garlic bread and beer but not cheesecake)} = \frac{10}{125} = \frac{2}{25}$

iv $\text{P(None)} = \frac{54}{125}$

3 a



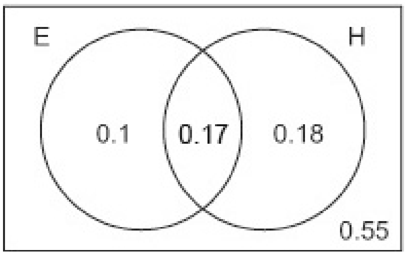
b i $\text{P(Plays piano)} = \frac{89}{275}$

ii $\text{P(At least 2)} = \frac{64+9+29+1}{275} = \frac{103}{275}$

iii $\text{P(Plays exactly one)} = \frac{20+15+35}{275} = \frac{14}{55}$

iv $\text{P(Plays none)} = \frac{102}{275}$

4

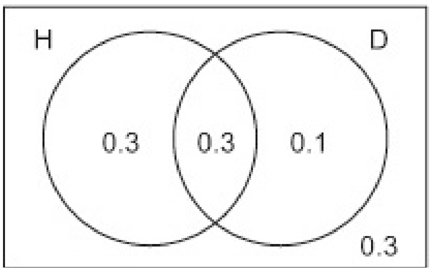


a $$\begin{aligned}\text{P}(E \cap H) &= \text{P}(E) + \text{P}(H) - \text{P}(E \cup H)\\ &= 0.27 + 0.35 - 0.45\\ &= 0.17\end{aligned}$$

b $\text{P(Blonde hair but not Blue eyes)} = 0.35 - 0.17 = 0.18$

c $\text{P(Neither)} = 1 - \text{P}(E \cup H) = 1 - 0.45 = 0.55$

5



5 a $\mathrm{P}(H \cap D) = \mathrm{P}(H) + \mathrm{P}(D) - \mathrm{P}(H \cup D)$
$= 0.6 + 0.4 - 0.7$
$= 0.3$

b P(Hiya only) $= 0.6 - 0.3 = 0.3$

6 a $\mathrm{P}(B) = x + 0.1 + 0.2 = 0.45$

So $x = 0.45 - 0.3 = 0.15$.

b $y = 1 - (0.35 + 0.15 + 0.1 + 0.2 + 0.05) = 1 - 0.85 = 0.15$

7 $\mathrm{P}(M) = 0.32 + p$

$\mathrm{P}(P) = p + q + 0.07$

As P(M) = $\mathrm{P}(P)$, $0.32 + p = p + q + 0.07$.

So, rearranging, $q = 0.32 - 0.07 = 0.25$.

$p = 1 - (0.32 + 0.25 + 0.07 + 0.13 + 0.1) = 0.13$

$p = 0.13$, $q = 0.25$

Challenge

$\mathrm{P}(B) = p + q + 0.05$

$\mathrm{P}(A) = 0.15 + p$

As $\mathrm{P}(B) = 2\mathrm{P}(A)$, $p + q + 0.05 = 2(0.15 + p)$, or $p + q + 0.05 = 0.3 + 2p$

So our first equation relating p and q is: $q = 0.25 + p$

As P(not C) = 0.83

$0.15 + p + q + 0.2 = 0.83$, so our second equation results: $p + q = 0.48$

Using substitution to solve simultaneously:

$p + (0.25 + p) = 0.48$, so $2p = 0.23$ and therefore $p = 0.115$

$q = 0.25 + 0.115 = 0.365$

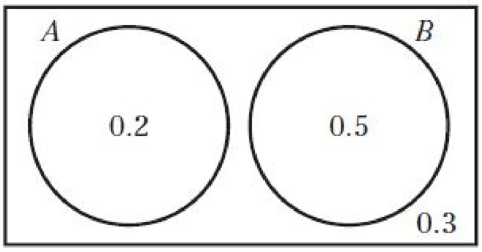
P(C) = 1 – P(not C) = 1 − 0.83 = 0.17

Hence $r + 0.05 = 0.17$, so $r = 0.12$

$p = 0.115$, $q = 0.365$, $r = 0.12$

Probability 5C

1 a



b $\mathrm{P}(A \cup B) = 0.7$

c $\mathrm{P}(A' \cap B') = 0.3$

2 $\mathrm{P}(\text{Sum of 4}) = \frac{3}{36} = \frac{1}{12}$

$\mathrm{P}(\text{Same number}) = \frac{6}{36} = \frac{1}{6}$

$\mathrm{P}(\text{Sum of 4}) + \mathrm{P}(\text{Same number}) = \frac{1}{12} + \frac{1}{6} = \frac{1}{4}$

$\mathrm{P}(\text{Sum of 4 or same number}) = \frac{8}{36} = \frac{2}{9}$

$\mathrm{P}(\text{Sum of 4}) + \mathrm{P}(\text{Same number}) \neq \mathrm{P}(\text{Sum of 4 or same number})$,
so the events are not mutually exclusive.

Alternatively: A roll of 2 followed by another roll of 2 fits both conditions, so the intersection is not empty, and the events are not mutually exclusive.

3 P(A and B) = P(A) × P(B) = 0.5 × 0.3 = 0.15

4 P(A and B) = P(A) × P(B)

P(B) = P(A and B) ÷ P(A) = 0.045 ÷ 0.15 = 0.3

5 a The closed curves representing bricks and trains do not overlap and so they are mutually exclusive.

b P(B and F) = $\dfrac{1}{3+1+4+6+2+5} = \dfrac{1}{21}$

P(B) × P(F) = $\dfrac{3+1}{21} \times \dfrac{1+4+6}{21} = \dfrac{4}{21} \times \dfrac{11}{21} = \dfrac{44}{441}$

As P(B and F) ≠ P(B) × P(F), 'plays with bricks' and 'plays with action figures' are not independent events.

6 a 0.4 + x + 0.3 + 0.05 = 1

x = 0.25

6 b P(A and B) = x = 0.25

P(A) × P(B) = 0.65 × 0.55 = 0.3575

As P(A and B) ≠ P(A) × P(B), the two events ‘like pasta’ and ‘like pizza’ are not independent.

7 a P(S and T) = P(S) – P(S but not T) = 0.3 − 0.18 = 0.12

P(S) × P(T) = 0.3 × 0.4 = 0.12

As P(S and T) = P(S) × P(T), S and T are independent events.

b i P(S and T) = 0.12, as above.

ii P(neither S nor T) = 1 – P(S or T) = 1 – (P(S but not T) + P(T)) = 1 – (0.18 + 0.4) = 0.42

8 P(W and X) = P(W) – P(W and not X) = 0.5 − 0.25 = 0.25

P(X) = 1 – (P(W and not X) + P(neither W nor X)) = 1 – (0.25 + 0.3) = 0.45

P(W) × P(X) = 0.5 × 0.45 = 0.225

As P(W and X) ≠ P(W) × P(X), the two events W and X are not independent.

9 a P(A or R) = P(A) + P(R) = 0.6 because A and R are mutually exclusive.

0.2 + 0.25 + x = 0.6, so x = 0.15

$y = 1 - (0.2 + 0.25 + 0.15 + 0.1) = 0.3$

$(x, y) = (0.15, 0.3)$

b P(R and F) = x = 0.15

P(R) × P(F) = 0.4 × 0.45 = 0.18

As P(R and F) ≠ P(R) × P(F), the two events R and F are not independent.

10 P(A and B) = p

P(A) × P(B) = (0.42 + p) × (p + 0.11)

$= (p + 0.42)(p + 0.11)$

As the events A and B are independent, P(A and B) = P(A) × P(B), so

$(p + 0.42)(p + 0.11) = p$

$p^2 + 0.53p + 0.0462 = p$

$p^2 - 0.47p + 0.0462 = 0$, a quadratic in p, which we can solve with the quadratic formula

10 $p = \dfrac{0.47 \pm \sqrt{(-0.47)^2 - 4(1)(0.0462)}}{2(1)}$

$p = \dfrac{0.47 \pm 0.19}{2}$

$p = 0.33$ or 0.14

When $p = 0.14$, $q = 1 - (0.42 + 0.14 + 0.11) = 0.33$

When $p = 0.33$, $q = 1 - (0.42 + 0.33 + 0.11) = 0.14$

$(p, q) = (0.14, 0.33)$ or $(0.33, 0.14)$

Challenge

a Set P(A) = p and P(B) = q

As A and B are independent events, P(A and B) = P(A) × P(B) = pq

P(A and not B) = P(A) − P(A and B) = $p - pq$, and notice P(not B) = 1 – P(B) = 1 - q

Then P(A) × P(not B) = $p(1 - q) = p - pq$ = P(A and not B)

As P(A and not B) = P(A) × P(not B), the events A and 'not B' are independent.

b Still using p and q as above,

P(not A and not B) = 1 − P(A or B)

P(A or B) = P(A) + P(B) − P(A and B)

Meaning P(not A and not B) = 1 − P(A) − P(B) + P(A and B) = $1 - p - q + pq = (1 - p)(1 - q)$

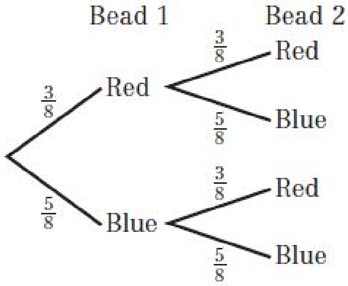
Remember P(not A) = $1 - p$ and P(not B) = $1 - q$

So P(not A) × P(not B) = $(1 - p)(1 - q)$ = P(not A and not B)

As P(not A and not B) = P(not A) × P(not B), the events 'not A' and 'not B' are independent.

Probability 5D

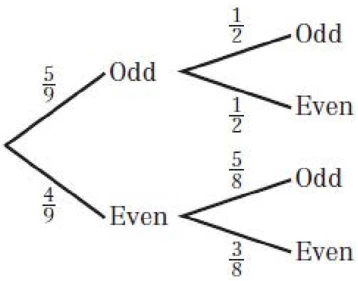
1 a



b $\text{P(both blue)} = \frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$

c $\text{P(second blue)} = \frac{3}{8} \times \frac{5}{8} + \frac{5}{8} \times \frac{5}{8} = \frac{5}{8}$

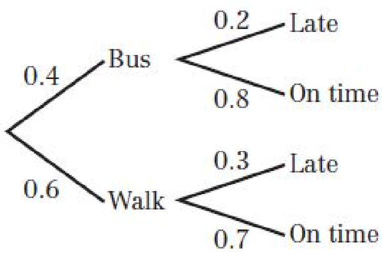
2 a



b $\text{P(both even)} = \frac{4}{9} \times \frac{3}{8} = \frac{12}{72} = \frac{1}{6}$

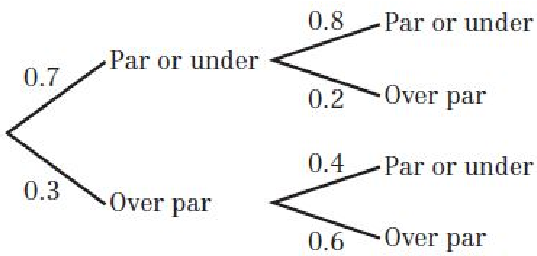
c $\text{P(different parity)} = \text{P(odd then even)} + \text{P(even then odd)} = \frac{5}{9} \times \frac{1}{2} + \frac{4}{9} \times \frac{5}{8} = \frac{40}{72} = \frac{5}{9}$

3 a



b P(late) = P(bus and late) + P(walk and late) = $0.4 \times 0.2 + 0.6 \times 0.3 = 0.26$

4 a

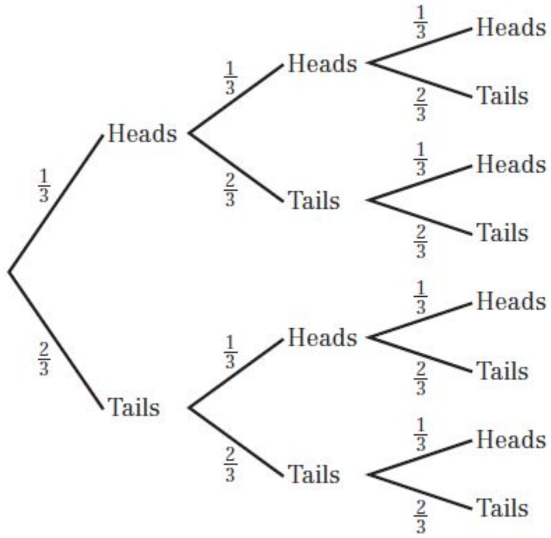


4 b The events are not independent.

c P(par or under on exactly one hole) = P(par or under then over par) + P(over par then par or under)

= 0.7 × 0.2 + 0.3 × 0.4 = 0.26

5 a



b $\text{P(HHH)} = \frac{1}{3}\times\frac{1}{3}\times\frac{1}{3} = \frac{1}{27}$

c $\text{P(one H only)} = \text{P(HTT)} + \text{P(THT)} + \text{P(TTH)} = \frac{1}{3}\times\frac{2}{3}\times\frac{2}{3}+\frac{2}{3}\times\frac{1}{3}\times\frac{2}{3}+\frac{2}{3}\times\frac{2}{3}\times\frac{1}{3} = \frac{4+4+4}{27} = \frac{4}{9}$

d $\text{P(HHH or TTT)} = \frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \frac{1+8}{27} = \frac{1}{3}$

So P(HHH or TTT in both trials) $= \frac{1}{3}\times\frac{1}{3} = \frac{1}{9}$

6 a P(both yellow) = P(first Y) × P(second Y) $= \frac{6}{13}\times\frac{5}{12} = \frac{30}{156} = \frac{5}{26}$

b P(third Y) $= \frac{4}{11}$

c P(all different) = P(BRY) + P(BYR) + P(RYB) + P(RBY) + P(YBR) + P(YRB)

$= \frac{4}{13}\times\frac{3}{12}\times\frac{6}{11} + \frac{4}{13}\times\frac{6}{12}\times\frac{3}{11} + \frac{3}{13}\times\frac{6}{12}\times\frac{4}{11} + \frac{3}{13}\times\frac{4}{12}\times\frac{6}{11} + \frac{6}{13}\times\frac{4}{12}\times\frac{3}{11} + \frac{6}{13}\times\frac{3}{12}\times\frac{4}{11}$

$= 6\times\left(\frac{4\times3\times6}{13\times12\times11}\right) = \frac{432}{1716} = \frac{36}{143}$

Probability, Mixed Exercise 5

1 a P(*RRB* or *RRG*) = $\left(\frac{7}{15}\times\frac{7}{15}\times\frac{3}{15}\right)+\left(\frac{7}{15}\times\frac{7}{15}\times\frac{5}{15}\right)$

$= \frac{392}{3375}$

b P(*RBG*) + P(*RGB*) + P(*BGR*) + P(*BRG*) + P(*GBR*) + P(*GRB*)

$=\left(\frac{7}{15}\times\frac{3}{15}\times\frac{5}{15}\right)+\left(\frac{7}{15}\times\frac{5}{15}\times\frac{3}{15}\right)+\left(\frac{3}{15}\times\frac{5}{15}\times\frac{7}{15}\right)+\left(\frac{3}{15}\times\frac{7}{15}\times\frac{5}{15}\right)+\left(\frac{5}{15}\times\frac{3}{15}\times\frac{7}{15}\right)+\left(\frac{5}{15}\times\frac{7}{15}\times\frac{3}{15}\right)$

$= 6\times\left(\frac{7\times3\times5}{15^3}\right) = \frac{630}{3375} = \frac{14}{75}$

2 a P(*HHH*) = 0.341 × 0.341 × 0.341 = 0.0397 (to 3 s.f.)

b P(*NNN*) = 0.659 × 0.659 × 0.659 = 0.286 (to 3 s.f.)

c P(at least one *H*) = 1 − P(*NNN*) = 1 − 0.28619118 = 0.714 (to 3 s.f.)

3 a P(female) = $\frac{8+13+19+30+26+32}{250} = \frac{128}{250} = \frac{64}{125}$

b P(s < 35) = $\frac{7+8+15+13+18+19}{250} = \frac{80}{250} = \frac{8}{25}$

c P(male with score between 25 and 34) = $\frac{15+18}{250} = \frac{33}{250}$

d Using interpolation:

Number of students passing $=\frac{40-37}{40-35}\times(25+30)+30+26+27+32$

$=\left(\frac{3}{5}\times55\right)+30+26+27+32=148$

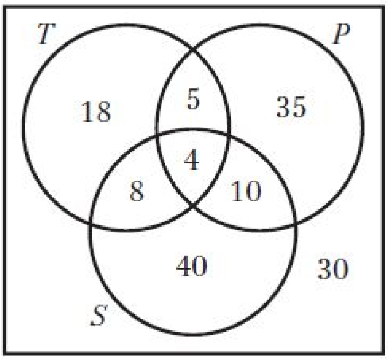
P(pass) = $\frac{148}{250} = \frac{74}{125}$

The assumption is that the marks between 35 and 40 are uniformly distributed.

4 a P(> 3) = $\frac{0.5\times50+0.5\times30+2\times2}{1\times6+0.5\times50+0.5\times30+2\times2} = \frac{44}{50} = \frac{22}{25}$

b P(< 3.75) = $\frac{(1\times6)+(0.5\times50)+0.5\times(0.5\times30)}{50} = \frac{38.5}{50} = 0.77$

5 a



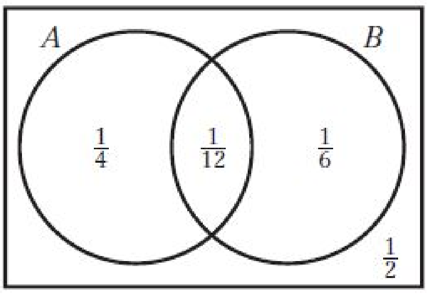
b i $\text{P(None)} = \frac{30}{150} = \frac{1}{5}$

ii $\text{P(No more than one)} = \frac{30+40+18+35}{150} = \frac{123}{150} = \frac{41}{50}$

6 a P(A and B) = P(A) + P(B) – P(A or B or both) = $\frac{1}{3} + \frac{1}{4} - \frac{1}{2} = \frac{1}{12}$

P(A and not B) = $\frac{1}{3} - \frac{1}{12} = \frac{1}{4}$

P(B and not A) = $\frac{1}{4} - \frac{1}{12} = \frac{1}{6}$



b P(A and B) = $\frac{1}{12}$

P(A) × P(B) = $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

As P(A and B) = P(A) × P(B), A and B are independent events.

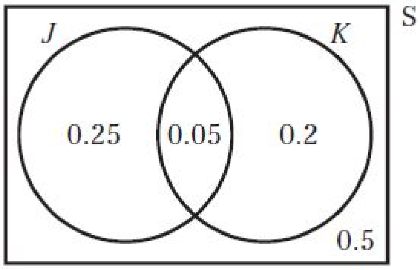
7 a Cricket and swimming do not overlap so are mutually exclusive.

b P(C and F) = $\frac{13}{38}$

P(C) × P(F) = $\frac{21}{38} \times \frac{22}{38} = \frac{462}{1444} = \frac{231}{722}$

As P(C and F) ≠ P(C) × P(F), the events 'likes cricket' and 'likes football' are not independent.

8 a



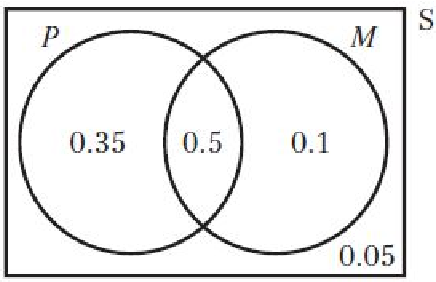
b P(J and K) = 0.05

P(J) × P(K) = 0.3 × 0.25 = 0.075

As P(J and K) ≠ P(J) × P(K), the events J and K are not independent.

9 a $\text{P}(\text{Phone and MP3}) = 0.85 + 0.6 - (1 - 0.05) = 0.5 = 50\%$

b



c P(only P) = 0.35

d P(P and M) = 0.5

P(P) × P(M) = 0.85 × 0.6 = 0.51

As P(P and M) ≠ P(P) × P(M), the events P and M are not independent.

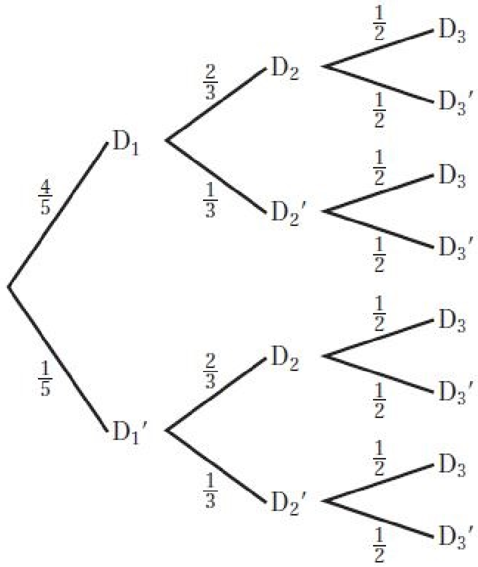
10 $x = 1 - (0.3 + 0.4 + 0.15) = 0.15$

P(A and B) = x = 0.15

P(A) × P(B) = 0.45 × 0.55 = 0.2475

As P(A and B) ≠ P(A) × P(B), the events A and B are not independent.

11 a



11 b i $\text{P}(D_1D_2D_3) = \frac{4}{5}\times\frac{2}{3}\times\frac{1}{2} = \frac{4}{15}$

ii Where D means a diamond and D' means no diamond,

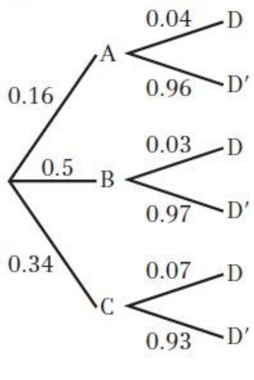
P (exactly one diamond) = P(D, D', D') + P(D', D, D') + P(D', D', D)

$$= \left(\frac{4}{5}\times\frac{1}{3}\times\frac{1}{2}\right)+\left(\frac{1}{5}\times\frac{2}{3}\times\frac{1}{2}\right)+\left(\frac{1}{5}\times\frac{1}{3}\times\frac{1}{2}\right)=\frac{7}{30}$$

11 c P(at least two diamonds) = 1 – P(at most one diamond) = 1 – (P(none) + P(exactly one diamond))

$$= 1 - \left(\frac{1}{5}\times\frac{1}{3}\times\frac{1}{2}+\frac{7}{30}\right) = 1 - \frac{4}{15} = \frac{11}{15}$$

12 a



b i $\text{P}(B \text{ and defective}) = 0.5\times 0.03 = 0.015$

ii $\text{P}(\text{defective}) = 0.16\times 0.04 + 0.5\times 0.03 + 0.34\times 0.07 = 0.0452$

Challenge

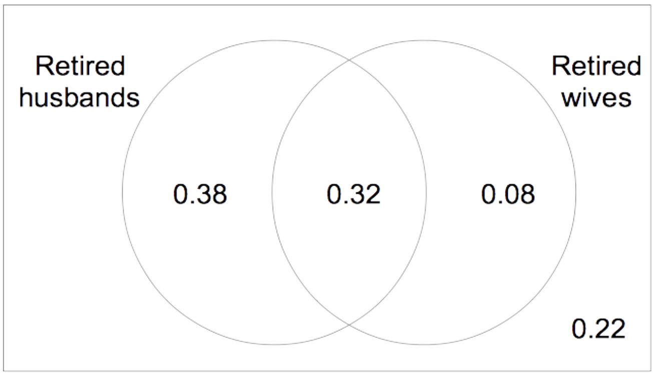
The probability that a wife is retired is 0.4.

Given that she is retired, the probability that her husband is also retired is 0.8.

Hence the probability that both are retired is $0.4 \times 0.8 = 0.32$.

The probability that a husband is retired is 0.7.

From this data you can deduce the following Venn diagram of the probabilities:



Let H = husband retired, H′ = husband not retired, W = wife retired, W′ = wife not retired.

The permutations where only one husband and only one wife is retired are:

Couple 1	Probability	Couple 2	Probability	Combined probability
H W′	0.38	H′ W	0.08	0.38×0.08
H′ W	0.08	H W′	0.38	0.08×0.38
H W	0.32	H′ W′	0.22	0.32×0.22
H′ W′	0.22	H W	0.32	0.22×0.32

$$\text{P(only one husband and only one wife is retired)} = (0.38 \times 0.08 + 0.32 \times 0.22) \times 2 = 0.2016$$

Statistical distributions 6A

1 a This is not a discrete random variable, since height is a continuous quantity.

b This is a discrete random variable, since it is takes whole number values at random.

c This is not a discrete random variable, since the number of days in a given week is always 7; the result is predetermined and so not random.

2 {0, 1, 2, 3, 4}

3 a (2, 2), (2, 3), (3, 2), (3, 3)

b i

x	4	5	6
$\mathbf{P}(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

ii

$$\mathrm{P}(X = x) = \begin{cases} \frac{1}{4}, \text{ if } x = 4, 6 \\ \frac{1}{2}, \text{ if } x = 5 \\ 0, \text{ otherwise} \end{cases}$$

4

$$\frac{1}{3}+\frac{1}{3}+k+\frac{1}{4}=1$$
$$k=1-\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{4}\right)$$
$$=1-\frac{11}{12}$$
$$=\frac{1}{12}$$

5

x	1	2	3	4
$\mathbf{P}(X = x)$	k	$2k$	$3k$	$4k$

$$k+2k+3k+4k=1$$
$$10k=1$$
$$k=\frac{1}{10}$$

6 a

x	1	2	3	4
$\mathbf{P}(X = x)$	k	k	$3k$	$3k$

6 a Using the fact that the probabilities add up to 1:

$$k + k + 3k + 3k = 1$$
$$8k = 1$$
$$k = \tfrac{1}{8}$$

b The probability distribution is:

x	1	2	3	4
P(X = x)	0.125	0.125	0.375	0.375

$P(X > 1) = 0.125 + 0.375 + 0.375 = 0.875$

7 a

x	−2	−1	0	1	2
P(X = x)	0.1	0.1	β	β	0.2

The probabilities add up to 1.

$$0.1 + 0.1 + \beta + \beta + 0.2 = 1$$
$$2\beta = 1 - 0.4 = 0.6$$
$$\beta = 0.3$$

b

x	−2	−1	0	1	2
P(X = x)	0.1	0.1	0.3	0.3	0.2

c $P(-1 \leqslant X < 2) = 0.1 + 0.3 + 0.3 = 0.7$

8 $\frac{1}{4} - a + a + \frac{1}{2} + a = 1$

$\frac{3}{4} + a = 1$

$a = \frac{1}{4}$

9 a $P(X = 1) = \frac{1}{50}$

since each of the 50 individual outcomes is equally likely.

b $P(X \geqslant 28) = 1 - \frac{27}{50} = \frac{23}{50}$

9 c $P(13 < X < 42) = P(14 \leqslant X \leqslant 41) = \frac{28}{50} = \frac{14}{25}$

10 a $P(1 < X \le 3) = P(X = 2) + P(X = 3) = \frac{1}{2} + \frac{1}{8} = \frac{5}{8}$

b $P(X < 2) = P(X = 0) + P(X = 1) = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$

c $P(X > 3) = 0$

11 a

s	1	2	3	4
P(*S* = *s*)	$\frac{2}{3}$	$\frac{1}{3}\times\frac{2}{3}=\frac{2}{9}$	$\frac{1}{3}\times\frac{1}{3}\times\frac{2}{3}=\frac{2}{27}$	$\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\times\frac{2}{3}+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}=\frac{1}{27}$

b $P(S > 2) = \frac{2}{27} + \frac{1}{27} = \frac{1}{9}$

12 a

x	0	1	2	3	4	5
P(*X* = *x*)	$0.6^5 = 0.07776$	$0.4 \times 0.6^4 \times 5 = 0.2592$	$0.4^2 \times 0.6^3 \times 10 = 0.3456$	$0.4^3 \times 0.6^2 \times 10 = 0.2304$	$0.4^4 \times 0.6 \times 5 = 0.0768$	$0.4^5 = 0.01024$

b

y	0	1	2	3	4	5
P(*Y* = *y*)	$0.8^5 = 0.32768$	$0.2 \times 0.8^4 \times 5 = 0.4096$	$0.2^2 \times 0.8^3 \times 10 = 0.2048$	$0.2^3 \times 0.8^2 \times 10 = 0.0512$	$0.2^4 \times 0.8 \times 5 = 0.0064$	$0.2^5 = 0.00032$

c

z	1	2	3	4	5
P(*Z* = *z*)	0.4	$0.4 \times 0.6 = 0.24$	$0.4 \times 0.6^2 = 0.144$	$0.4 \times 0.6^3 = 0.0864$	$0.4 \times 0.6^4 + 0.6^5 = 0.1296$

13 a

x	2	3	4
P(*X* = *x*)	$\frac{1}{2}$	$\frac{2}{9}$	$\frac{1}{8}$

$\frac{1}{2} + \frac{2}{9} + \frac{1}{8} = \frac{61}{72}$
The sum of the probabilities is not 1.

13 b

x	2	3	4
P(X = x)	$\frac{k}{4}$	$\frac{k}{9}$	$\frac{k}{16}$

$$\frac{k}{4} + \frac{k}{9} + \frac{k}{16} = 1$$

$$\frac{61k}{144} = 1$$

$$k = \frac{144}{61} = 2\frac{22}{61}$$

Challenge

x	1	2	3	4	5	6	7	8
P(X = x)	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

y	2	3	6
P(Y = y)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

P($X > Y$) = P($X > 2$ and $Y = 2$) + P($X > 3$ and $Y = 3$) + P($X > 6$ and $Y = 6$)

$$= \frac{6}{8} \times \frac{1}{2} + \frac{5}{8} \times \frac{1}{3} + \frac{2}{8} \times \frac{1}{6} = \frac{5}{8}$$

Statistical distributions 6B

1 a $P(X=2)=\binom{8}{2}\times\left(\frac{1}{3}\right)^2\times\left(\frac{2}{3}\right)^6$

$= 0.273$ (to 3 s.f.)

b $P(X=5)=\binom{8}{5}\times\left(\frac{1}{3}\right)^5\times\left(\frac{2}{3}\right)^3$

$= 0.0683$ (to 3 s.f.)

c $P(X\le 1)=P(X=1)+P(X=0)$

$$=8\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^7+\left(\frac{2}{3}\right)^8$$
$$=\left(\frac{2}{3}\right)^7\left(\frac{8}{3}+\frac{2}{3}\right)$$
$$=\left(\frac{2}{3}\right)^7\times\frac{10}{3}$$

$= 0.195$ (to 3 s.f.)

2 a $P(T=5)=\binom{15}{5}\times\left(\frac{2}{3}\right)^5\times\left(\frac{1}{3}\right)^{10}=0.00670$ (to 3 s.f.)

b $P(T=10)=\binom{15}{10}\times\left(\frac{2}{3}\right)^{10}\times\left(\frac{1}{3}\right)^5=0.214$ (to 3 s.f.)

c $P(3\le T\le 4)=P(T=3)+P(T=4)=\binom{15}{3}\times\left(\frac{2}{3}\right)^3\times\left(\frac{1}{3}\right)^{12}+\binom{15}{4}\times\left(\frac{2}{3}\right)^4\times\left(\frac{1}{3}\right)^{11}$

$$=0.00025367...+0.00152206...$$
$$=0.00178 \text{ (to 3 s.f.)}$$

3 a $X=$ 'number of defective bolts in a sample of 20'

$X\sim B(20, 0.01)$

$n=20$
$p=0.01$

Assume bolts are defective independently of one another.

3 b $X =$ 'number of times wait or stop in 6 lights'

$X \sim \text{B}(6, 0.52)$

$n = 6$
$p = 0.52$

Assume the lights operate independently and the time lights are on/off is constant.

c $X =$ 'number of aces in Stephanie's next 30 serves'

$X \sim \text{B}(30, \frac{1}{8})$

$n = 30$
$p = \frac{1}{8}$

Assume serving an ace occurs independently and the probability of an ace is constant.

4 a $X =$ 'number of people in class of 14 who are Rh–'

$X \sim \text{B}(14, 0.15)$ is a reasonable model if we assume that being Rh– is independent from pupil to pupil - so no siblings.

b This is not binomial since the number of trials or tosses is not known and fixed. The probability of a head at each toss is constant $(p = 0.5)$ but there is no value for *n*.

c Assuming, reasonably, that the colours of the cars are independent,

$X =$ 'number of red cars out of 15'

$X \sim \text{B}(15, 0.12)$

5 a Let $X =$ 'number of balloons that do not burst'

$\text{P}(X = 0) = (0.95)^{20}$

$= 0.358$ (to 3 s.f.)

b Let Y = 'number of balloons that do burst'

$$\text{P}(Y = 2) = \binom{20}{2}(0.95)^{18}(0.05)^2$$

$= 0.189$ (to 3 s.f.)

6 a There are two possible outcomes of each trial: faulty or not faulty. There are a fixed number of trials, 10, and fixed probability of success: 0.08. Assuming each member in the sample is independent, a suitable model is $X \sim \mathrm{B}(10, 0.08)$.

b $\mathrm{P}(X = 4) = \binom{10}{4}(0.08)^4(0.92)^6 = \frac{10!}{4!6!}(0.08)^4(0.92)^6 = 0.00522$ (to 3 s.f.)

7 a Assumptions are that there is a fixed sample size, that there are only two outcomes for the genetic marker (present or not present), and that there is a fixed probability of people having the marker.

b $X \sim \mathrm{B}(50, 0.04)$

$\mathrm{P}(X = 6) = \binom{50}{6}(0.04)^6(0.96)^{44} = \frac{50!}{6!44!}(0.04)^6(0.96)^{44} = 0.0108$ (to 3 s.f.)

8 a The random variable can take two values, 6 or not 6. There are a fixed number of trials (15) and a fixed probability of success (0.3). We are assuming that each roll of the dice is independent. A suitable model is $X \sim \mathrm{B}(15, 0.3)$.

b $X \sim \mathrm{B}(15, 0.3)$

$\mathrm{P}(X = 4) = \binom{15}{4}(0.3)^4(0.7)^{11} = \frac{15!}{4!11!}(0.3)^4(0.7)^{11} = 0.219$ (to 3 s.f.)

c $\mathrm{P}(X \leq 2) = \mathrm{P}(X = 0) + \mathrm{P}(X = 1) + \mathrm{P}(X = 2)$

$$= (0.7)^{15} + \binom{15}{1}(0.3)^1(0.7)^{14} + \binom{15}{2}(0.3)^2(0.7)^{13}$$

$$= (0.7)^{15} + \frac{15!}{1!14!}(0.3)^1(0.7)^{14} + \frac{15!}{2!13!}(0.3)^2(0.7)^{13}$$

$= 0.127$ (to 3 s.f.)

Statistical distributions 6C

1 $X \sim \mathrm{B}(9, 0.2)$

a $\mathrm{P}(X \leq 4) = 0.9804$ (tables)

b $\mathrm{P}(X < 3) = \mathrm{P}(X \leq 2) = 0.7382$ (tables)

c $\mathrm{P}(X \geq 2) = 1 - \mathrm{P}(X \leq 1) = 1 - 0.4362 = 0.5638$ (tables)

d $\mathrm{P}(X = 1) = \mathrm{P}(X \leq 1) - \mathrm{P}(X = 0) = 0.4362 - 0.1342 = 0.3020$ (tables)

2 $X \sim \mathrm{B}(20, 0.35)$

a $\mathrm{P}(X \leq 10) = 0.9468$ (tables)

b $\mathrm{P}(X > 6) = 1 - \mathrm{P}(X \leq 6) = 1 - 0.4166 = 0.5834$ (tables)

c $\mathrm{P}(X = 5) = \mathrm{P}(X \leq 5) - \mathrm{P}(X \leq 4) = 0.2454 - 0.1182 = 0.1272$ (tables)

d $\mathrm{P}(2 \leq X \leq 7) = \mathrm{P}(X \leq 7) - (X \leq 1) = 0.6010 - 0.0021 = 0.5989$ (tables)

3 **a** Using the binomial cumulative function on a calculator where $x = 19$, $n = 40$ and $p = 0.47$,

$\mathrm{P}(X < 20) = \mathrm{P}(X \leq 19) = 0.5888$

b Using the binomial cumulative function on a calculator where $x = 16$, $n = 40$ and $p = 0.47$,

$\mathrm{P}(X > 16) = 1 - \mathrm{P}(X \leq 16) = 0.7662$

c Using the binomial cumulative function on a calculator where $x = 10$ and 15, $n = 40$ and $p = 0.47$,

$\mathrm{P}(11 \leq X \leq 15) = \mathrm{P}(X \leq 15) - \mathrm{P}(X \leq 10) = 0.1478 - 0.0036 = 0.1442$

d Using the binomial cumulative function on a calculator where $x = 10$ and 16, $n = 40$ and $p = 0.47$,

$\mathrm{P}(10 < X < 17) = \mathrm{P}(X \leq 16) - \mathrm{P}(X \leq 10) = 0.2338 - 0.0036 = 0.2302$

4 **a** Using the binomial cumulative function on a calculator where $x = 20$, $n = 37$ and $p = 0.65$,

$\mathrm{P}(X > 20) = 1 - \mathrm{P}(X \leq 20) = 0.8882$

b Using the binomial cumulative function on a calculator where $x = 26$, $n = 37$ and $p = 0.65$,

$\mathrm{P}(X \leq 26) = 0.7992$

c Using the binomial cumulative function on a calculator where $x = 19$ and 14, $n = 37$ and $p = 0.65$,

$\mathrm{P}(15 \leq X < 20) = \mathrm{P}(X \leq 19) - \mathrm{P}(X \leqslant 14) = 0.06061 - 0.00068 = 0.05993$

4 **d** Using the binomial cumulative function on a calculator where $x = 23$ and 22, $n = 37$ and $p = 0.65$,

$\text{P}(X = 23) = \text{P}(X \leq 23) - \text{P}(X \leq 22) = 0.4184 - 0.2926 = 0.1258$

5 $X =$ 'number of heads'
$X \sim \text{B}(8, 0.5)$ (coins are fair so $p = 0.5$)

a $\text{P}(X = 0) = (0.5)^8 = 0.0039$ (tables)

b $\text{P}(X \geq 2) = 1 - \text{P}(X \leq 1) = 1 - 0.0352 = 0.9648$ (tables)

c $\text{P}(X \geq 5) = 1 - \text{P}(X \leq 4) = 1 - 0.6367 = 0.3633$ (tables)

6 $X =$ 'number of plants with blue flowers on tray of 15'

a $\text{P}(X = 4) = \text{P}(X \leq 4) - \text{P}(X \leq 3) = 0.6865 - 0.4613 = 0.2252$ (tables)

b $\text{P}(X \leq 3) = 0.4613$ (tables)

c $\text{P}(3 \leq X \leq 6) = \text{P}(X \leq 6) - \text{P}(X \leq 2) = 0.9434 - 0.2361 = 0.7073$ (tables)

7 $X \sim \text{B}(50, 0.40)$

a $\text{P}(X \leq 13) = 0.0280$

$\text{P}(X \leq 14) = 0.0540$ (tables)

$\therefore k = 13$

b $\text{P}(X \leq 27) = 0.9840$

$\Rightarrow \text{P}(X > 27) = 0.0160 > 0.01$

$\text{P}(X \leq 28) = 0.9924$

$\Rightarrow \text{P}(X > 28) = 0.0076 < 0.01$

$\therefore r = 28$

8 $X \sim \text{B}(40, 0.10)$

a $\text{P}(X = 0) = 0.0148 < 0.02$

$\text{P}(X \leq 1) = 0.0805 > 0.02$ (tables)

$\text{P}(X < 1) = 0.0148 < 0.02$

$\therefore k = 1$

8 b $P(X \leq 8) = 0.9845$ (tables)

$\Rightarrow P(X > 8) = 0.0155 > 0.01$

$P(X \leq 9) = 0.9949$

$\Rightarrow P(X > 9) = 0.0051 < 0.01$

$r = 9$

c $P(k \leq X \leq r) = P(X \leq r) - P(X \leq k-1)$

$= P(X \leq 9) - P(X = 0)$

$= 0.9949 - 0.0148$

$= 0.9801$

9 a A suitable distribution is $X \sim B$ (10, 0.30). Assumptions: There are two possible outcomes of each trial listen or don't listen. There is a fixed number of trials, 10, and fixed probability of success: 0.3. Each member in the sample is assumed to listen independently.

b $P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.8497 = 0.1503$ (tables)

c $P(X \leq 6) = 0.9894$ so $P(X \geq 7) = 1 - 0.9894 = 0.0106 > 0.01$

$P(X \leq 7) = 0.9984$ so $P(X \geq 8) = 1 - 0.9984 = 0.0016 < 0.01$ (tables)

Therefore $s = 8$ is the smallest such value.

10 X = number of defects in 50 components

$X \sim B(50, 0.05)$

a $P(X < 2) = P(X \leq 1) = 0.2794$ (tables)

b $P(X > 5) = 1 - P(X \leq 5) = 1 - 0.9622 = 0.0378$ (tables)

c Seek smallest d such that $P(X > d) < 0.05$

$P(X \leq 4) = 0.8964$ so $P(X > 4) = 0.1036 > 0.05$

$P(X \leq 5) = 0.9622$ so $P(X > 5) = 0.0378 < 0.05$

$\therefore d = 5$

Statistical distributions, Mixed Exercise 6

1 a

x	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{3}{21}$	$\frac{4}{21}$	$\frac{5}{21}$	$\frac{6}{21}$

b $P(2 < X \leq 5) = P(X=3)+P(X=4)+P(X=5) = \frac{3}{21}+\frac{4}{21}+\frac{5}{21} = \frac{12}{21} = \frac{4}{7}$

2 a $0.1+0.2+0.3+r+0.1+0.1=1$

$r = 1-0.8$
$\quad = 0.2$

b $P(-1 \leq X < 2) = P(-1)+P(0)+P(1) = 0.2+0.3+0.2 = 0.7$

3 a

x	1	2	3	4
$P(X=x)$	$\frac{2}{26}$	$\frac{5}{26}$	$\frac{8}{26}$	$\frac{11}{26}$

b $P(2 < X \leq 4) = P(X=3)+P(X=4) = \frac{19}{26}$

4 a For a discrete uniform distribution, the probability of choosing each counter must be equal.

b i $P(X=5) = \frac{1}{16}$

ii The prime numbers are 2, 3, 5, 7, 11 and 13

$P(X \text{ is prime}) = \frac{6}{16} = \frac{3}{8}$

iii $P(3 \leq X < 11) = \frac{8}{16} = \frac{1}{2}$

5 a

y	1	2	3	4	5
$P(Y=y)$	$\frac{1}{k}$	$\frac{2}{k}$	$\frac{3}{k}$	$\frac{4}{k}$	$\frac{5}{k}$

$\frac{1}{k}+\frac{2}{k}+\frac{3}{k}+\frac{4}{k}+\frac{5}{k}=1$

$\frac{15}{k}=1, k=15$

5 b

y	1	2	3	4	5
$\mathrm{P}(Y = y)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15} = \frac{1}{5}$	$\frac{4}{15}$	$\frac{5}{15} = \frac{1}{3}$

c $\mathrm{P}(Y > 3) = \mathrm{P}(Y = 4) + \mathrm{P}(Y = 5) = \frac{4}{15} + \frac{5}{15} = \frac{9}{15} = \frac{3}{5}$

6 a

t	0	1	2	3	4
$\mathrm{P}(T = t)$	$\frac{81}{256}$	$\frac{108}{256}$	$\frac{54}{256}$	$\frac{12}{256}$	$\frac{1}{256}$

b $\mathrm{P}(T < 3) = \mathrm{P}(T = 0) + \mathrm{P}(T = 1) + \mathrm{P}(T = 2) = \frac{243}{256}$

c

S	1	2	3	4	5
$\mathrm{P}(S = s)$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{9}{64}$	$\frac{27}{256}$	$\frac{81}{256}$

d $\mathrm{P}(S > 2) = \mathrm{P}(S = 3) + \mathrm{P}(S = 4) + \mathrm{P}(S = 5) = \frac{9}{16}$

7 a $\mathrm{P}(X = 20) = \binom{30}{20}(0.73)^{20}(0.27)^{10} = \frac{30!}{20!10!}(0.73)^{20}(0.27)^{10} = 0.114$ (to 3 s.f.)

b Using the binomial cumulative function on a calculator where $x = 13$, $n = 30$ and $p = 0.73$,

$\mathrm{P}(X \leq 13) = 0.000580$ (to 3 s.f.)

c Using the binomial cumulative function on a calculator where $x = 11$ and 25, $n = 30$ and $\mathrm{p} = 0.73$,

$\mathrm{P}(11 < X \leq 25) = \mathrm{P}(X \leq 25) - \mathrm{P}(X \leq 11) = 0.937302995 - 0.000033512 = 0.937$ (to 3 s.f.)

8 a Sequence is: H H H H H T

Probability: $\left(\frac{2}{3}\right)^5 \times \frac{1}{3} = \frac{32}{729}$ or 0.0439 (to 3 s.f.)

b Let X = ‘number of tails in the first 8 tosses’, then

$$P(X=2)=\binom{8}{2}\left(\frac{1}{3}\right)^2\times\left(\frac{2}{3}\right)^6=0.273 \text{ (to 3 s.f.)}$$

9 X = number of patients waiting more than $\frac{1}{2}$ hour

$X \sim B(12, 0.3)$

a $P(X=0)=(0.7)^{12}=0.01384...=0.0138$ (to 3 s.f.)

b $P(X>2)=1-P(X\le 2)=1-0.2528=0.7472=0.747$ (to 3 d.p.)

10 a i There are n independent trials.
ii n is a fixed number.
iii The outcome of each trial is a success or a failure.
iv The probability of success at each trial is constant.
v The outcome of any trial is independent of any other trial.

b X = number of successes

$X \sim B(10, 0.05)$

$P(X\ge 2)=1-P(X\le 1)=1-0.9139=0.0861$ (to 3 s.f.)

c Y_n = 'number of successes in n houses'

$Y_n \sim B(n, 0.05)$

Looking for smallest n such that $P(Y_n\ge 1)>0.99$ or, equivalently, $P(Y_n=0)<0.01$.

$P(Y_n=0)=0.95^n<0.01$

So $n=90$ using logarithms.

11 X = 'number of correctly answered questions' and $X\sim B(10, 0.5)$

a $P(X=10)=(0.5)^{10}=0.00097656...=0.000977$ (to 3 s.f.)

b $P(X\ge 8)=1-P(X\le 7)=1-0.9453=0.0547$ (to 3 s.f. using tables)

12

x	1	2	3	4	5	6
$P(X=x)$	p	p	p	p	$2p$	p

$7p=1\Rightarrow p=\frac{1}{7}$

a Sequence: $\bar{5}\ \bar{5}\ \bar{5}\ \bar{5}\ \bar{5}\ 5$

Probability: $\left(\frac{5}{7}\right)^5\left(\frac{2}{7}\right) = 0.0531$ (to 3 s.f.)

12 b $Y =$ 'number of 5s in 8 throws'

$Y \sim B(8, \frac{2}{7})$

$P(Y = 3) = \binom{8}{3} \times \left(\frac{2}{7}\right)^3 \times \left(\frac{5}{7}\right)^5 = 0.24285 = 0.243$ (to 3 s.f.)

13 $X =$ 'number of green chairs in sample of 10'

a $X \sim B(10, 0.15)$

b $P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.9901 = 0.0099$ (tables)

c $P(X = 2) = P(X \leq 2) - P(X \leq 1) = 0.8202 - 0.5443 = 0.2759$ (tables)

14 $X =$ 'number of yellow beads in sample of 20' and assume $X \sim B(20, 0.45)$

a $P(X < 12) = P(X \leq 11) = 0.8692$ (tables)

b $P(X = 12) = P(X \leq 12) - P(X \leq 11) = 0.9420 - 0.8692 = 0.0728$ (tables)

15 a $P(X \geq 10) = 1 - P(X \leq 9) = 1 - 0.1275 = 0.8725$ (tables)

b $P(X \geq 10 \text{ in 7 out of 12 sets}) = \binom{12}{7}(0.8725)^7(0.1275)^5$

$= 0.0103$ (to 3 s.f.)

c Let $Y =$ ‘number of sets out of 12 that she hits the bullseye with at least 50% of her arrows’, then

$Y \sim B(12, 0.8725)$

Using the binomial cumulative function on a calculator where $x = 5$, $n = 12$ and $p = 0.8725$,

$P(Y < 6) = P(Y \leq 5) = 0.0002407$

Challenge

$Y \sim B(18, 0.25)$

$P(Y \geq 11) = 1 - P(Y \leq 10) = 1 - 0.9988 = 0.0012$ (tables)

Hypothesis testing 7A

1 a A hypothesis is a statement made about the value of a population parameter. A hypothesis test uses a sample or an experiment to determine whether or not to reject the hypothesis.

b The null hypothesis (H_0) is what we assume to be correct and the alternative hypothesis (H_1) tells us about the parameter if our assumption is shown to be wrong.

c The test statistic is used to test the hypothesis. It could be the result of the experiment or statistics from a sample.

2 a one-tailed test

b two-tailed test

c one-tailed test

3 a The test statistic is the number of sixes rolled in the 60 trials.

b $H_0 : p = \frac{1}{6}$

c $H_1 : p > \frac{1}{6}$

4 a Shell is describing what her experiment wants to test rather than the test statistic. The test statistic is the number of times she gets a head in 100 tosses.

b $H_0 : p = \frac{1}{2}$

c $H_1 : p \neq \frac{1}{2}$

5 a A suitable test statistic is the number of faulty articles found in a sample of 100.

b $H_0 : p = 0.1$ $\quad$ $H_1 : p < 0.1$

c If the probability of that number being 8 or less is less than 5%, the null hypothesis is rejected.

6 a A suitable test statistic is the number of supporters found in a sample of 20.

b H_0: p = 0.55 $\quad$ H_1: p < 0.55

c If the probability of that number being 7 is 2% or more, the null hypothesis is accepted.

Hypothesis testing 7B

1 **a** The critical value is the first value to fall inside of the critical region.

b A critical region is a region of the probability distribution which, if the test statistic falls within it, would cause you to reject the null hypothesis.

c The acceptance region is the area in which we accept the null hypothesis.

2 B (10, 0.2)

$$\begin{aligned}P(X \geq 4) &= 1 - P(X \leq 3)\\ &= 1 - 0.8791\\ &= 0.1209 > 0.05\end{aligned}$$

$$\begin{aligned}P(X \geq 5) &= 1 - P(X \leq 4)\\ &= 1 - 0.9672\\ &= 0.0328 < 0.05\end{aligned}$$

The critical value is $x = 5$ and the critical region is $X \geq 5$ since $P(X \geq 5) = 0.0328 < 0.05$

3 B (20, 0.15)

$$P(X \leq 1) = 0.1756 > 0.05$$
$$P(X = 0) = 0.0388 < 0.05$$

The critical value is $x = 0$ and the critical region is $X = 0$

4 **a** B (20, 0.4)

$$P(X \leq 4) = 0.0510 > 0.025$$
$$P(X \leq 3) = 0.0160 < 0.025$$

The critical value is $x = 3$

$$P(X \geq 13) = 1 - P(X \leq 12) = 1 - 0.9790 = 0.0210 < 0.025$$
$$P(X \geq 12) = 1 - P(X \leq 11) = 1 - 0.9435 = 0.0565 > 0.025$$

The critical value is $x = 13$
The critical region is $X \geq 13$ and $X \leq 3$

b The actual significance level is 0.021 + 0.016 = 0.037 = 3.7%

5 B (20, 0.18)

$B(X = 0) = 0.0189 < 0.05$
$B(X \leq 1) = 0.1018 > 0.05$

The critical value is $x = 0$
The critical region is $X = 0$

6 **a** B (10, 0.22)

$P(X < 5) = 0.952$
$P(X \geq 5) = 0.0478 > 0.005$

The critical value is $x = 5$
The critical region is $X \geq 5$

b The actual significance level is 0.0478 = 4.78%

7 **a** The test statistic is the number of components in the sample that fail.

b H_0: $p = 0.3$
H_1: $p < 0.3$

c Assume H_0 is true then $X \sim B(20, 0.3)$

$P(X \leq 2) = 0.0355$ (closer to 0.05)
$P(X \leq 3) = 0.1071$

The critical region is $X \leq 2$

d 0.0355 = 3.55%

8 **a** The test statistic is the number of seedlings that survive.

H_0: $p = \frac{1}{3}$,
H_1: $p > \frac{1}{3}$

b Assume H_0 is true then $X \sim B(36, \frac{1}{3})$

Using a calculator

$P(X \geq 16) = 1 - P(X \leq 15) = 1 - 0.8906 = 0.1094 > 0.1$

$P(X \geq 17) = 1 - P(X \leq 16) = 1 - 0.9416 = 0.0584 < 0.1$

The critical region is $X \geq 17$

c 0.0584 = 5.84%

9 **a** In a given time, the number of customers choosing lasagne out of the total number.

H_0: $p = 0.2$
H_1: $p \neq 0.2$

b Assume H_0 is true then $X \sim B(25, 0.2)$

Consider the lower tail:

$P(X \leq 0) = 0.0038$
$P(X \leq 1) = 0.0274$ (closer to 0.025)

Consider the upper tail:

$P(X \geq 9) = 1 - P(X \leq 8) = 1 - 0.9532 = 0.0468$
$P(X \geq 10) = 1 - P(X \leq 9) = 1 - 0.9827 = 0.0173$ (closer to 0.025)

The critical region is $X \leq 1$ and $X \geq 10$.

c The probability of incorrectly rejecting H_o is 0.0274 + 0.0173 = 0.0447 = 4.47%

Challenge

a Assume H_0 is true then $X \sim B(50, 0.7)$

Consider the lower tail:

$P(X \leq 29) = 0.0478$ (closer to 0.05)
$P(X \leq 30) = 0.0848$

Consider the upper tail:

$P(X \geq 41) = 1 - P(X \leq 40) = 1 - 0.9598 = 0.0402$ (closer to 0.05)
$P(X \geq 40) = 1 - P(X \leq 39) = 1 - 0.9211 = 0.0789$

The critical region is $X \leq 29$ and $X \geq 41$

b The probability of one observation falling within the critical region is 0.0478 + 0.0402 = 8.8%

The probability of two observations falling within the critical region is $0.088^2 = 0.007744$ = 0.77%

The probability that Chloe has incorrectly rejected H_0 is 0.77%

Hypothesis testing 7C

1 Distribution, B(10, 0.25)

$H_0 : p = 0.25 \qquad H_1 : p > 0.25$

$$\begin{aligned} P(X \geq 5) &= 1 - P(X \leq 4) \\ &= 1 - 0.9219 \\ &= 0.0781 > 0.05 \end{aligned}$$

There is insufficient evidence to reject H_0 so there is no reason to doubt $p = 0.25$

2 Distribution, B(10, 0.40)

$H_0 : p = 0.40 \qquad H_1 : p < 0.40$

$P(X \leq 1) = 0.0464 < 0.05$

There is sufficient evidence to reject H_0 so $p < 0.04$

3 Distribution, B(20, 0.30)

$H_0 : p = 0.30 \qquad H_1 : p > 0.30$

$$\begin{aligned} P(X \geq 10) &= 1 - P(X \leq 9) \\ &= 1 - 0.9520 \\ &= 0.0480 < 0.05 \end{aligned}$$

There is sufficient evidence to reject H_0 so $p > 0.3$

4 Distribution, B(20, 0.45)

$H_0 : p = 0.45 \qquad H_1 : p < 0.45$

$P(X \leq 3) = 0.0049 < 0.01$

There is sufficient evidence to reject H_0 so $p < 0.45$

5 Distribution, B(20, 0.28)

$H_0 : p = 0.28 \qquad H_1 : p < 0.28$

p-value = $P(X \leq 2) = 0.0526 > 0.05$

There is insufficient evidence to reject H_0 so there is no reason to doubt $p = 0.28$

6 Distribution, B(8, 0.32)

$H_0 : p = 0.32 \qquad H_1 : p < 0.32$

$$\begin{aligned} P(X \geq 7) &= 1 - P(X \leq 6) \\ &= 1 - 0.9980 \\ &= 0.002 < 0.05 \end{aligned}$$

There is sufficient evidence to reject H_0 so $p < 0.32$

7 Distribution, B(12, $\frac{1}{6}$)

$H_0 : p = \frac{1}{6} \qquad H_1 : p < \frac{1}{6}$

$P(X \leq 1) = 0.3813 > 0.05$

There is insufficient evidence to reject H_0 so there is no evidence that the probability of a 6 on this dice is less than $\frac{1}{6}$

8 a Distribution, B(n, 0.68)

Reasons:
- Fixed number of trials.
- Outcomes of the trials are independent.
- There are two outcomes, success and failure.
- The probability of success is constant.

b Distribution, B(10, 0.68)

$H_0 : p = 0.68 \qquad H_1 : p < 0.68$

$P(X \leq 3) = 0.0155 < 0.05$

There is sufficient evidence to reject H_0 so $p < 0.68$. The treatment is not as effective as claimed.

9 a X is the number of seeds in the trial for which the germination method was successful.
p is the probability of success for each seed.

$X \sim B(20, p)$

H_0: $p = 0.4 \qquad H_1$: $p > 0.4$

$P(X \geq 12) = 1 - P(X \leq 11) = 1 - 0.9435 = 0.0565 \geq 0.05$
$P(X \geq 13) = 1 - P(X \leq 12) = 1 - 0.9790 = 0.021 \leq 0.05$

The critical region is $X \geq 13$

9 b 14 lies within the critical region, so we can reject the null hypothesis. There is evidence that the new technique has improved the number of plants that germinate.

10 a The test statistic is the number of people who support the candidate.

H_0: $p = 0.35$ H_1: $p > 0.35$

b $X \sim B(50, 0.35)$

$P(X \geq 23) = 1 - P(X \leq 22) = 1 - 0.9290 = 0.071 > 0.05$
$P(X \geq 24) = 1 - P(X \leq 23) = 1 - 0.9604 = 0.0396 < 0.05$

The critical region is $X \geq 24$

c 28 lies in the critical region, so we can reject the null hypothesis. There is evidence that the candidate's level of popularity has increased.

Hypothesis testing 7D

1 $H_0: p = 0.5$ $H_1: p \neq 0.5$

If H_0 is true $X \sim B(30, 0.5)$

Expected value would be $30 \times 0.5 = 15$.
The observed value, 10, is less than this so consider $P(X \leq 10)$

$P(X \leq 10) = 0.0494 > 0.025$ (two-tailed)

There is insufficient evidence to reject H_0 so there is no reason to doubt $p = 0.5$

2 $H_0: p = 0.3$ $H_1: p \neq 0.3$

If H_0 is true $X \sim B(25, 0.3)$

Expected value would be $25 \times 0.3 = 7.5$.
The observed value, 10, is more than this so consider $P(X \geq 10)$

$P(X \geq 10) = 0.1894... > 0.05$ (two-tailed)

There is insufficient evidence to reject H_0 so there is no reason to doubt $p = 0.3$

3 $H_0: p = 0.75$ $H_1: p \neq 0.75$

If H_0 is true $X \sim B(10, 0.75)$

Expected value would be $10 \times 0.75 = 7.5$.
The observed value, 9, is more than this so consider $P(X \geq 9)$

$P(X \geq 9) = 0.2440... > 0.025$ (two-tailed)

There is insufficient evidence to reject H_0 so there is no reason to doubt $p = 0.75$

4 $H_0: p = 0.6$ $H_1: p \neq 0.6$

If H_0 is true $X \sim B(20, 0.6)$

Expected value would be $20 \times 0.6 = 12$
The observed value, 1, is less than this so consider $P(X \leq 1)$

$P(X \leq 1) = 0.00000034.. < 0.005$ (two-tailed)

Reject H_0. There is evidence that $p \neq 0.6$.

5 H_0: $p = 0.02$ H_1: $p \neq 0.02$

If H_0 is true $X \sim B(50, 0.02)$

Expected value would be $50 \times 0.02 = 1$
The observed value, 4, is more than this so consider $P(X \geq 4)$

$P(X \geq 4) = 0.01775... > 0.01$ (two-tailed)

There is insufficient evidence to reject H_0 so there is no reason to doubt $p = 0.02$

6 The probability that an unbiased coin lands on heads is 0.5
X is the number of times the coin being tested lands on heads
p is the probability that the coin being tested lands on heads.

$H_0 : p = 0.5$ $H_1 : p \neq 0.5$

If H_0 is true $X \sim B(20, 0.5)$

Expected value would be $20 \times 0.5 = 10$
The observed value, 6, is less than this so consider $P(X \leq 6)$

$P(X \leq 6) = 0.0577 > 0.025$ (two-tailed)

There is insufficient evidence to reject H_0 so there is no reason to think that the coin is biased.

7 a $H_0 : p = 0.20$ $H_1 : p \neq 0.20$

If H_0 is true $X \sim B(20, 0.20)$

$P(X \leq 1) = 0.0692$
$P(X \leq 0) = 0.0115$ (closer to 0.025)

critical value $= 0$

$P(X \geq 9) = 1 - P(X \leq 8) = 1 - 0.9900 = 0.0100$
$P(X \geq 8) = 1 - P(X \leq 7) = 1 - 0.9679 = 0.0321$ (closer to 0.025)

Critical region $X = 0$ and $X \geq 8$

b Actual significance level is $0.0115 + 0.0321 = 0.0436 = 4.36\%$

c $X = 8$ is in the critical region. There is enough evidence to reject H_0. The hospital's proportion of complications differs from the national figure.

8 a The probability that a glass bowl made using the original process is cracked is 0.1
X is the number of bowls in the sample using the new process that are cracked.
p is the probability that a bowl made using the new process is cracked.

$H_0 : p = 0.1 \qquad H_1 : p \neq 0.1$

If H_0 is true $X \sim B(20, 0.1)$

Expected value would be $20 \times 0.1 = 2$
The observed value, 1, is less than this so consider $P(X \leq 1)$

$P(X \leq 2) = 0.3917... > 0.05$ (two-tailed)

There is insufficient evidence to reject H_0 so there is no reason to think that the proportion of cracked bowls has changed.

b Double the calculated probability to find the p-value
p- value = 0.3917…+ 0.3917… = 0.7835

9 The probability that a carrot grown in the original fertiliser is longer than 7 cm is 0.25
X is the number of carrots in the sample grown in the new fertiliser that are longer than 7 cm.
p is the probability that a carrot grown in the new fertiliser is longer than 7 cm.

$H_0 : p = 0.25 \qquad H_1 : p \neq 0.25$

If H_0 is true $X \sim B(30, 0.25)$

Expected value would be $30 \times 0.25 = 7.5$
The observed value, 13, is more than this so consider $P(X \geq 13)$

$P(X \geq 13) = 0.02159... < 0.025$ (two-tailed)

There is evidence to reject H_0. Therefore, there is reason to doubt $p = 0.25$.
So the probability of a carrot being longer than 7 cm has changed.

10 The probability that a standard blood test diagnoses the disease is 0.96
X is the number of patients correctly diagnosed in the sample using the new process.
p is the probability that a patient is correctly diagnosed using the new process.

$H_0 : p = 0.96 \qquad H_1 : p \neq 0.96$

If H_0 is true $X \sim B(75, 0.96)$

Expected value would be $75 \times 0.96 = 72$
The observed value, 63, is less than this so consider $P(X \leq 63)$

$P(X \leq 63) = 0.0000417... < 0.05$ (two-tailed)

There is evidence to reject H_0. Therefore, there is reason to doubt $p = 0.96$.
So the new test does not have the same probability of success as the old test.

Hypothesis testing, Mixed Exercise 7

1 $X \sim \text{B}(10, 0.20)$

$\text{H}_0 : p = 0.20 \qquad \text{H}_1 : p > 0.20$

$$\begin{aligned}\text{P}(X \geq 3) &= 1 - \text{P}(X \leq 2)\\ &= 1 - 0.6778\\ &= 0.3222 > 0.05\end{aligned}$$

There is insufficient evidence to reject H_0
There is no evidence that the trains are late more often.

2 $X \sim \text{B}(5, 0.5)$

$\text{H}_0 : p = 0.50 \qquad \text{H}_1 : p > 0.50$

$$\begin{aligned}\text{P}(X \geq 4) &= 1 - \text{P}(X \leq 3)\\ &= 1 - 0.8125\\ &= 0.1875 > 0.05\end{aligned}$$

There is insufficient evidence to reject H_0
There is insufficient evidence that the company's claims are true.

3 **a** Fixed number; independent trials; two outcomes (pass or fail); p constant for each car.

b $X \sim \text{B}(5, 0.30)$

$\text{P(all pass)} = 0.70^5 = 0.16807$

c $X \sim \text{B}(10, 0.30)$

$\text{H}_0 : p = 0.30 \qquad \text{H}_1 : p < 0.30$

$\text{P}(X \leq 2) = 0.3828 > 0.05$

There is insufficient evidence to reject H_0.
There is no evidence that the garage fails fewer than the national average.

4 a $X \sim B(50, 0.1)$

$H_0 : p = 0.10 \qquad H_1 : p \neq 0.10$

$P(X \leq 1) = 0.0338$ (closer to 0.025)
$P(X = 0) = 0.0052$
critical value = 1

$P(X \geq 9) = 1 - P(X \leq 8) = 1 - 0.9421 = 0.0579$
$P(X \geq 10) = 1 - P(X \leq 9) = 1 - 0.9755 = 0.0245$ (closer to 0.025)

critical value = 10

Critical region $X \leq 1$ and $X \geq 10$

b Actual significance level = 0.0338 + 0.0245 = 0.0583 = 5.83%

c $X \sim B(20, 0.1)$

$H_0 : p = 0.1 \qquad H_1 : p > 0.1$

$$\begin{aligned} P(X \geq 4) &= 1 - P(X \leq 3) \\ &= 1 - 0.8670 \\ &= 0.133 > 0.1 \end{aligned}$$

p-value $= 0.133$

Accept H_0. There is no evidence that the proportion of defective articles has increased.

5 $X \sim B(20, 0.5)$
$H_0 : p = 0.50 \qquad H_1 : p \neq 0.50$

8 used Oriels powder.

$P(X \leq 8) = 0.2517 > 0.025$

There is insufficient evidence to reject H_0.
There is no evidence that the claim is wrong.

6 $X \sim B(50, 0.2)$

a $P(X \leq 4) = 0.0185$ (closer to 0.025)
$P(X \leq 5) = 0.0480$

$c_1 = 4$

$P(X \geq 16) = 1 - P(X \leq 15) = 1 - 0.9692 = 0.0308$ (closer to 0.025)
$P(X \geq 17) = 1 - P(X \leq 16) = 1 - 0.9856 = 0.0144$

$c_2 = 16$

Critical region $X \leq 4$ and $X \geq 16$

b Actual significance level = 0.0185 + 0.0308 = 0.0493 = 4.93%

c This is not in the critical region. Therefore, there is insufficient evidence to reject H_0. There is no evidence to suggest that the proportion of people buying that certain make of computer differs from 0.2.

7 a i A hypothesis is a statement made about the value of a population parameter. A hypothesis test uses a sample or an experiment to determine whether or not to reject the hypothesis.

ii The critical value is the first value to fall inside of the critical region.

iii The acceptance region is the region where we accept the null hypothesis.

b H_0: $p = 0.2$ H_1: $p \neq 0.2$

If H_0 is true $X \sim B(20, 0.2)$

Let c_1 and c_2 be the two critical values so $P(X \leq c_1) \leq 0.05$ and $P(X \geq c_2) \leq 0.05$

For the lower tail:

$P(X = 0) = 0.0115 < 0.05$
$P(X \leq 1) = 0.0692 > 0.05$

So $c_1 = 0$

For the upper tail:

$P(X \geq 7) = 1 - P(X \leq 6) = 1 - 0.9133 = 0.0978 > 0.05$
$P(X \geq 8) = 1 - P(X \leq 7) = 1 - 0.9679 = 0.0321 < 0.05$

So $c_2 = 8$

So the critical region is $X = 0$ and $X \geq 8$

c Actual significance level = 0.0115 + 0.0321 = 0.0436 = 4.36%

7 d As 7 does not lie in the critical region, H_0 is not rejected. Therefore, the proportion of times that Johan is late for school has not changed.

8 X is the number of days with zero or a trace of rain.

$X \sim B(30, 0.5)$

$H_0: p = 0.5 \qquad H_1: p > 0.5$

$P(X \geq 19) = 1 - P(X \leq 18) = 1 - 0.8998 = 0.1002 > 0.05$
$P(X \geq 20) = 1 - P(X \leq 19) = 1 - 0.9506 = 0.0494 < 0.05$

The critical region is $X \geq 20$

21 lies in the critical region, so we can reject the null hypothesis. There is evidence that the likelihood of a rain-free day in 2015 has increased.

9 a $H_0: p = 0.35 \qquad H_1: p \neq 0.35$

If H_0 is true $X \sim B(30, 0.35)$

Let c_1 and c_2 be the two critical values so $P(X \leq c_1) \leq 0.025$ and $P(X \geq c_2) \leq 0.025$

For the lower tail:

$P(X \leq 5) = 0.0233 < 0.025$
$P(X \leq 6) = 0.0586 > 0.025$

So $c_1 = 5$

For the upper tail:

$P(X \geq 16) = 1 - P(X \leq 15) = 1 - 0.9699 = 0.0301,\ 0.0301 - 0.025 = 0.0051$
$P(X \geq 17) = 1 - P(X \leq 16) = 1 - 0.9876 = 0.0124,\ 0.025 - 0.0124 = 0.0126$

So $c_2 = 16$

So the critical region is $X \leq 5$ and $X \geq 16$

b Actual significance test is $0.0233 + 0.0301 = 0.0534 = 5.34\%$

c $X = 4$ lies in the critical region so there is enough evidence to reject H_0.

10 a $X \sim B(20, 0.85)$

b $P(X = 16) = \binom{20}{16} 0.85^{16} 0.15^4 = 0.18$

10 c X is the number of patients that recover
p is the probability that a patient recovers

$\text{H}_0 : p = 0.85 \qquad \text{H}_1 : p < 0.85$

If H_0 is true $X \sim \text{B}(30, 0.85)$

Expected value would be $30 \times 0.85 = 25.5$
The observed value, 20, is less than this so consider $\text{P}(X \leq 20)$

p-value $= \text{P}(X \leq 20) = 0.009657... < 0.05$ (one-tailed)

There is evidence to reject H_0. The percentage of patients who recover after treatment with the new ointment is lower than 85%.

Large Data Set

1 a X is the number of days with a recorded daily mean temperature greater than 15 °C.

$X \sim (10, 0.163)$

H_0: $p = 0.163 \qquad \text{H}_1$: $p > 0.163$

$\text{P}(X \geq 4) = 1 - \text{P}(X \leq 3) = 1 - 0.935 = 0.065 > 0.05$
$\text{P}(X \geq 5) = 1 - \text{P}(X \leq 4) = 1 - 0.9959 = 0.0141 < 0.05$

The critical region is $X \geq 5$

b A random sample of temperatures, 15.6, 17.3, 12.5, 14.1, 11.9, 14.5, 13.0, 9.1, 14.1, 10.0, 9.3 has 2 days with a daily mean temperature greater than 15 °C.

c 2 does not lie in the critical region so H_0 is not rejected. Therefore, there is no reason to suggest that $p \neq 0.163$.

2 X is the number of days with daily mean temperature greater than 25 °C.

$X \sim (10, 0.23)$

H_0: $p = 0.23 \qquad \text{H}_1$: $p > 0.23$

$\text{P}(X \geq 5) = 1 - \text{P}(X \leq 4) = 1 - 0.9431 = 0.0569 > 0.05$
$\text{P}(X \geq 6) = 1 - \text{P}(X \leq 5) = 1 - 0.9870 = 0.0130 < 0.05$

The critical region is $X \geq 6$

A random sample of temperatures 17.5, 18.9, 25.9, 27.7, 30.4, 26.6, 27.4, 27.0, 19.4 and 13.9 has 6 days with a mean temperature greater than 25 °C.

6 does lie in the critical region so H_0 is rejected. Therefore, there is reason to suggest that $p > 0.23$.

Review Exercise 1

1 a A census observes every member of a population.
A disadvantage of a census is it would be time-consuming to get opinions from all the employees.
OR It would be difficult/time-consuming to process the large amount of data from a census.

b Opportunity sampling.

c It is not a random sample. The sample only includes cleaners, there are no types of other employees such as managers.
The first 50 cleaners to leave may be in the same group/shift so may share the same views.

d i Allocate a number from 1–550 to all employees.
For a sample of 50, you need every eleventh person since 550 ÷ 50 = 11.
Select the first employee using a random number from 1 - 11, then select every eleventh person from the list; e.g. if person 8 is then the sample is 8, 19, 30, 41...

ii For this sample, you need $\frac{55}{550}\times 50 = 5$ managers and $\frac{495}{550}\times 50 = 45$ cleaners.
Label the managers 1-55 and the cleaners 1-495.
Use random numbers to select 5 managers and 45 cleaners.

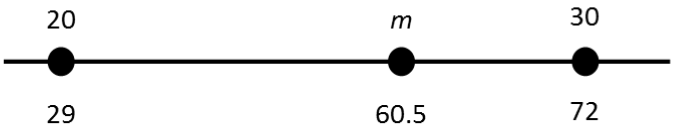
2 a Opportunity sampling is using a sample that is available at the time the study is carried out.
It is unlikely to provide a representative sample of the weather in May.

b mean = $\frac{\sum h}{n} = \frac{100+91+77+83+86}{5} = \frac{437}{5} = 87.4$

c The daily maximum relative humidity is how saturated the air saturation is with water vapour. Mist and fog occur only when the relative humidity is above 95%. Since 87.4% is less than this, Joanna is correct. 'However 5 days is likely not to be a representative sample for the whole of May.

3 a There are 120 observations, so the median is the 60.5th value. This lies in the class $20 < x \leq 30$.

Using interpolation:



$$\frac{m-20}{30-20} = \frac{60.5-29}{72-29}$$
$$\frac{m-20}{10} = \frac{31.5}{43}$$
$m = 27.3$

The median distance is 27.3 miles.

3 b mean = $\dfrac{\sum fx}{\sum f} = \dfrac{3610}{120} = 30.1$

$$\sigma = \sqrt{\frac{\sum fx^2}{n} - \left(\frac{\sum fx}{n}\right)^2}$$

$$= \sqrt{\frac{141\,600}{120} - \left(\frac{3610}{120}\right)^2}$$

$$= \sqrt{1180 - 30.1^2}$$

$$= \sqrt{275} = 16.6$$

The mean distance travelled is 30.1 miles, with a standard deviation of 16.6 miles.

4 $x = 10s + 1$

$s = \dfrac{x-1}{10}$

coded mean, $\bar{x} = \dfrac{\sum x}{n} = \dfrac{947}{30} = 31.6$

actual mean, $\bar{s} = \dfrac{31.6-1}{10} = 3.06$ hours

coded standard deviation, $\sigma_x = \sqrt{\dfrac{33\,065.37}{30}} = 33.2$

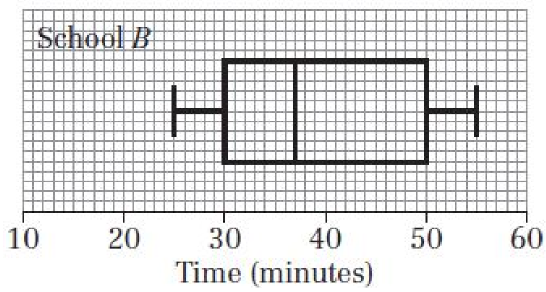
actual standard deviation, $\sigma_s = \dfrac{33.2}{10} = 3.32$ hours

5 a i 37 minutes

ii Upper quartile / third quartile / 75th percentile

b The crosses represent outliers; i.e. observations that are very different from the others and need to be treated with caution. These two children took a lot longer so probably walked.

c Defining an outlier as an observation that lies either 1.5 × interquartile range (IQR) above the upper quartile (Q_3) or 1.5 × IQR below the lower quartile (Q_1):
IQR = 20, Q_1 = 30, Q_3 = 50
$30 - 1.5 \times 20 = 0$ therefore no outliers
$50 + 1.5 \times 20 = 80$ therefore no outliers



d The median time for children from school A is less than that of children from school B.
The interquartile range is less for school A, although the overall range is greater and there are also outliers (there are none for school B).

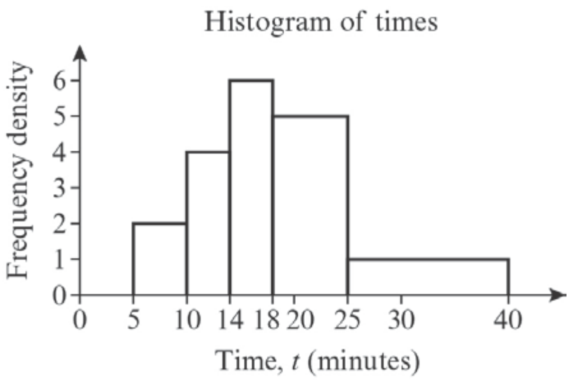
5 d Children from school A generally took less time than those from school B.

Both plots have a positive skew, showing that there was more variation in the time taken by slower children than in the time taken by the faster children.

75% of those from A took less than 37 minutes whereas only 50% of those from B completed in this time.

50% of those from A took less than 30 minutes whereas only 25% of those from B completed in this time.

6 a Missing frequencies:
$18 \le x < 25$: area $= 7 \times 5 = 35$
$25 \le x < 40$: area $= 15 \times 1 = 15$
Missing frequency densities:
$10 \le x < 14$: height = 16 ÷ 4 = 4
$14 \le x < 18$: height = 24 ÷ 4 = 6



b Sample size, n = 100
The number who took longer than 20 minutes is given by the shaded area:
$(25-20)\times 5 + (40-25)\times 1 = 40$
40 ÷ 100 = 0.4
The probability that a person chosen at random took more than 20 minutes to swim 500 m is 0.4

c Mid points are 7.5, 12, 16, 21.5, 32.5
$\sum f = 100$
$\sum ft = (10\times 7.5) + (16\times 12) + (24\times 16) + (35\times 21.5) + (15\times 32.5)$
$= 1891$
$\frac{\sum ft}{\sum f} = \frac{1891}{100} = 18.91$
The mean time taken is 18.9 minutes.

d $\sum ft^2 = (10\times 7.5^2) + (16\times 12^2) + (24\times 16^2) + (35\times 21.5^2) + (15\times 32.5^2) = 41033$
$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$
$$= \sqrt{\frac{41\,033}{100} - \left(\frac{1891}{100}\right)^2}$$

6 d $\sigma = \sqrt{410.33 - 357.59}$
$= \sqrt{52.74}$
$= 7.26$

The standard deviation of t is approximately 7.26 minutes (3 s.f.)

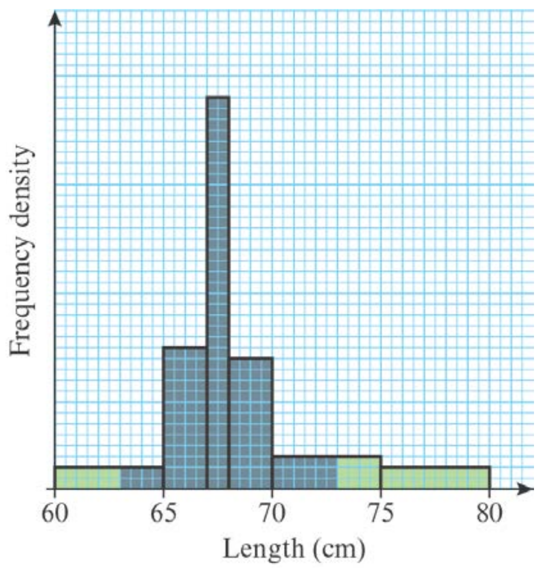
e median, Q_2 is the $\frac{100+1}{2} = 50.5$th term
There are 50 values less than 18 minutes and 50 values greater than or equal to 18 minutes.
The median is therefore 18 minutes.

7 Area of 65 to 67 cm class = 26
Frequency density = 26 ÷ 2 = 13

Using this information:

Length, l (cm)	Frequency	Class width	Frequency density
60 to 65	10	5	2
65 to 67	26	2	13
67	36	1	36
68 to 70	24	2	12
70 to 75	15	5	3
75 to 80	10	5	2

The number of owls with wing length between 63 and 73 cm is given by the shaded area on the graph.



$$P(63 \le l \le 73) = \frac{(2\times2)+26+36+24+(3\times3)}{10+26+36+24+15+10}$$
$$= \frac{99}{121}$$
$$= 0.82$$

8 a 20th percentile : $\frac{20}{100}\times 31 = 6.2$

$$\frac{P_{20}-13}{16-13}=\frac{6.2-1}{8-1}$$

$$P_{20}-13=\frac{5.2\times 3}{7}$$

$$P_{20}=2.23+13= 15.23$$

80th percentile : $\frac{80}{100}\times 31 = 24.8$

$$\frac{P_{80}-19}{22-19}=\frac{24.8-21}{31-21}$$

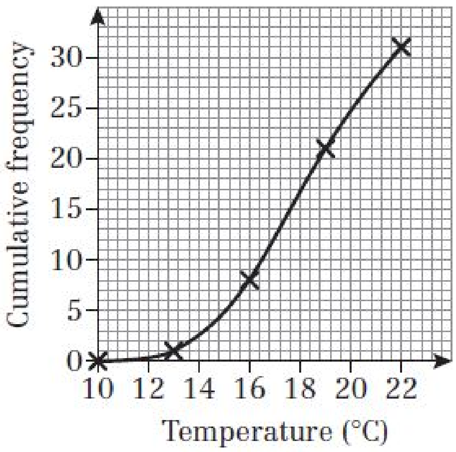
$$P_{80}-19=\frac{3.8\times 3}{10}$$

$$P_{80}=1.14+19$$

$$=20.14$$

$P_{80} - P_{20} = 20.14 - 15.23 = 4.9$ (2 s.f.)

b Cumulative frequencies are 1, 8, 21, 31



c $P_{80} - P_{20} = 19.8 - 15.4 = 4.8$
The 20% to 80% interpercentile range is slightly bigger using interpolation than using the diagram. Interpolation is likely to be more accurate.

d Using the graph:
31 − 5 = 26 days

OR Using interpolation:

$$\frac{15-13}{16-13}=\frac{d-1}{8-1}$$

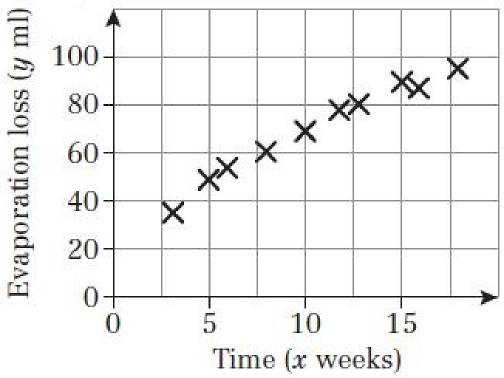
$$\frac{2}{3}=\frac{d-1}{7}$$

$$d=\frac{2\times 7}{3}+1$$

$$=5.67$$

31 − 5.67 = 25.33, so 25 days

9 a



b Points lie close to a straight line.

c For every extra week in storage, another 3.90 ml of chemical evaporates.

d The prediction for 19 weeks is likely to be reasonably reliable as it is close to the range investigated.
The prediction for 35 weeks is likely to be unreliable, since the time is well outside range of x and there is no evidence that model will continue to hold.

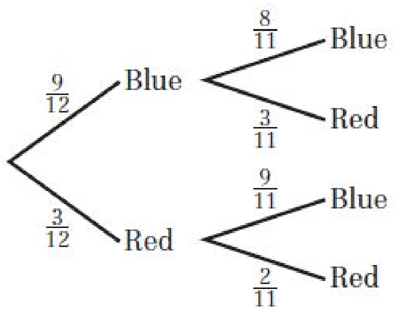
10 a mean + 2 standard deviations = 15.3 + 2 × 10.2 = 35.7
45 > 35.7 so t = 45 is an outlier

b A temperature of 45 °C is very high so it is likely this value was recorded incorrectly. Therefore, this outlier should be omitted from the data.

c In the regression equation, 2.81 represents the number of additional ice creams (in hundreds) sold each month for each degree Celsius increase in average temperature.

d A temperature of 2 °C is outside the range of the data so a value calculated using the equation for the regression line involves extrapolation and is likely to be inaccurate.

11 a



b P (second ball red) = P(blue then red) + P(red then red)

$$= \frac{9}{12} \times \frac{3}{11} + \frac{3}{12} \times \frac{2}{11}$$

$$= \frac{27+6}{132} = \frac{1}{4}$$

The probablility the second ball is red is 0.25.

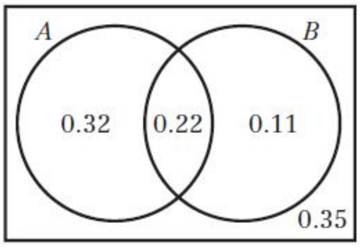
11 c P (balls are different colours) = P(blue then red) + P(red then blue)

$$= \frac{9}{12} \times \frac{3}{11} + \frac{3}{12} \times \frac{9}{11}$$

$$= \frac{27+27}{132} = \frac{54}{132}$$

The probability the balls are different colours is 0.409.

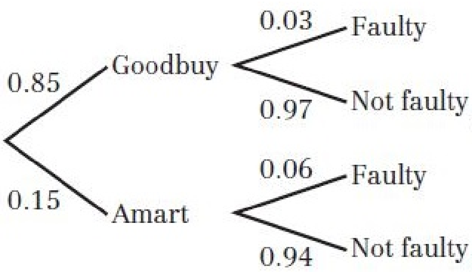
12 a P(*A* or *B*) = P(*A*) + P(B) − P(*A* and *B*)
0.65 = 0.32 + 0.11 − P(*A* and *B*)
P(*A* and *B*) = 0.65 − 0.32 − 0.11= 0.22
P(neither *A* nor *B*) = 1 – 0.65 = 0.35



b $\mathrm{P}(A) = 0.32 + 0.22 = 0.54$
$\mathrm{P}(B) = 0.33$

c For independence P(*A* and *B*) = P(*A*) × P(B)
Here: P(*A*) × P(B) = 0.54 × 0.33 = 0.1782
$0.1782 \neq 0.22$
So these events are not independent.

13 a



b *G* = Goodbuy, *A* = Amart, *NF* = Not faulty
P(*NF*) = P(*G* and *NF*) + P(*A* and *NF*)

$$= (0.85 \times 0.97) + (0.15 \times 0.94)$$

$$= 0.9655$$

14 a Comics and Television are mutually exclusive preferences as the sets do not overlap.

b P(*C* and *B*) = $\dfrac{13}{38} = 0.34$

P(*C*) × P(*B*) = $\dfrac{21}{38} \times \dfrac{11}{19} = \dfrac{231}{722} = 0.32$

$0.34 \neq 0.32$ so these preferences are not independent.

15 a For four coin tosses there are $2 \times 2 \times 2 \times 2 = 2^4 = 16$ possible outcomes.
There is only way of achieving 0 or 4 heads (= 0 tails).
1 head or 3 heads (= 1 tail) may appear on the 1st, 2nd, 3rd or 4th toss; i.e. in one of 4 ways.

Therefore, if X is the number of heads in the outcome:

No. of heads, x	0	1	2	3	4
P(X = x)	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{n}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

$$\frac{1}{16}+\frac{4}{16}+\frac{n}{16}+\frac{4}{16}+\frac{1}{16}=1$$
$$10+n=1$$
$$n=6$$

The probability of an equal number of heads and tails is $\frac{6}{16}=\frac{3}{8}=0.375$

Alternative solution:
Let X be the random variable 'the number of heads'.
$X \sim \mathrm{B}(4, 0.5)$

$$\mathrm{P}(X=2)=\binom{4}{2}0.5^2\times0.5^2$$
$$=\frac{4!}{2!2!}0.5^2\times0.5^2$$
$$=0.375$$

b $\mathrm{P}(X=0 \text{ or } 4)=\mathrm{P}(X=0)+\mathrm{P}(X=4)$
$$=\frac{1}{16}+\frac{1}{16}=\frac{1}{8}$$
$$=0.125$$

c $\mathrm{P}(HHT)=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$
$$=0.125$$

16 a Let X be the random variable 'the number of faulty bolts'.
$X \sim \mathrm{B}(20, 0.3)$

$$\mathrm{P}(X=2)=\binom{20}{2}\times(0.3)^2(0.7)^{18}$$
$$=\frac{20!}{2!18!}(0.3)^2(0.7)^{18}$$
$$=0.0278$$

b $\mathrm{P}(X>3)=1-\mathrm{P}(X\le3)$
From calculator or tables, $\mathrm{P}(X \le 3) = 0.1071$
$\mathrm{P}(X>3)=1-0.1071$
$$=0.8929$$

16 c Let Y be the random variable 'number of bags containing more than 3 faulty bolts'.
Y ~ B(10,0.8929)

$$P(Y=6)=\binom{10}{6}\times(0.8929)^6(0.1071)^4$$
$$=\frac{10!}{6!4!}(0.8929)^6(0.1071)^4$$
$$=0.0140$$

17a

x	1	2	3	4	5	6
P (X = x)	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$
P (X = x)	0.0278	0.0833	0.1389	0.1944	0.25	0.3056

b $P(2<X\leq 5)=P(Y=3)+P(Y=4)+P(Y=5)$
$$P(Y=3)+P(Y=4)+P(Y=5)=\tfrac{21}{36}$$
$$=\tfrac{7}{12}$$
$$=0.5833...$$

18 a $P(X = x) = 0.2$

b The spinner has 3 odd numbers and two even numbers so:
$P(X = \text{even}) = 0.4$
$P(X = \text{odd}) = 0.6$

y	1	2	3	4
P(Y = y)	0.6	$0.4 \times 0.6 = 0.24$	$0.4^2 \times 0.6 = 0.096$	$0.4^3 \times 0.6 + 0.4^4 = 0.064$

c $P(Y > 2) = P(Y = 3) + P(Y = 4) = 0.096 + 0.064 = 0.16$

19 For $X \sim B(15, 0.32)$, using calculator or tables:

a $P(X = 7) = 0.101$ (3 s.f.)

b $P(X \leq 4) = 0.448$ (3 s.f.)

c $P(X < 8) = P(X \leq 7) = 0.929$ (3 s.f.)

d $P(X > 6) = 1 - P(X \leq 5) = 1 - 0.6607 = 0.339$ (3 s.f.)

20 a H_0: p = 0.3 H_1: p ≠ 0.3
Significance level 2.5%
If H_0 is true $X \sim B(40, 0.3)$
Let c_1 and c_2 be the two critical values.
$P(X \leq c_1) = 0.0125$ and $P(X \geq c_2) = 0.0125$
For the lower tail:
Finding values of P(X) from tables or calculator, those either side of 0.0125 are:
$P(X \leq 5) = 0.0086$
$P(X \leq 6) = 0.0238$
$0.0125 - 0.0086 = 0.0039$ ($X \leq 5$) and $0.0238 - 0.0125 = 0.0113$ ($X \leq 6$)
So $c_1 = 5$ as this gives the value closest to 0.0125.

20 a In the same way, for the upper tail:
$P(X \geq 19) = 1 - P(X \leq 18) = 1 - 0.9852 = 0.0148$
$P(X \geq 20) = 1 - P(X \leq 19) = 1 - 0.9937 = 0.0063$
$0.0148 - 0.0125 = 0.0023$ ($X \geq 19$) and
$0.0125 - 0.0063 = 0.0062$ ($X \geq 20$)
So $c_2 = 19$
$P(X \leq 5)$ and $P(X \geq 19)$, so the critical region is $0 \leq X \leq 5$ and $19 \leq X \leq 40$

b The probability of incorrectly rejecting the null hypothesis is the same as the probability that X falls within the critical region.
$P(X \leq 5) + P(X \geq 19) = 0.0086 + 0.0148 = 0.0234$

21 a $X \leq B(10, 0.75)$ where X is the random variable 'number of patients who recover when treated'.

b Using tables or calculator:
$P(X = 6) = 0.146$
OR

$$\begin{aligned} P(X = 6) &= P(X \leq 6) - P(X \leq 5) \\ &= 0.9219 - 0.7759 \\ &= 0.146 \end{aligned}$$

OR, Calculating:

$$\begin{aligned} P(X = 6) &= \binom{10}{6} \times (0.75)^6 (0.25)^4 \\ &= \frac{10!}{6!4!} \times (0.75)^6 (0.25)^4 \\ &= 0.146 \end{aligned}$$

c $H_0 : p = 0.75$
$H_1 : p < 0.75$
$X \sim B(20, 0.75)$

$$\begin{aligned} P(X \leq 13) &= 1 - 0.7858 \\ &= 0.2142 \end{aligned}$$

As the probability is greater than 5%, there is insufficient evidence to reject the null hypothesis that 75% of patients will recover. Therefore the doctor's belief that fewer patients than this will recover.

d Using tables/calculator to find values either side of $1 - 0.01 = 0.99$:
$P(X \leq 9) = 1 - 0.9961 = 0.0039 < 0.01$
i.e. if 9 patients recover, the null hypothesis is accepted.
$P(X \leq 10) = 1 - 0.9861 = 0.0139 > 0.01$
i.e. if 10 patients recover, the null hypothesis is rejected.
Therefore, no more than 9 patients should recover for the test to be significant at this level.

22 a H_0: p = 0.3 i.e the fertiliser will have no effect.
H_1: p > 0.3 i.e. the fertiliser will increase the probability of the tomatoes having a dimeter > 4 cm.

b Significance level 5%
If H_0 is true $X \sim B(40, 0.3)$

22 b Let c be the critical value.
$P(X \geq 17) = 1 - P(X \leq 16) = 1 - 0.9367 = 0.0633 > 0.05$
$P(X \geq 18) = 1 - P(X \leq 17) = 1 - 0.9680 = 0.032 < 0.05$
So $c = 18$
$P(X \geq 18)$, so the critical region is $18 \leq X \leq 40$

c Actual significance level = 0.032 = 3.2%

d The observed value of 18 lies in the critical region so it is reasonable to reject the null hypothesis that the fertiliser has no effect. Dhiriti's claim that the fertiliser works is supported.

Challenge

1 $P(C) = \dfrac{z+7}{50}$

$P(A) = \dfrac{y+1}{50}$

$\dfrac{z+7}{50} = 3\left(\dfrac{y+1}{50}\right)$

$z + 7 = 3y + 3$
$z + 4 = 3y$ **(1)**

$P(\text{not } B) = 0.76 = \dfrac{38}{50}$

$P(\text{not } B) = \dfrac{y+z+18}{50}$

So $\dfrac{y+z+18}{50} = \dfrac{38}{50}$

$y + z + 18 = 38$
$y = 20 - z$ **(2)**
Use **(2)** to substitute for y in **(1)**:
$z + 4 = 3(20 - z)$
$z + 4 = 60 - 3z$
$4z = 56$
$z = 14$
Substituting this value for z in **(2)**:
$y = 20 - 14 = 6$
Referring to the diagram:
$x = 50 - (6 + 1 + 7 + 14 + 18) = 4$
$x = 4, y = 6, z = 14$

2 a Significance level 10%
If H_0 is true $X \sim B(30, 0.65)$
Let c be the critical value.
$P(X \leq 15) = 0.0652$
$P(X \leq 16) = 0.1263$
$0.1 - 0.0652 = 0.0348$ $(X \leq 15)$
$0.1263 - 0.1 = 0.0263$ $(X \leq 16)$
So $c = 16$
$P(X \leq 16)$, so the critical region is $0 \leq X \leq 16$

b $P(X \leq 16) = 0.1263$
$P(X \leq 16 \text{ and } X \leq 16) = 0.1263^2 = 0.0160$

Modelling in mechanics 8A

1 a i $x = 0$ gives $h = 0.36 \text{ x } 0 - 0.003 \text{ x } (0)^2 = 0$

Height = 0 m

ii $x = 100$ m gives $h = 0.36 \times 100 - 0.003 \times (100)^2 = 36 - 30$

Height = 6 m

b $x = 200$ m gives $h = 0.36 \times 200 - 0.003 \times (200)^2 = 72 - 120$

Height = −48 m

c The model is not valid for this distance as it predicts the ball would be 48 m below ground level: the ball has already hit the ground at this point.

2 a 90 m (as this is the height when $t = 0$)

b i $t = 3$ gives $h = -5 \times (3)^2 + 15 \times 3 + 90 = -45 + 45 + 90$

Height above sea level = 90 m

ii $t = 5$ gives $h = -5 \times (5)^2 + 15 \times 5 + 90 = -125 + 75 + 90$

Height above sea level = 40 m

c $t = 20$ gives $h = -5 \times (20)^2 + 15 \times 20 + 90 = -2000 + 300 + 90 = -1610$

Height = 1610 m below sea level

d The prediction is incorrect because this height is below sea level, where the model is probably no longer valid, because forces acting on the ball will be different.

3 a When $h = 4$,

$4 = 2 + 1.1x - 0.1x^2$ or rearranging, $0 = -0.1x^2 + 1.1x - 2$, so using the quadratic formula,

$$x = \frac{-(1.1) \pm \sqrt{(1.1)^2 - 4 \times (-0.1) \times (-2)}}{2 \times (-0.1)}$$

$$= \frac{-1.1 \pm \sqrt{0.41}}{-0.2}$$

$$\approx \frac{-1.1 \pm 0.6403}{-0.2}$$

So $x = 2.30$ or 8.70 (to 3 s.f.)

The ball is 4 m above the ground after it has travelled both 2.30 m and 8.70 m horizontally.

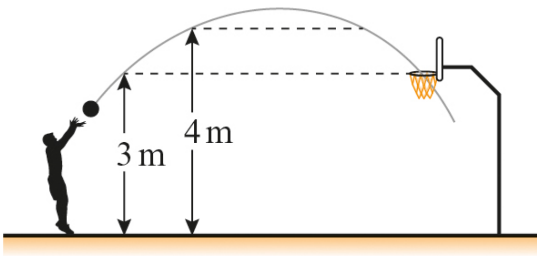
3 b When $h = 3$,

$3 = 2 + 1.1x - 0.1x^2$ or rearranging, $0 = -0.1x^2 + 1.1x - 1$, and dividing by -0.1,

$0 = x^2 - 11x + 10 = (x - 1)(x - 10)$

So $x = 1$ or 10, and the ball is at height 3 m after it has travelled both 1 m and 10 m horizontally.

At the shorter distance, the ball will be travelling upward (see diagram), so $k = 10$.



c If $x > 10$ m the equation is no longer valid as the ball will have gone through (or past) the net, and there would possibly be new forces acting on the ball.

4 a When $t = 1$, $d = 13.2$

So, substituting, $d = kt^2$ becomes $13.2 = k \times 1^2$ and therefore $k = 13.2$.

Our completed equation is $d = 13.2t^2$. When $t = 10$,

$d = 13.2 \times 10^2 = 1320$

The distance travelled is 1320 m.

b Clearly, the model is valid for positive values of t only. We are also unsure of what happens after t = 10, and we therefore can't use this model past that point. The model is valid for $0 \leq t \leq 10$.

5 $h \geq 0$ means that $0.36x - 0.003x^2 = x(0.36 - 0.003x) \geq 0$

We are assuming that $x \geq 0$ after the ball is struck, so we need the bracket to be non-negative:

$0.36 \geq 0.003x$

$$x \leq \frac{0.36}{0.003} = 120$$

So the model is valid for $0 \leq x \leq 120$.

6 When the stone enters the sea, $h = 0$

$0 = -5t^2 + 15t + 90$, or, dividing by -5, $0 = t^2 - 3t - 18 = (t - 6)(t + 3)$

So, $t = -3$ or 6 and the stone hits the sea 6 seconds after it is thrown, since the model is valid only for the time **after** the stone is thrown at t = 0.

Therefore the model is valid for $0 \leq t \leq 6$.

Modelling in mechanics 8B

1 a Modelling the ball as a particle means we can ignore the rotational effect of any external forces that are acting on it and the effects of air resistance, and assume all the mass acts at a single point.

b Assuming air resistance is negligible means we can ignore the frictional effects of the air on the football.

2 a Modelling the puck as a particle means we can ignore the rotational effect of any external forces that are acting on it and the effects of air resistance, and assume all the mass acts at a single point.

b Modelling the ice as smooth means we can ignore any friction between the puck and the ice.

3 Modelling an object as a particle means that the effect of air resistance is ignored but, for a parachute, this force is significant.

4 a Modelling the fishing rod as a light rod means we can assume it has no mass or thickness and is rigid and unbending.

b While the mass of the rod may be negligible in comparison with the reel or any fish it is designed to catch (justifying the ‘light’ assumption), and narrow compared to its length (allowing it to be treated as a one-dimensional object) rigidity is not a desirable property of fishing rods, so it is not appropriate to consider it as a rod.

5 a Model the golf ball as a particle, and ignore the effects of air resistance.

b Model the child and sledge as a single particle, consider the hill to be smooth, and ignore the effects of air resistance.

c Model the objects as particles, the string as light and inextensible, and the pulley as smooth.

d Model the suitcase and handle as a single particle, consider the path to be smooth, and ignore friction between the wheels and their holdings.

Modelling in mechanics 8C

1 a $65 \text{ km h}^{-1} = \frac{65 \times 1000}{60 \times 60} \text{ m s}^{-1} = 18.1 \text{ m s}^{-1}$ (to 3 s.f.)

b $15 \text{ g cm}^{-2} = \frac{15 \div 1000}{1 \div (100 \times 100)} \text{ kg m}^{-2} = 150 \text{ kg m}^{-2}$

c 30 cm per minute $= \frac{30 \div 100}{60} \text{ m s}^{-1} = 5 \times 10^{-3} \text{ m s}^{-1}$

d $24 \text{ g m}^{-3} = \frac{24}{1000} \text{ kg m}^{-3} = 2.4 \times 10^{-2} \text{ kg m}^{-3}$

e $4.5 \times 10^{-2} \text{ g cm}^{-3} = \frac{4.5 \times 10^{-2} \div 1000}{1 \div (100 \times 100 \times 100)} \text{ kg m}^{-3} = 45 \text{ kg m}^{-3}$

f $6.3 \times 10^{-3} \text{ kg cm}^{-2} = \frac{6.3 \times 10^{-3}}{1 \div (100 \times 100)} \text{ kg m}^{-2} = 63 \text{ kg m}^{-2}$

2 a A: normal reaction, B: forward thrust, C: weight, D: friction

b A: buoyancy, B: forward thrust, C: weight, D: drag/water resistance

c A: normal reaction, B: friction, C: weight, D: tension

d A: normal reaction, B: weight, C: friction

Modelling in mechanics 8D

1 **a** 2.1 m s^{-1}

b 500 m

c -1.8 m s^{-1}

d -2.7 m s^{-1}

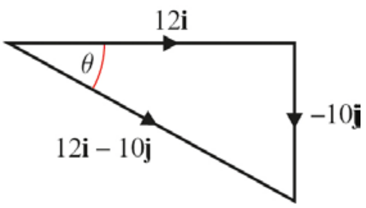
e -750 m

f 2.5 m s^{-1}

2 **a** speed $|\mathbf{v}| = \sqrt{12^2 + 10^2} = \sqrt{244}$

The speed of the car is 15.6 m s^{-1} (to 3 s.f.)

b Let the acute angle made with **i** be θ, then



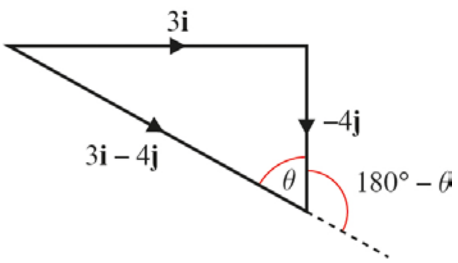
$\tan\theta = \frac{10}{12} = 0.8333$ so $\theta = 39.8°$ (to 3 s.f.)

The direction of motion of the car is 39.8° from the **i** vector.

3 **a** $|\mathbf{a}| = \sqrt{3^2 + 4^2} = \sqrt{25}$

The magnitude of the acceleration is 5 m s^{-2}.

b Let the acute angle made with **j** be θ



$\tan\theta = \frac{3}{4} = 0.75$ so $\theta = 36.9°$ (to 3 s.f.)

Angle required = $180° - \theta = 180° - 36.9° = 143.1°$

The direction of the acceleration is 143° from the **j** vector.

4 a $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$$\overrightarrow{AC} = \begin{pmatrix} 10 \\ 3 \end{pmatrix} + \begin{pmatrix} -7 \\ 12 \end{pmatrix} = \begin{pmatrix} 3 \\ 15 \end{pmatrix}$$

$$\left|\overrightarrow{AC}\right| = \sqrt{3^2 + 15^2} = \sqrt{234}$$

The magnitude of the displacement is 15.3 m (to 3 s.f.)

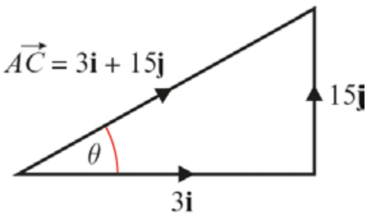
b $\left|\overrightarrow{AB}\right| = \sqrt{10^2 + 3^2} = \sqrt{109}$

$$\left|\overrightarrow{BC}\right| = \sqrt{7^2 + 12^2} = \sqrt{193}$$

$$\left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right| = \sqrt{109} + \sqrt{193} = 24.3$$

The girl cycles 24.3 km (to 3 s.f.)

c Let the acute angle made with **i** be θ



$$\tan\theta = \frac{15}{3} = 5 \text{ so } \theta = 78.7° \text{ (to 3 s.f.)}$$

The direction of motion of the car is 78.7° from the **i** vector.

Modelling in mechanics, Mixed Exercise 8

1 a $x = 2$ gives

$$h = \frac{1}{10}\left(24\times 2 - 3\times(2)^2\right)$$
$$= \frac{1}{10}\left(48-12\right) = 3.6$$

When it is 2 m horizontally from where it is hit, the ball is at a vertical height of 3.6 m.

b When $h = 2.1$,

$2.1 = \frac{1}{10}\left(24x - 3x^2\right)$ or rearranging, $0 = 24x - 21 - 3x^2$, and dividing by -3,

$0 = x^2 - 8x + 7 = (x - 1)(x - 7)$

So $x = 1$ or 7.

The ball is at a height of 2.1m when it is at a horizontal distance of 1 m and again at 7 m.

c Model becomes valid when the ball is hit, i.e. $x = 0$.

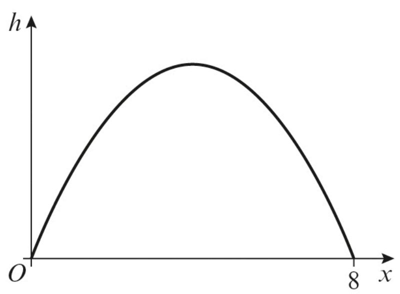
The model remains valid while the ball is above the ground, i.e. $h \geqslant 0$

$\frac{1}{10}\left(24x - 3x^2\right) \geq 0$ or factorising, $0.3x(8 - x) \geq 0$

We know that $x \geq 0$ for our region, so we need the bracket to be non-negative, i.e. $x \leq 8$.

The model is valid for $0 \leq x \leq 8$.

d The equation for this model produces a curve of the form shown:



Since the curve is symmetrical, maximum height occurs when $x = 4$, at which point:

$$h = \frac{1}{10}\left(24\times 4 - 3\times(4)^2\right)$$
$$= \frac{1}{10}\left(96-48\right) = 4.8$$

The maximum height of the ball is 4.8 m.

2 a $x = 2$ gives

$h = 10 - 0.58 \times (2)^2 = 7.68$

When the horizontal distance from the end of the board is 2 m, the diver is at a height of 7.68 m.

b When the diver hits the water, $h = 0$

$0 = 10 - 0.58x^2$

$0.58x^2 = 10$

$$x = \sqrt{\frac{10}{0.58}} \approx \sqrt{17.24}$$

The diver enters the water 4.15 m from the end of the board.

c By modelling the diver as a particle, we can ignore air resistance and the rotational effects of external forces. The mass of the diver is assumed to be concentrated at a single point.

d The pool is only 4.5 m deep so the model could not be accurate for large values of x.
Also, the motion is likely to be different in the water than in the air.

3 a Model the man on skis as a particle. This allows one to ignore the rotational effect of any forces that are acting on the man as well as any effects due to air resistance.

Consider the slope to be smooth: that there is no friction between the skis and the slope.

b Model the yo-yo as a particle. This allows one to ignore the rotation of the yo-yo and air resistance. We assume that the mass of the yo-yo is concentrated at a point.

Consider the string to be light and inextensible. This allows one to ignore the weight of the string and assume it does not stretch, thereby affecting the acceleration of attached objects.

Model the yo-yo as smooth, that is, assume there is no friction between the yo-yo and the string.

4 a 2.5 km per minute $= \dfrac{2.5\times1000}{60}$ m s^{-1} $= 41.7$ m s^{-1} (to 3 s.f.)

b $0.6 \text{ kg cm}^{-2} = \dfrac{0.6}{1\div(100\times100)} \text{ kg m}^{-2} = 6000 \text{ kg m}^{-2}$

c $1.2 \times 10^3 \text{ g cm}^{-3} = \dfrac{1.2\times10^3\times(100\times100\times100)}{1000} \text{ kg m}^{-3} = 1.2\times10^6 \text{ kg m}^{-3}$

5 a Model the ball as a particle.

Assume the floor is smooth.

5 b i The velocity will be positive as the positive direction is defined as such.

ii In real life the acceleration would be negative, as the ball always slows down. However, if we assume there is no friction, then the ball would move at constant velocity.

6 a Velocity is positive, displacement is positive

b Velocity is negative, displacement is positive

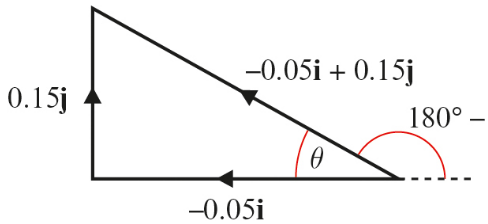
c Velocity is negative, displacement is negative

7 a $|\mathbf{a}| = \sqrt{0.05^2 + 0.15^2} = \sqrt{0.025} = 0.158$ (to 3 s.f.)

The magnitude of the acceleration is 0.158 m s^{-2}.

b Let the acute angle made with **i** be θ, then

$\tan\theta = \dfrac{0.15}{0.05} = 3$ so $\theta = 71.6°$ (to 3 s.f.)



Angle required = 180° − θ = 180° − 71.6° = 108° (to 3 s.f.)

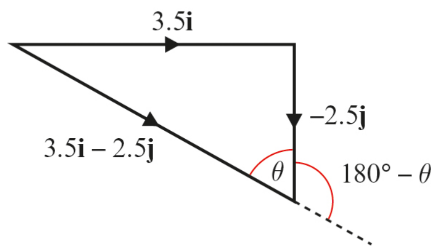
The direction of the acceleration is 108° from the **i** vector.

8 a $|\mathbf{v}| = \sqrt{3.5^2 + 2.5^2} = \sqrt{18.5} = 4.30$ (to 3 s.f.)

The speed of the toy car is 4.30 m s^{-1}.

b Let the acute angle made with **j** be θ, then

$\tan\theta = \dfrac{3.5}{2.5} = 1.4$ so $\theta = 54.5°$ (to 3 s.f.)



Angle required = 180° − θ = 180° − 54.5° = 126° (to 3 s.f.)

The direction of the acceleration is 126° from the **j** vector.

9 a $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$

$\overrightarrow{PR} = \begin{pmatrix} 100 \\ 80 \end{pmatrix} + \begin{pmatrix} 50 \\ -30 \end{pmatrix} = \begin{pmatrix} 150 \\ 50 \end{pmatrix}$

$\left|\overrightarrow{PR}\right| = \sqrt{150^2 + 50^2} = \sqrt{25000} = 158$ (to 3 s.f.)

The magnitude of the displacement is 158 m.

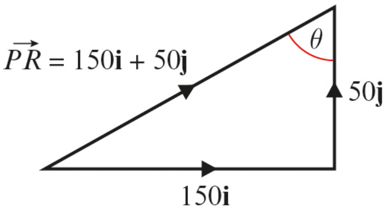
b $\left|\overrightarrow{PQ}\right| = \sqrt{100^2 + 80^2} = \sqrt{16400} = 128$ (to 3 s.f.)

$\left|\overrightarrow{QR}\right| = \sqrt{50^2 + 30^2} = \sqrt{3400} = 58.3$ (to 3 s.f.)

$\left|\overrightarrow{PQ}\right| + \left|\overrightarrow{QR}\right| = \sqrt{16400} + \sqrt{3400} = 186$ (to 3 s.f.)

The plane travels a total distance of 186 m.

c Let the acute angle made with **j** be θ



$\tan\theta = \frac{150}{50} = 3$ so $\theta = 71.6°$ (to 3 s.f.)

The direction of motion of the car is 71.6° from the **j** vector.

Constant acceleration 9A

1 a *A* displacement = 40 km, time = 0.5 h and $\frac{40}{0.5} = 80$

So the average velocity is 80 km h^{-1}.

B displacement = 20 km, time = 0.5 h and $\frac{20}{0.5} = 40$

So the average velocity is 40 km h^{-1}.

C displacement = 0 km, time = 0.5 h and $\frac{0}{0.5} = 0$

So the average velocity is 0 km h^{-1}.

D displacement = 40 km, time = 1 h and $\frac{40}{1} = 40$

So the average velocity is 40 km h^{-1}.

E displacement = −100 km, time = 1.5 h and $\frac{100}{1.5} = -66.7$ (to 3 s.f.)

So the average velocity is -66.7 km h^{-1}.

b The average velocity for the whole journey is 0 km h^{-1} as the overall displacement is 0 km.

c Total distance travelled = 200 km
Total time taken = 4 h
average speed = $\frac{200}{4} = 50\text{ km h}^{-1}$

2 a For first section of the journey: average velocity = 60 km h^{-1}, time taken = 2.5 h

displacement = 2.5 × 60 = 150 km

This is 6 squares on the vertical axis, so one square is $\frac{150}{6} = 25\text{ km}$

total displacement shows as 7.5 squares = 7.5 × 25 = 187.5 km

b Time for whole journey = 3.75 h
average velocity = $\frac{187.5}{3.75} = 50\text{ km h}^{-1}$

3 a displacement = 12 km, time = 1 h
average velocity = $\frac{12}{1} = 12\text{ km h}^{-1}$

b Sarah passed her home at 12:45.

c For the penultimate stage: displacement = −12 + (−3) = −15 km, time = 1.5 h
average velocity = $\frac{-15}{1.5} = -10\text{ km h}^{-1}$

For the final stage: displacement = 3 km, time = 1 h
average velocity = $\frac{3}{1} = 3\text{ km h}^{-1}$

3 d Total distance travelled = 30 km
Total time taken = 4 h
average speed = $\frac{30}{4} = 7.5\ \text{km h}^{-1}$

4 a Reading from the graph:

maximum height = 2.5 m
time taken to reach this = 0.75 s

b When it reaches the highest point, the velocity of the ball is 0 m s^{-1}.

c i The velocity of the ball is positive (upwards) and decreases (the ball is decelerating) until it reaches 0 at the highest point.

ii The velocity of the ball is negative (downwards), and increases (the ball is accelerating) until it hits the ground at the same speed at which it was launched.

Constant acceleration 9B

1 a $a = \frac{9}{4} = 2.25$

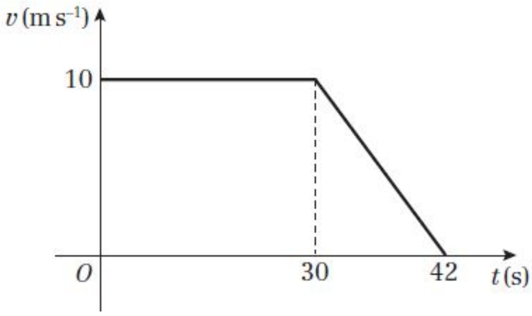
The athlete accelerates at a rate of 2.25 m s^{-2}.

b $s = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(8+12)\times 9 = 90$

The displacement of the athlete after 12 s is 90 m.

2 a



b $s = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(30+42)\times 10 = 360$

The distance from A to B is 360 m.

3 a $a = \frac{8}{20} = 0.4$

The acceleration of the cyclist is 0.4 m s^{-2}.

b $a = -\frac{8}{15} = -0.533$ (to 3 s.f.)

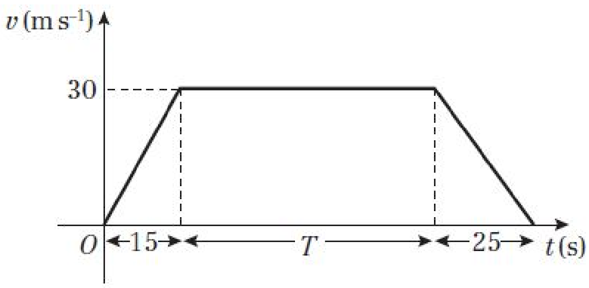
The deceleration of the cyclist is 0.533 m s^{-2}.

c $s = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(40+75)\times 8 = 460$

After 75 s, the distance from the starting point of the cyclist is 460 m.

4 a



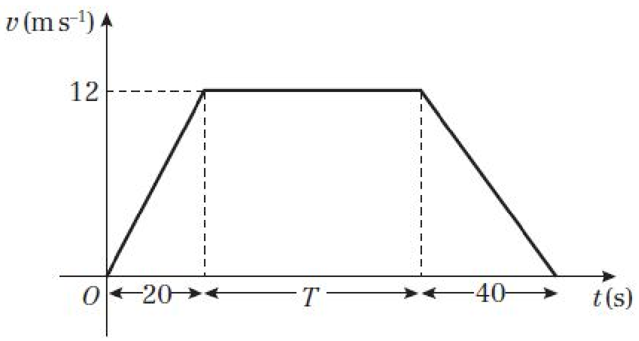
4 b $s = \frac{1}{2}(a+b)h$

$$2400 = \tfrac{1}{2}(T + (15 + T + 25)) \times 30$$
$$= 15(2T + 40)$$
$$2T + 40 = \tfrac{2400}{15} = 160$$

$$T = \frac{160 - 40}{2} = 60$$

The time taken to travel from S to F is (15 + T + 25) = 100 s.

5 a The velocity after 20 s is given by



velocity = acceleration x time $= 0.6 \times 20 = 12$

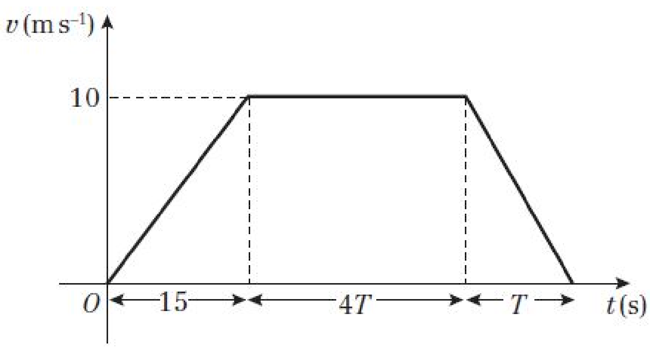
b $s = \frac{1}{2}(a+b)h$

$$4200 = \tfrac{1}{2}(T + (20 + T + 40)) \times 12$$
$$= 6(2T + 60)$$
$$2T + 60 = \frac{4200}{6} = 700$$
$$T = \frac{700 - 60}{2} = 320$$

c While at constant velocity: v = 12 m s^{-1}, t = 320 s

distance travelled = 12 × 320 = 3840 m

6 a



6 b $s = \frac{1}{2}(a+b)h$

$$480 = \tfrac{1}{2}(4T + (15 + 4T + T))10$$
$$= 5 \times (15 + 9T)$$
$$9T + 15 = \frac{480}{5} = 96$$

$$T = \frac{96-15}{9} = 9$$

Total time travelling = 15 + 5T = 15 + (5 × 9) = 60

The particle travels for a total of 60 s.

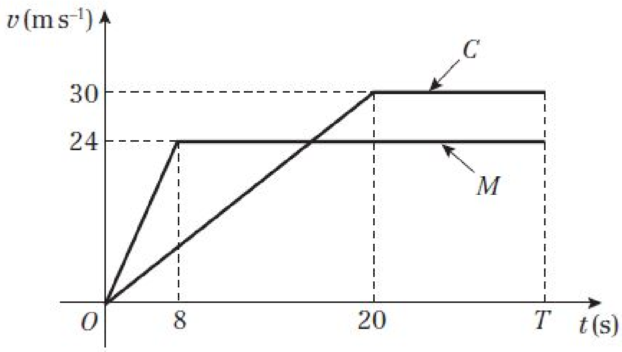
7 a Area = trapezium + rectangle + triangle

$$100 = \tfrac{1}{2}(u+10)\times 3 + 7\times 10 + \tfrac{1}{2}\times 2\times 10$$
$$= \tfrac{3}{2}(u+10) + 70 + 10$$
$$\tfrac{3}{2}(u+10) = 100 - 70 - 10 = 20$$
$$u = 20\times\tfrac{2}{3} - 10$$
$$= \tfrac{10}{3}$$

b $a = \dfrac{10-\frac{10}{3}}{3} = \frac{20}{9} = 2.22$ (to 3 s.f.)

The acceleration of the particle is 2.22 m s^{-2}.

8 a For M, velocity = acceleration x time = $3\times 8 = 24$



b Let C overtake M at time T seconds.

The distance travelled by M is given by

$$s = \frac{1}{2}(8\times 24) + 24\times(T-8)$$
$$= 24(T-4)$$

8 b The distance travelled by C is given by

$$s = \tfrac{1}{2}(a+b)h = \tfrac{1}{2}(T-20+T)\times 30$$
$$= 15\,(2T-20)$$

At the point of overtaking the distances are equal.

$$\begin{aligned} 24(T-4) &= 15(2T-20) \\ 24T-96 &= 30T-300 \\ 6T &= 204 \\ T &= \tfrac{204}{6} = 34 \end{aligned}$$

$$\begin{aligned} s &= 24(T-4) \\ &= 24(34-4) = 720 \end{aligned}$$

The distance of the pedestrian from the road junction is 720 m.

Challenge

a The object changed direction after 6 s, as this is when the velocity changed from positive to negative.

b While travelling at positive velocity:

$$s_p = \tfrac{1}{2}(1+6)\times 3 = \tfrac{1}{2}\times 21 = 10.5$$

While travelling at negative velocity:

$$s_n = \tfrac{1}{2}(4+2)\times 2 = \tfrac{1}{2}\times 12 = 6$$

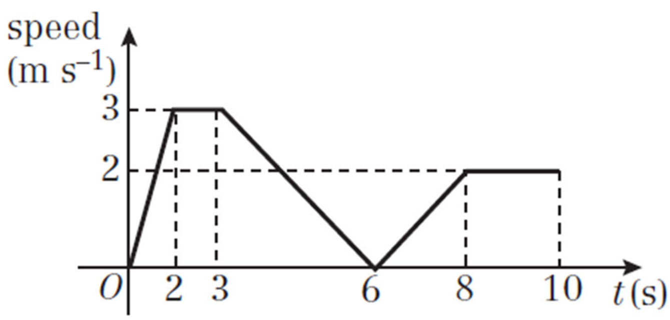
The total distance travelled by the object = $s_p + s_n = 10.5 + 6 = 16.5$ m

c i Using the value calculated in **b**, after 6 s the displacement of the object is $s_p = 10.5$ m.

ii In the first 6 seconds, displacement is positive.
In the last 4 seconds, displacement is negative.

Hence, using the values calculated in **b**, total displacement = $s_p + (-s_n) = 10.5 + (-6) = 4.5$ m.

d



Constant acceleration 9C

1 $a = 3, u = 2, t = 6, v = ?$

$$v = u + at$$
$$= 2 + 3 \times 6 = 2 + 18 = 20$$

The velocity of the particle at time $t = 6$ s is 20 m s^{-1}.

2 $u = 10, v = 0, t = 16, a = ?$

$$v = u + at$$

$$0 = 10 + a \times 16$$
$$a = -\tfrac{10}{16} = -0.625$$

The deceleration of the car is 0.625 m s^{-1}.

3 $s = 360, t = 15, v = 28, u = ?$

$$s = \left(\frac{u+v}{2}\right)t$$
$$360 = \frac{u+28}{2} \times 15$$
$$u = \frac{360 \times 2}{15} - 28$$
$$= 20$$

The velocity of the car at the first sign post is 20 m s^{-1}.

4 a $a = 0.5, u = 3, t = 12, v = ?$

$$v = u + at$$
$$= 3 + 0.5 \times 12 = 3 + 6 = 9$$

The velocity of the cyclist at B is 9 m s^{-1}.

b $u = 3, v = 9, t = 12, s = ?$

$$s = \left(\frac{u+v}{2}\right)t$$
$$= \left(\frac{3+9}{2}\right) \times 12 = 6 \times 12 = 72$$

The distance from A to B is 72 m.

5 a $s = 24, t = 6, v = 5, u = ?$

$$s = \left(\frac{u+v}{2}\right)t$$

$$24 = \left(\frac{u+5}{2}\right)\times 6$$

$$u = \frac{24\times 2}{6} - 5 = 3$$

The velocity of the particle at A is 3 m s^{-1}.

b $u = 3, v = 5, t = 6, a = ?$

$$v = u + at$$
$$5 = 3 + 6a$$
$$a = \frac{5-3}{6} = \frac{1}{3} = 0.333 \text{ (to 3 s.f.)}$$

The acceleration of the particle is 0.333 m s^{-2}.

6 a $a = -1.2, t = 6, v = 2, u = ?$

$$v = u + at$$
$$2 = u - 1.2\times 6 = u - 7.2$$
$$u = 2 + 7.2 = 9.2$$

The speed of the particle at A is 9.2 m s^{-1}.

b $u = 9.2, v = 2, t = 6, s = ?$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{9.2+2}{2}\right)\times 6 = 11.2\times 3 = 33.6$$

The distance from A to B is 33.6 m.

7 a $72 \text{ km h}^{-1} = 72\times 1000 \text{ m h}^{-1} = \frac{72\times 1000}{3600} \text{ m s}^{-1} = 20 \text{ m s}^{-1}$

$u = 20, a = -0.6, t = 25, v = ?$

$$v = u + at = 20 - 0.6\times 25 = 20 - 15 = 5 \text{ m s}^{-1}$$

$$5 \text{ m s}^{-1} = \frac{5\times 3600}{1000} \text{ km h}^{-1} = 18 \text{ km h}^{-1}$$

The speed of the train as it passes the second signal is 18 km h^{-1}

b $u = 20, v = 5, t = 25, s = ?$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{20+5}{2}\right) \times 25 = 12.5 \times 25 = 312.5$$

The distance between the signals is 312.5 m.

8 a $a = -4, u = 32, v = 0, t = ?$

$$v = u + at$$
$$0 = 32 - 4t$$
$$t = \frac{32}{4} = 8$$

The time taken for the particle to move from A to B is 8 s.

b $u = 32, v = 0, t = 8, s = ?$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{32+0}{2}\right)\times 8 = 16\times 8 = 128$$

The time between A and B is 128 m.

9 a $u = 16,\ t = 40,\ v = 0,\ a = ?$

$$v = u + at$$
$$0 = 16 + 40a$$
$$a = \frac{-16}{40} = -0.4$$

The deceleration between A and B is 0.4 m s^{-2}.

b $u = 16, t = 40, v = 0,\ s = ?$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{16+0}{2}\right)\times 40 = 8\times 40 = 320$$

The distance from the bottom of the hill to the point where the skier comes to rest is 320 m.

10 a $u = 2, v = 7, t = 20, a = ?$

$$v = u + at$$
$$7 = 2 + 20a$$
$$a = \frac{7-2}{20} = 0.25$$

The acceleration of the particle is 0.25 m s^{-2}.

b From B to C, $u = 7,\ v = 11,\ a = 0.25,\ t = ?$

$$v = u + at$$
$$11 = 7 + 0.25t$$
$$t = \frac{11-7}{0.25} = 16$$

10 b The time taken for the particle to move from B to C is 16 s.

c The time taken to move from A to C is (20 + 16) = 36 s

From A to C, $u=2$, $v=11$, $t=36$, $s=?$

$$s=\left(\frac{u+v}{2}\right)t=\left(\frac{2+11}{2}\right)\times 36=6.5\times 36=234$$

The distance between A and C is 234 m.

11 a From A to B, $a=1.5$, $u=1$, $t=12$, $v=?$

$$v=u+at=1+1.5\times 12=1+18=19$$

The velocity of the particle at B is 19 m s^{-1}.

b From B to C, $u=19$, $v=43$, $t=10$, $a=?$

$$v=u+at$$
$$43=19+10a$$
$$a=\frac{43-19}{10}=2.4$$

The acceleration from B to C is 2.4 m s^{-2}.

c The distance from A to B, $u=1$, $v=19$, $t=12$, $s=?$

$$s=\left(\frac{u+v}{2}\right)t=\left(\frac{1+19}{2}\right)\times 12=10\times 12=120$$

The distance from B to C, $u=19$, $v=43$, $t=10$, $s=?$

$$s=\left(\frac{u+v}{2}\right)t=\left(\frac{19+43}{2}\right)\times 10=31\times 10=310$$

The distance from A to C is (120 + 310) = 430 m.

12 a $u=0$, $v=5$, $t=20$, $a=x$

$$v=u+at$$
$$5=0+20x$$
$$x=\tfrac{5}{20}=0.25$$

b While accelerating, $u=0$, $v=5$, $t=20$, $s=?$

$$s=\left(\frac{u+v}{2}\right)t=\left(\frac{0+5}{2}\right)\times 20=2.5\times 20=50$$

12 b While decelerating, $u = 5,\ v = 0,\ a = -\frac{1}{2}, x = -0.125,\ t = ?$

$$\begin{aligned} v &= u + at \\ 0 &= 5 - 0.125t \\ t &= \frac{5}{0.125} = 40 \end{aligned}$$

Now, $u = 5,\ v = 0,\ t = 40,\ s = ?$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{5+0}{2}\right) \times 40 = 2.5 \times 40 = 100$$

The total distance travelled is the distance travelled while accelerating added to the distance travelled while decelerating = (50 + 100) = 150 m.

13 a From A to B

$$\begin{aligned} v &= u + at \\ 30 &= 20 + at_1 \\ at_1 &= 10 \qquad (\mathbf{1}) \end{aligned}$$

From B to C

$$\begin{aligned} v &= u + at \\ 45 &= 30 + at_2 \\ at_2 &= 15 \qquad (\mathbf{2}) \end{aligned}$$

Dividing (**1**) by (**2**),

$$\frac{at_1}{at_2} = \frac{10}{15}$$

$$\frac{t_1}{t_2} = \frac{2}{3} \quad \text{as required.}$$

b From the result in part **a**

$$\begin{aligned} t_2 &= \tfrac{3}{2}t_1 \\ t_1 + t_2 &= t_1 + \tfrac{3}{2}t_1 = \tfrac{5}{2}t_1 = 50 \\ t_1 &= \tfrac{2}{5} \times 50 = 20 \end{aligned}$$

From A to B, $u = 20,\ v = 30,\ t = 20,\ s = ?$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{20+30}{2}\right) \times 20 = 25 \times 20 = 500$$

The distance from A to B is 500 m.

Challenge

a Distance s is the same for both particles: AB.

For the first particle: $u = 3$, $v = 5$**,** time taken is t seconds

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{3+5}{2}\right)t = 4t \qquad \textbf{(1)}$$

For the second particle: $u = 4$, $v = 8$, time taken is $(t - 1)$ seconds, because the particle starts 1 second later than the first and arrives at the same time)

$$s = \left(\frac{u+v}{2}\right)(t-1) = \left(\frac{4+8}{2}\right)(t-1) = 6(t-1) = 6t-6 \qquad \textbf{(2)}$$

$4t = 6t - 6$ (**1**) and (**2**)

$t = 3$

The time for the first particle to get from A to B is 3 s.

b Substituting this value of t into equation (**1**):

$s = 4t = 4 \times 3 = 12$

The distance between A and B is 12 m.

[Check by substituting into equation (**2**): $s = 6t - 6 = 6 \times 3 - 6 = 12$]

Constant acceleration 9D

1 $a = 2.5,\ u = 3,\ s = 8,\ v = ?$

$$v^2 = u^2 + 2as = 3^2 + 2 \times 2.5 \times 8 = 9 + 40 = 49$$
$$v = \sqrt{49} = 7$$

The velocity of the particle as it passes through B is $7\,\mathrm{m\,s^{-1}}$.

2 $u = 8,\ t = 6,\ s = 60,\ a = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$60 = 8 \times 6 + \tfrac{1}{2} \times a \times 6^2 = 48 + 18a$$
$$a = \frac{60 - 48}{18} = \frac{2}{3}$$

The acceleration of the car is $0.667\,\mathrm{m\,s^{-2}}$ (to 3 s.f.)

3 $u = 12,\ v = 0,\ s = 36,\ a = ?$

$$v^2 = u^2 + 2as$$
$$0^2 = 12^2 + 2 \times a \times 36 = 144 + 72a$$
$$a = -\tfrac{144}{72} = -2$$

The deceleration is $2\ \mathrm{m\,s^{-2}}$.

4 $u = 15,\ v = 20,\ s = 500,\ a = ?$ $\quad 54\,\mathrm{km\,h^{-1}} = \dfrac{54 \times 1000}{3600}\mathrm{m\,s^{-1}} = 15\,\mathrm{m\,s^{-1}}$

$$72\,\mathrm{km\,h^{-1}} = \frac{72 \times 1000}{3600}\mathrm{m\,s^{-1}} = 20\,\mathrm{m\,s^{-1}}$$

$$v^2 = u^2 + 2as$$
$$20^2 = 15^2 + 2 \times a \times 500$$
$$400 = 225 + 1000a$$
$$a = \frac{400 - 225}{1000} = 0.175$$

The acceleration of the train is $0.175\,\mathrm{m\,s^{-2}}$.

5 a $s = 48,\ u = 4,\ v = 16,\ a = ?$

$v^2 = u^2 + 2as$

$16^2 = 4^2 + 2 \times a \times 48$

$256 = 16 + 96a$

$a = \frac{256 - 16}{96} = 2.5$

The acceleration of the particle is $2.5\,\mathrm{m\,s^{-2}}$.

b $u = 4,\ v = 16,\ a = 2.5,\ t = ?$

$v = u + at$

$16 = 4 + 2.5t$

$t = \frac{16 - 4}{2.5} = 4.8$

The time taken to move from A to B is 4.8 s.

6 a $a = 3, s = 38, t = 4, u = ?$

$s = ut + \frac{1}{2}at^2$

$38 = 4u + \frac{1}{2} \times 3 \times 4^2 = 4u + 24$

$u = \frac{38 - 24}{4} = 3.5$

The initial velocity of the particle is $3.5\,\mathrm{m\,s^{-1}}$.

b $a = 3, t = 4, u = 3.5, v = ?$

$v = u + at = 3.5 + 3 \times 4 = 15.5$

The final velocity of the particle is $15.5\,\mathrm{m\,s^{-1}}$.

7 a $u = 18,\ v = 0,\ a = -3,\ s = ?$

$v^2 = u^2 + 2as$

$0^2 = 18^2 + 2 \times (-3) \times s = 324 - 6s$

$s = \frac{324}{6} = 54$

The distance travelled as the car decelerates is 54 m.

7 b $u = 18,\ v = 0,\ a = -3,\ t = ?$

$$v = u + at$$
$$0 = 18 - 3t$$
$$t = \tfrac{18}{3} = 6$$

The time taken for the car to decelerate is 6 s.

8 a $u = 12,\ v = 0,\ a = -0.8,\ s = ?$

$$v^2 = u^2 + 2as$$
$$0^2 = 12^2 + 2\times(-0.8)\times s = 144 - 1.6s$$
$$s = \tfrac{144}{1.6} = 90$$

The distance moved by the stone is 90 m.

b Half the distance in **a** is 45 m.

$u = 12,\ a = -0.8,\ s = 45,\ v = ?$

$$v^2 = u^2 + 2as$$
$$= 12^2 + 2\times(-0.8)\times 45 = 144 - 72 = 72$$
$$v = \sqrt{72} = 8.49 \text{ (to 3 s.f.)}$$

The speed of the stone is $8.49\,\text{m}\,\text{s}^{-1}$.

9 a $a = 2.5,\ u = 8,\ s = 40,\ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$40 = 8t + 1.25t^2$$
$$0 = 1.25t^2 + 8t - 40$$
$$t = \frac{-(8) \pm \sqrt{(8)^2 - 4\times(1.25)\times(-40)}}{2\times(1.25)}$$
$$t = \frac{-8 + \sqrt{264}}{2.5} = 3.30 \text{ (to 3 s.f.)}$$

The time taken for the particle to move from O to A is 3.30 s.

b $a = 2.5,\ u = 8,\ s = 40,\ v = ?$

$$v^2 = u^2 + 2as$$
$$= 8^2 + 2\times 2.5\times 40 = 264$$
$$v = \sqrt{264} = 16.2 \text{ (to 3 s.f.)}$$

The speed of the particle at A is $16.2\,\text{m}\,\text{s}^{-1}$.

10 a $a=-2,\ s=32,\ u=12,\ t=?$

$$\begin{aligned} s &= ut+\tfrac{1}{2}at^2 \\ 32 &= 12t-t^2 \\ t^2-12t+32 &= (t-4)(t-8)=0 \end{aligned}$$

So $t = 4$ or $t = 8$.

b When $t = 4$,

$v=u+at=12-2\times 4=4$

The velocity is $4\,\mathrm{m\,s^{-1}}$ in the direction $\overrightarrow{AB}$.

When $t = 8$,

$v=u+at=12-2\times 8=-4$

The velocity is $4\,\mathrm{m\,s^{-1}}$ in the direction $\overrightarrow{BA}$.

11 a $a=-5,\ u=12,\ s=8,\ t=?$

$$\begin{aligned} s &= ut+\tfrac{1}{2}at^2 \\ 8 &= 12t-2.5t^2 \\ 2.5t^2-12t+8 &= 0 \\ 5t^2-24t+16 &= (5t-4)(t-4)=0 \end{aligned}$$

So $t = 0.8$ or $t = 4$.

b $a=-5,\ u=12,\ s=-8,\ v=?$

$$\begin{aligned} v^2 &= u^2+2as \\ &= 12^2+2\times(-5)\times(-8) \\ &= 144+80=224 \\ v &= \sqrt{224}=-15.0 \text{ (to 3 s.f.)} \end{aligned}$$

The velocity at $x=-8$ is $-15.0\,\mathrm{m\ s^{-1}}$.

12 a $a=-4,\ u=14,\ s=22.5,\ t=?$

$$\begin{aligned} s &= ut+\tfrac{1}{2}at^2 \\ 22.5 &= 14t-2t^2 \\ 2t^2-14t+22.5 &= 0 \\ 4t^2-28t+45 &= (2t-5)(2t-9)=0 \end{aligned}$$

The difference between the times is $(4.5-2.5)\ \mathrm{s}=2\ \mathrm{s}$.

12 b The maximum distance is reached when P reverses direction.
$a = -4,\ u = 14,\ v = 0,\ t = ?$

$v = u + at$

$0 = 14 - 4t \Rightarrow t = \frac{14}{4} = 3.5$

Find the displacement when $t = 3.5$.

$$s = ut + \tfrac{1}{2}at^2$$
$$= 14 \times 3.5 - 2 \times 3.5^2 = 24.5$$

Between $t = 2.5$ and $t = 4.5$ the particle moves back and forward.

Hence total distance travelled $= 2 \times (24.5 - 22.5)$ m $= 4$ m.

13 a From B to C, $u = 14,\ v = 20,\ s = 300,\ a = ?$

$$v^2 = u^2 + 2as$$
$$20^2 = 14^2 + 2 \times a \times 300$$
$$a = \frac{20^2 - 14^2}{600} = 0.34$$

The acceleration of the car is 0.34 m s^{-2}.

b From A to C, $v = 20,\ s = 400,\ a = 0.34,\ u = ?$

$$v^2 = u^2 + 2as$$
$$20^2 = u^2 + 2 \times 0.34 \times 400 = u^2 + 272$$
$$u^2 = 400 - 272 = 128$$
$$u = \pm\sqrt{128} = \pm 8\sqrt{2}$$

Assuming the car is not in reverse at A, $u = +8\sqrt{2}$

$$v = u + at$$
$$20 = 8\sqrt{2} + 0.34t$$
$$t = \frac{20 - 8\sqrt{2}}{0.34} = 25.5 \text{ (to 3 s.f.)}$$

The time taken for the car to travel from A to C is 25.5 s.

14 a For P, $a = 2$, $u = 4$

$$s = ut + \tfrac{1}{2}at^2$$
$$= 4t + \tfrac{1}{2} \times 2t^2 = 4t + t^2$$

The displacement of P is $(4t + t^2)$ m.

For Q, $a = 3.6$, $u = 3$

Q has been moving for $(t - 1)$ seconds since passing through A, so

$$s = u(t-1) + \tfrac{1}{2}a(t-1)^2$$
$$= 3(t-1) + 1.8(t-1)^2 = 1.8t^2 - 0.6t - 1.2$$

The displacement of Q is $(1.8t^2 - 0.6t - 1.2)$ m.

b P and Q meet when $s_P = s_Q$, so, from **a**:

$$4t + t^2 = 1.8t^2 - 0.6t - 1.2$$
$$0.8t^2 - 4.6t - 1.2 = 0$$

Divide throughout by 0.2:

$$4t^2 - 23t - 6 = 0$$
$$(t-6)(4t+1) = 0$$

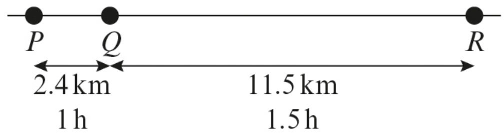
Rejecting a negative solution for time, $t = 6$.

c Substitute $t = 6$ into the equation for one of the displacements (here P):

$$s = 4t + t^2 = 4 \times 6 + 6^2 = 60$$

The distance of A from the point where the particles meet is 60 m.

15



a Let the velocity as the competitor passes point Q be v_Q

For PQ, $s = 2.4$, $t = 1$, $v = v_Q$

$$s = vt - \tfrac{1}{2}at^2$$
$$2.4 = v_Q \times 1 - \tfrac{1}{2}(a \times 1^2) = v_Q - \tfrac{1}{2}a$$
$$v_Q = 2.4 + 0.5a$$

For QR, $s = 11.5$, $t = 1.5$, $u = v_Q$

$$s = ut + \tfrac{1}{2}at^2$$
$$11.5 = v_Q \times 1.5 + \tfrac{1}{2}a \times 1.5^2 = 1.5v_Q + 1.125a$$

Substituting for v_Q:

$$11.5 = 1.5(2.4 + 0.5a) + 1.125a$$
$$= 3.6 + 0.75a + 1.125a$$
$$11.5 - 3.6 = (0.75 + 1.125)a$$
$$a = \frac{11.5 - 3.6}{0.75 + 1.125} = \frac{7.9}{1.875} = 4.21 \text{ (to 3 s.f.)}$$

The acceleration is 4.21 km h^{-2}.

$$4.21 \text{ km h}^{-2} = \frac{4.21 \times 1000}{3600 \times 3600} \text{ m s}^{-2} = 3.25 \times 10^{-4} \text{ m s}^{-2} \text{ (to 3 s.f.)}$$

So her acceleration is 3.25 x 10^{-4} m s^{-2}.

b For PQ, $s = 2.4$, $t = 1$, $a = 4.21$, $u = ?$, using exact figures

$$s = ut + \tfrac{1}{2}at^2$$
$$2.4 = u \times 1 + \tfrac{1}{2} \times \frac{7.9}{1.875} \times 1^2$$
$$u = 0.293 \text{ (to 3 s.f.)}$$

$$0.293 \text{ km h}^{-1} = \frac{0.293 \times 1000}{3600} \text{ m s}^{-1} = 0.0815 \text{ m s}^{-1} \text{ (to 3 s.f.)}$$

Constant acceleration 9E

1 a Take downwards as the positive direction.

$s = 28$, $u = 0$, $a = 9.8$, $t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$28 = 0 \times t + \tfrac{1}{2} \times 9.8 \times t^2 = 4.9t^2$$
$$t = \sqrt{\frac{28}{4.9}} = 2.4 \text{ (to 2 s.f.)}$$

The time taken for the diver to hit the water is 2.4 s.

b $v^2 = u^2 + 2as$
$$v^2 = 0 + 2 \times 9.8 \times 28 = 548.8$$
$$v = \sqrt{548.8} = 32.4 \text{ (to 3 s.f.)}$$

When the diver hits the water, he is travelling at 32.4 m s^{-1}.

2 Take upwards as the positive direction.

$u = 20,\ a = -9.8,\ s = 0,\ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$0 = 20t - 4.9t^2 = t(20 - 4.9t), \quad t \neq 0$$
$$t = \tfrac{20}{4.9} = 4.1 \text{ (to 2 s.f.)}$$

The time of flight of the particle is 4.1 s.

3 Take downwards as the positive direction.

$u = 18,\ a = 9.8,\ t = 1.6,\ s = ?$

$$s = ut + \tfrac{1}{2}at^2 = 18 \times 1.6 + 4.9 \times 1.6^2 = 41 \text{ (to 2 s.f.)}$$

The height of the tower is 41 m.

4 a Take upwards as the positive direction.

$u = 24,\ a = -9.8,\ v = 0,\ s = ?$

$$v^2 = u^2 + 2as$$
$$0^2 = 24^2 - 2 \times 9.8 \times s$$
$$s = \frac{24^2}{2 \times 9.8} = 29 \text{ (to 2 s.f.)}$$

The greatest height reached by the pebble above the point of projection is 29 m.

4 b $u = 24,\ a = -9.8,\ v = 0,\ t = ?$

$v = u + at$
$0 = 24 - 9.8t$
$t = \frac{24}{9.8} = 2.4$ (to 2 s.f.)

The time taken to reach the greatest height is 2.4 s.

5 a Take upwards as the positive direction.

$u = 18,\ a = -9.8,\ s = 15,\ v = ?$

$v^2 = u^2 + 2as = 18^2 - 2 \times 9.8 \times 15 = 30$
$v = \sqrt{30} = \pm 5.5$ (to 2 s.f.)

The speed of the ball when it is 15 m above its point of projection is 5.5 m s^{-1}.

b $u = 18,\ a = -9.8,\ s = -4,\ v = ?$

$v^2 = u^2 + 2as = 18^2 + 2 \times (-9.8) \times (-4) = 324 + 78.4 = 402.4$
$v = -\sqrt{402.2} = -20$ (to 2 s.f.)

The speed with which the ball hits the ground is 20 m s^{-1}.

6 a Take downwards as the positive direction.

$s = 80,\ u = 4,\ a = 9.8,\ v = ?$

$v^2 = u^2 + 2as$
$= 4^2 + 2 \times 9.8 \times 80 = 1584$
$v = \sqrt{1584} = 40$ (to 2 s.f.)

The speed with which P hits the ground is 40 m s^{-1}.

b $u = 4,\ a = 9.8,\ v = \sqrt{1584},\ t = ?$

$v = u + at$
$\sqrt{1584} = 4 + 9.8t$
$t = \dfrac{\sqrt{1584} - 4}{9.8} = 3.7$ (to 2 s.f.)

The time P takes to reach the ground is 3.7 s.

7 a Take upwards as the positive direction.

$v = -10,\ a = -9.8,\ t = 5,\ u = ?$

$$\begin{aligned} v &= u + at \\ -10 &= u - 9.8 \times 5 \\ u &= 9.8 \times 5 - 10 = 39 \end{aligned}$$

The speed of projection of P is 39 m s^{-1}.

b $u = 39,\ v = 0,\ a = -9.8,\ s = ?$

$$\begin{aligned} v^2 &= u^2 + 2as \\ 0^2 &= 39^2 - 2 \times 9.8 \times s \\ s &= \frac{1521}{2 \times 9.8} = 78 \text{ (to 2 s.f.)} \end{aligned}$$

The greatest height above X attained by P during its motion is 78 m.

8 Take upwards as the positive direction.

$u = 21,\ t = 4.5,\ a = -9.8,\ s = ?$

$$s = ut + \tfrac{1}{2}at^2 = 21 \times 4.5 - 4.9 \times 4.5^2 = -4.7 \text{ (to 2 s.f.)}$$

The height above the ground from which the ball was thrown is 4.7 m.

9 Take upwards as the positive direction.

Find time when stone is instantaneously stationery:

$v = 0,\ u = 16,\ a = -9.8,\ t = ?$

$v = u + at$

$0 = 16 - 9.8t$

$t = \frac{16}{9.8} = 16.326... = 1.6$ s (to 1 d.p.)

So the stone is instantaneously stationary at 1.6 s

Find time of flight:

$s = -3,\ u = 16,\ a = -9.8,\ t = ?$

$s = ut + \frac{1}{2}at^2$

$-3 = 16t - 4.9t^2$

$4.9t^2 - 16t - 3 = 0$, so using the quadratic formula,

$$t = \frac{-(-16) \pm \sqrt{(-16)^2 - 4 \times (4.9) \times (-3)}}{2 \times (4.9)}$$

$t = 3.4431... = 3.4$ (to 1 d.p.) as we may discount the negative answer.

So the time of flight of the stone is 3.4 s.

Find speed when stone hits the ground:

$v = ?,\ u = 16,\ a = -9.8,\ t = 3.4431...$

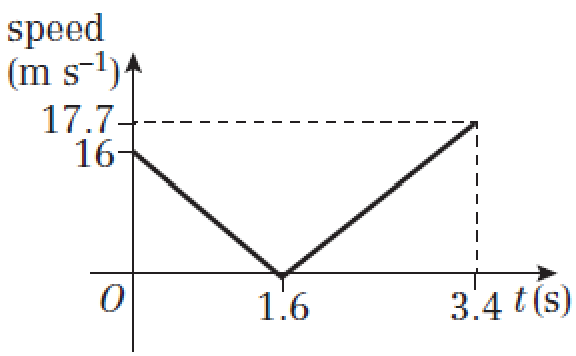
$v = u + at$

$v = 16 - 9.8 \times 3.4431...$

$v = -17.74... = -17.7$ ms^{-1} (to 1 d.p.)

So speed when stone hits the ground is 17.7 ms^{-1}

Sketch speed-time graph



10 Take upwards as the positive direction.

$u = 24.5,\ a = -9.8,\ s = 21,\ t = ?$

$s = ut + \frac{1}{2}at^2$
$21 = 24.5t - 4.9t^2$

$4.9t^2 - 24.5t + 21 = 0$

Using the quadratic formula,

$$t = \frac{-(-24.5) \pm \sqrt{(-24.5)^2 - 4\times(4.9)\times(21)}}{2\times(4.9)}$$

$= 1.1 \text{ or } 3.9$

The difference between these times is
$(3.9 - 1.1)\text{ s} = 2.8\text{ s}$

The total time for which the particle is 21 m or more above its point of projection is 2.8 s.

11 a Take upwards as the positive direction.

$v = \frac{1}{3}u,\ a = -9.8,\ t = 2,\ u = ?$

$v = u + at$
$\frac{1}{3}u = u - 9.8\times 2$
$\frac{2}{3}u = 19.6 \Rightarrow u = \frac{3}{2}\times 19.6 = 29.4$

$u = 29$ (to 2 s.f.)

b $u = 29.4,\ s = 0,\ a = -9.8,\ t = ?$

$s = ut + \frac{1}{2}at^2$
$0 = 29.4t - 4.9t^2 = t(29.4 - 4.9t),\ t \neq 0$
$t = \frac{29.4}{4.9} = 6$

The time from the instant that the particle leaves O to the instant that it returns to O is 6 s.

12 For A, take downwards as the positive direction, $s_A = ut + \frac{1}{2}at^2 = 5t + 4.9t^2$

For B, take upwards as the positive direction, $s_B = ut + \frac{1}{2}at^2 = 18t - 4.9t^2$

$s_A + s_B = 46$

$(5t + 4.9t^2) + (18t - 4.9t^2) = 46$

$23t = 46 \Rightarrow t = 2$

Substitute $t = 2$ into $s_A = 5t + 4.9t^2$

$s_A = 5 \times 2 + 4.9 \times 2^2 = 29.6 = 30$ (to 2 s.f.)

The distance of the point where A and B collide from the point where A was thrown is 30 m.

13 a Find the speed, u_1 say, immediately before the ball strikes the floor.

$u = 0,\ a = 9.8,\ s = 10,\ v = u_1$
$v^2 = u^2 + 2as$
$u_1^2 = 0^2 + 2 \times 9.8 \times 10 = 196$
$u_1 = \sqrt{196} = 14$

The speed of the first rebound, u_2 say, is given by

$u_2 = \frac{3}{4}u_1 = \frac{3}{4} \times 14 = 10.5$

Find the maximum height, h_1 say, reached after the first rebound.

$u = 10.5,\ v = 0,\ a = -9.8,\ s = h_1$

$v^2 = u^2 + 2as$
$0^2 = 10.5^2 - 2 \times 9.8 \times h_1$

13 a $h_1 = \dfrac{10.5^2}{2\times 9.8} = 5.6$ (to 2 s.f.)

The greatest height above the floor reached by the ball the first time it rebounds is 5.6 m.

b Immediately before the ball strikes the floor for the second time, its speed is again $u_2 = 10.5$ by symmetry. The speed of the second rebound, u_3 say, is given by

$$u_3 = \tfrac{3}{4}u_2 = \tfrac{3}{4}\times 10.5 = 7.875$$

Find the maximum height, h_2 say, reached after the second rebound.

$u = 7.875,\ v = 0,\ a = -9.8,\ s = h_2$

$$v^2 = u^2 + 2as$$
$$0^2 = 7.875^2 - 2\times 9.8\times h_2$$
$$h_2 = \frac{7.875^2}{2\times 9.8} = 3.2 \text{ (to 2 s.f.)}$$

The greatest height above the floor reached by the ball the second time it rebounds is 3.2 m.

Challenge

1 a Take upwards as the positive direction.

For P, $s = ut + \tfrac{1}{2}at^2$ gives $s_P = 12t - 4.9t^2$

For Q, $s = ut + \tfrac{1}{2}at^2$

Q has been moving for 1 less second than P, so

$$s_Q = 20(t-1) - 4.9(t-1)^2$$

At the point of collision $s_P = s_Q$

$$\begin{aligned} 12t - 4.9t^2 &= 20(t-1) - 4.9(t-1)^2 \\ &= 20t - 20 - 4.9t^2 + 9.8t - 4.9 \end{aligned}$$

$24.9 = 17.8t \Rightarrow t = \tfrac{24.9}{17.8} = 1.4$ (to 2 s.f.)

The time between the instant when P is projected and the instant when P and Q collide is 1.4 s.

Challenge

1 b Substitute t into $s_P = 12t - 4.9t^2$ from part **a**

$s_P = 12t - 4.9t^2 \approx 12 \times 1.4 - 4.9 \times 1.4^2 = 7.2$ (to 2 s.f.)

The distance of the point where P and Q collide from O is 7.2 m.

2 Take downwards as positive.

For 1st stone: $u = 0$, $t = t_1$, $a = 9.8$, $s = h$

$s = ut + \frac{1}{2}at^2$

$h = 0 \times t_1 + \frac{1}{2} \times 9.8 \times t_1^2 = 4.9t_1^2$

For 2nd stone: $u = 25$, $t = t_1 - 2$, $a = 9.8$, $s = h$

$s = ut + \frac{1}{2}at^2$

$$\begin{aligned} h &= 25(t_1 - 2) + \tfrac{1}{2}(9.8 \times (t_1 - 2)^2) \\ &= 25t_1 - 50 + 4.9 \times (t_1^2 - 4t_1 + 4)) \\ &= 25t_1 - 50 + 4.9t_1^2 - 19.6t_1 + 19.6 \\ &= 4.9t_1^2 + 5.4t_1 - 30.4 \end{aligned}$$

Substituting for h from information for first stone:

$$\begin{aligned} 4.9t_1^2 &= 4.9t_1^2 + 5.4t_1 - 30.4 \\ 30.4 &= 5.4t_1 \\ t_1 &= \frac{30.4}{5.4} = 5.629 \end{aligned}$$

Putting this value into equation for first stone:

$h = 4.9 \times 5.629^2 = 4.9 \times 31.69 = 155$ (to 3 s.f.)

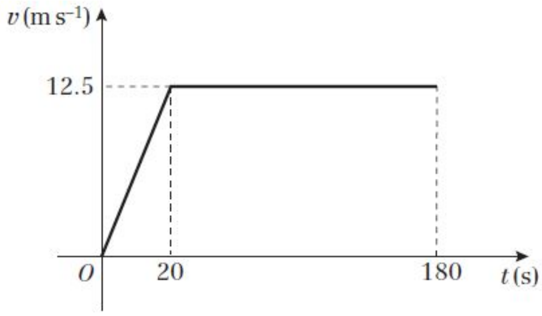
The height of the building is 155 m.

Constant acceleration, Mixed Exercise 9

1 a $45 \text{ km h}^{-1} = \dfrac{45 \times 1000}{3600} \text{ m s}^{-1}$

$= 12.5 \text{ m s}^{-1}$

$3 \text{ min} = 180 \text{ s}$

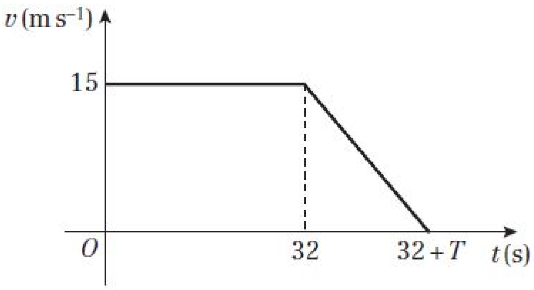


b $s = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(160+180) \times 12.5 = 2125$

The distance from A to B is 2125 m.

2 a



b $s = \frac{1}{2}(a+b)h$

$570 = \frac{1}{2}(32+32+T) \times 15$

$\frac{15}{2}(T+64) = 570$

$T + 64 = \dfrac{570 \times 2}{15} = 76$

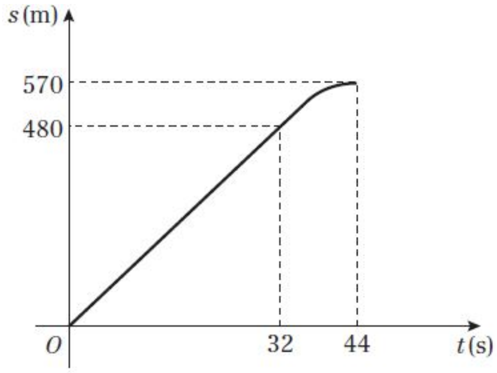
$T = 76 - 64 = 12$

c At $t = 32$, $s = 32 \times 15 = 480$

At $t = 44$, $s = 480 + \text{area of the triangle}$

$= 480 + \frac{1}{2} \times 12 \times 15 = 570$

2 c



3 a i Gradient of line = $\frac{v-u}{t}$

$a = \frac{v-u}{t}$

Rearranging: $v = u + at$

ii Shaded area is a trapezium

area = $\left(\frac{u+v}{2}\right)t$

$s = \left(\frac{u+v}{2}\right)t$

b i Rearrange $v = u + at$

$t = \frac{v-u}{a}$

Substitute into $s = \left(\frac{u+v}{2}\right)t$

$s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$

$2as = v^2 - u^2$

$v^2 = u^2 + 2as$

ii Substitute $v = u + at$ into $s = \left(\frac{u+v}{2}\right)t$

$s = \left(\frac{u+u+at}{2}\right)t$

$s = \left(\frac{2u}{2} + \frac{at}{2}\right)t$

$s = ut + \frac{1}{2}at^2$

3 b iii Substitute $u = v - at$ into $s = ut + \frac{1}{2}at^2$

$$s = (v - at)t + \tfrac{1}{2}at^2$$
$$s = vt - \tfrac{1}{2}at^2$$

4 $s = \frac{1}{2}(a+b)h$

$$152 = \tfrac{1}{2}(15+23)u = 19u$$
$$u = \frac{152}{19} = 8$$

5 $40 \text{ km h}^{-1} = \dfrac{40 \times 1000}{3600} \text{ m s}^{-1} = \dfrac{100}{9} \text{ m s}^{-1}$

$24 \text{ km h}^{-1} = \dfrac{24 \times 1000}{3600} \text{ m s}^{-1} = \dfrac{20}{3} \text{ m s}^{-1}$

$u = \frac{100}{9},\ v = \frac{20}{3},\ s = 240,\ a = ?$

$$v^2 = u^2 + 2as$$
$$\left(\tfrac{20}{3}\right)^2 = \left(\tfrac{100}{9}\right)^2 + 2 \times a \times 240$$
$$a = \frac{\left(\frac{20}{3}\right)^2 - \left(\frac{100}{9}\right)^2}{2 \times 240} = -0.165 \text{ (to 2 s.f.)}$$

The deceleration of the car is 0.165 m s^{-2}.

6 a $a = -2.5,\ u = 20,\ t = 12,\ s = ?$

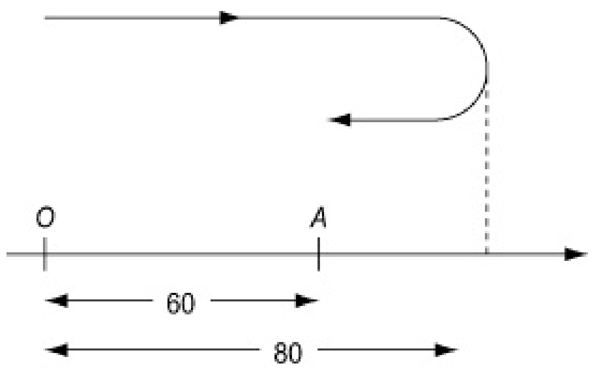
$$\begin{aligned} s &= ut + \tfrac{1}{2}at^2 \\ &= 20 \times 12 - \tfrac{1}{2} \times 2.5 \times 12^2 \\ &= 240 - 180 = 60 \end{aligned}$$

$OA = 60$ m

b The particle will turn round when $v = 0$

$a = -2.5,\ u = 20,\ v = 0,\ s = ?$

$$v^2 = u^2 + 2as$$
$$0^2 = 20^2 - 5s \Rightarrow s = 80$$



The total distance P travels is $(80 + 20)$ m $= 100$ m

7 $u = 6,\ v = 25,\ a = 9.8,\ t = ?$

$$v = u + at$$
$$25 = 6 + 9.8t$$
$$t = \frac{25-6}{9.8} = 1.9 \text{ (to 2 s.f.)}$$

The ball takes 1.9 s to move from the top of the tower to the ground.

8 Take downwards as the positive direction.

a $u = 0,\ s = 82,\ a = 9.8,\ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$82 = 0 + 4.9t^2$$
$$t = \sqrt{\frac{82}{4.9}} = 4.1 \text{ (to 2 s.f.)}$$

The time taken for the ball to reach the sea is 4.1 s.

b $u = 0,\ s = 82,\ a = 9.8,\ v = ?$

$$v^2 = u^2 + 2as$$
$$= 0 + 2\times 9.8\times 82 = 1607.2$$
$$v = \sqrt{1607.2} = 40 \text{ (to 2 s.f.)}$$

The speed at which the ball hits the sea is 40 m s^{-1}.

c Air resistance/wind/turbulence

9 **a** distance = area of triangle + area of rectangle + area of trapezium

$$451 = \tfrac{1}{2}\times 8\times 2u + 12\times 2u + \tfrac{1}{2}\times(u+2u)\times 6$$
$$= 8u + 24u + 9u = 41u$$
$$u = \tfrac{451}{41} = 11$$

b The particle is moving with speed less than u m s^{-1} for the first 4 s

$$s = \tfrac{1}{2}\times 4\times 11 = 22$$

The distance moved with speed less than u m s^{-1} is 22 m.

10 a From O to P, $u=18,\ t=12,\ v=24,\ a=?$

$u=18,\ t=12,\ v=24,\ a=?$

$$v=u+at$$
$$24=18+12a$$
$$a=\frac{24-18}{12}=\frac{1}{2}$$

From O to Q, $u=18,\ t=20,\ a=\frac{1}{2},\ v=?$

$$v=u+at$$
$$=18+\tfrac{1}{2}\times 20=28$$

The speed of the train at Q is 28 m s^{-1}.

b From P to Q

$u=24,\ v=28,\ t=8,\ s=?$

$$s=\left(\frac{u+v}{2}\right)t=\left(\frac{24+28}{2}\right)\times 8=208$$

The distance from P to Q is 208 m.

11 a $s=104,\ t=8,\ v=18,\ u=?$

$$s=\left(\frac{u+v}{2}\right)t$$
$$104=\left(\frac{u+18}{2}\right)\times 8=(u+18)\times 4=4u+72$$
$$u=\frac{104-72}{4}=8$$

The speed of the particle at X is 8 m s^{-1}

b $s=104,\ t=8,\ v=18,\ a=?$

$$s=vt-\tfrac{1}{2}at^2$$
$$104=18\times 8-\tfrac{1}{2}a\times 8^2=144-32a$$
$$a=\frac{144-104}{32}=1.25$$

The acceleration of the particle is 1.25 m s^{-2}.

11 c From X to Z, $u = 8,\ v = 24,\ a = 1.25,\ s = ?$

$$v^2 = u^2 + 2as$$
$$24^2 = 8^2 + 2 \times 1.25 \times s$$
$$s = \frac{24^2 - 8^2}{2.5} = 204.8$$

$XZ = 204.8$ m

12 a Take upwards as the positive direction.

$u = 21,\ s = -32,\ a = -9.8,\ v = ?$

$$v^2 = u^2 + 2as$$
$$= 21^2 + 2 \times (-9.8) \times (-32) = 441 + 627.2 = 1068.2$$
$$v = \sqrt{1068.2} = \pm 33 \text{ (to 2 s.f.)}$$

The velocity with which the pebble strikes the ground is -33 m s^{-1}.
The speed is 33 m s^{-1}.

b 40 m above the ground is 8 m above the point of projection.

$u = 21,\ s = 8,\ a = -9.8,\ t = ?$
$s = ut + \frac{1}{2}at^2$
$8 = 21t - 4.9t^2$

$0 = 4.9t^2 - 21t + 8$, so using the quadratic formula,

$$t = \frac{21 \pm \sqrt{21^2 - 4 \times 4.9 \times 8}}{9.8} = \frac{21 \pm \sqrt{284.2}}{9.8} = 3.86,\ 0.423 \text{ (to 3 s.f.)}$$

The pebble is above 40 m between these times: $3.863... - 0.423... = 3.4$ (to 2 s.f.)
The pebble is more than 40 m above the ground for 3.4 s.

12 c Take upwards as the positive direction.

$u = 21,\ a = -9.8$

$v = u + at = 21 - 9.8t \Rightarrow t = \dfrac{21 - v}{9.8}$

From part **a**, the pebble hits the ground when $v = -33$.

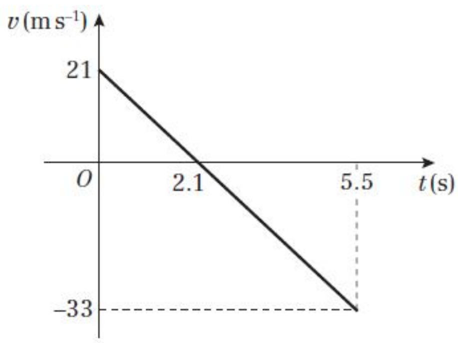
$t = \dfrac{21 - v}{9.8} = \dfrac{21 - (-33)}{9.8} = \dfrac{54}{9.8} = 5.5$ (to 2 s.f.)

This is shown on the graph at point $(5.5, -33)$

The graph crosses the t-axis when $v = 0$.

$t = \dfrac{21 - v}{9.8} = \dfrac{21 - 0}{9.8} = \dfrac{21}{9.8} = 2.1$ (to 2 s.f.)

So the graph passes through point $(2.1,\ 0)$



13 a $u = 12,\ v = 32,\ s = 1100,\ t = ?$

$$s = \left(\frac{u + v}{2}\right)t$$

$$1100 = \left(\frac{12 + 32}{2}\right)t = 22t \Rightarrow t = \frac{1100}{22} = 50$$

The time taken by the car to move from A to C is 50 s.

13 b Find a first.

From A to C, $u=12,\ v=32,\ t=50,\ a=?$

$$v=u+at$$
$$32=12+a\times 50$$
$$a=\frac{32-12}{50}=0.4$$

From A to B, $u=12,\ s=550,\ a=0.4,\ v=?$

$$v^2=u^2+2as$$
$$=12^2+2\times 0.4\times 550=584\Rightarrow v=24.2 \text{ (to 3 s.f.)}$$

The car passes B with speed 24.2 m s^{-1}.

14 Take upwards as the positive direction.

At the top:

$u=30,\ v=0,\ a=-9.8,\ t=?$

$$v=u+at$$
$$0=30-9.8t\Rightarrow t=\frac{30}{9.8}$$

The ball spends 2.4 seconds above h, thus (by symmetry) 1.2 seconds rising between h and the top.
So it passes h 1.2 seconds earlier, at $t=\frac{30}{9.8}-1.2=1.86$ (to 3 s.f.)

At h, $u=30,\ a=-9.8,\ t\approx 1.86, s=?$

$$s=ut+\tfrac{1}{2}at^2$$
$$=30\times 1.86+\tfrac{1}{2}(-9.8)\times 1.86^2=39 \text{ (to 2 s.f.)}$$

15 a $u=20,\ a=4,\ s=78,\ v=?$

$$v^2=u^2+2as$$
$$=20^2+2\times 4\times 78=1024$$
$$v=\sqrt{1024}=32$$

The speed of B when it has travelled 78 m is 32 m s^{-1}.

15 b Find time for B to reach the point 78 m from O.

$v = 32,\ u = 20,\ a = 4,\ t = ?$

$v = u + at$

$32 = 20 + 4t \Rightarrow t = \dfrac{32-20}{4} = 3$

For A, distance $=$ speed $\times$ time

$s = 30 \times 3 = 90$

The distance from O of A when B is 78 m from O is 90 m.

c At time t seconds, for A, $s = 30t$

for B, $s = ut + \frac{1}{2}at^2 = 20t + 2t^2$

On overtaking the distances are the same.

$20t + 2t^2 = 30t$

$t^2 - 5t = t(t-5) = 0$

$t = 5$ (at $t = 0$, A overtakes B)

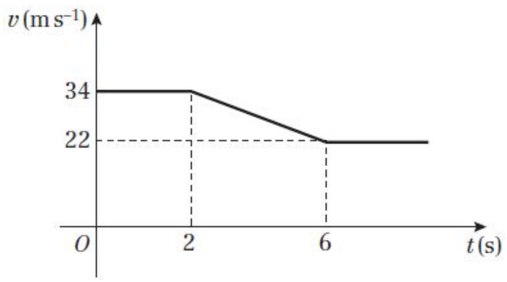
B overtakes A 5 s after passing O.

16 a To find time decelerating:

$u = 34,\ v = 22,\ a = -3,\ t = ?$

$v = u + at$

$22 = 34 - 3t \Rightarrow t = \dfrac{34-22}{3} = 4$

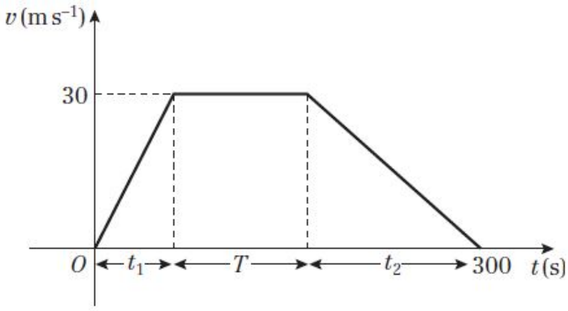


16 b distance = rectangle + trapezium

$$s = 34 \times 2 + \tfrac{1}{2}(22 + 34) \times 4$$
$$= 68 + 112 = 180$$

Distance required is 180 m.

17 a



b Acceleration is the gradient of a line.

For the first part of the journey, $3x = \dfrac{30}{t_1} \Rightarrow t_1 = \dfrac{30}{3x} = \dfrac{10}{x}$

For the last part of the journey, $-x = -\dfrac{30}{t_2} \Rightarrow t_2 = \dfrac{30}{x}$

$$t_1 + T + t_2 = 300$$

$\frac{10}{x} + T + \frac{30}{x} = 300 \Rightarrow \frac{40}{x} + T = 300,$ as required

c $s = \tfrac{1}{2}(a + b)h$

$$6000 = \tfrac{1}{2}(T + 300) \times 30 = 15T + 4500$$
$$T = \frac{6000 - 4500}{15} = 100$$

Substitute into the result in part **b**:

$$\tfrac{40}{x} + 100 = 300 \Rightarrow \tfrac{40}{x} = 200$$
$$x = \tfrac{40}{200} = 0.2$$

d From part **c**, $T = 100$

At constant velocity, distance = velocity × time $= 30 \times 100 = 3000$ (m)

The distance travelled at a constant speed is 3 km.

17 e From part **b**, $t_1 = \frac{10}{x} = \frac{10}{0.2} = 50$

Total distance travelled = 6 km (given) so halfway = 3 km = 3000 m

While accelerating, distance travelled is $\left(\frac{1}{2} \times 50 \times 30\right)$ m = 750 m.

At constant velocity, the train must travel a further 2250 m.

At constant velocity, time $= \dfrac{\text{distance}}{\text{velocity}} = \dfrac{2250}{30}$ s = 75 s

Time for train to reach halfway is $(50 + 75)$ s $= 125$ s

Challenge

Find the time taken by the first ball to reach 25 m below its point of projection (25 m above the ground). Take upwards as the positive direction.

$u = 10,\ s = -25,\ a = -9.8,\ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$-25 = 10t - 4.9t^2$$
$$0 = 4.9t^2 - 10t - 25$$
$$t = 10 \pm \frac{\sqrt{102 + 4 \times 4.9 \times 25}}{9.8}$$
$$= 3.5 \text{ (to 2 s.f.)}$$

As we discard the negative solution. Find the time taken by the second ball to reach 25 m below its point of projection. Take downwards as the positive direction.

$u = 0,\ s = 25,\ a = 9.8,\ t = ?$

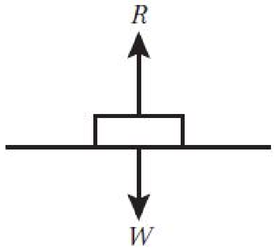
$$s = ut + \tfrac{1}{2}at^2$$
$$25 = 4.9t^2$$
$$t = 2.3 \text{ (to 2 s.f.)}$$

Combining the two results:

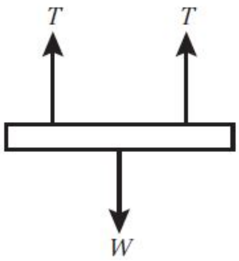
$T = 3.4989\ldots - 2.2587\ldots = 1.2$ (to 2 s.f. using exact figures)

Forces and motion 10A

1 R is the normal reaction of the table on the box.
W is the weight of the box.

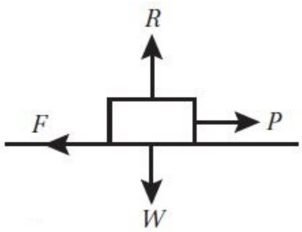


2 T is the tension in each of the ropes.
W is the weight of the bar.



3 W is the weight of the apple.

4 R is the normal reaction of the road on the car.
W is the weight of the car.
F is the sum of the frictional forces on the car.
P is the forward force produced by the car's engine.



5 W is the weight of the rescuer.
T is the tension in the rope.

6 Although its speed is constant, the satellite is continuously changing direction. This means the velocity changes. Therefore, there must be a resultant force acting on the satellite.

7 5 N

8 Since each particle is stationary, the overall force in each case is zero.

a Considering vertical forces:
$P - 10 = 0$
$P = 10$ N

8 b Considering horizontal forces only:
$P - 30 = 0$
$P = 30$ N

c Considering horizontal forces only:
$P + 1.5P - 50 = 0$
$2.5P = 50$
$P = 20$ N

9 a Since the platform is moving at constant velocity, the total vertical force is zero.
$T + T = 400$
$T = 200$
The tension in each rope is 200 N.

b If the tension in each rope is reduced by 50 N, there is a resultant downward force on the platform. It will therefore accelerate downward.

10 Since the particle is at rest, both horizontal and vertical forces must be balanced.
Considering horizontal forces only:
$p - 50 = 0$
$p = 50$
Considering vertical forces only:
$5q - (q + 10) - 3p = 0$
$4q - 10 - (3 \times 50) = 0$
$4q = 160$
$q = 40$
The values of p and q are 50 and 40 respectively.

11 Since the particle is moving with constant velocity, both horizontal and vertical forces must be balanced.
Considering horizontal forces only:
$2P + Q = 25$
$Q = 25 - 2P$
Considering vertical forces only:
$3P - 2Q = 20$
Substituting for Q:
$3P - 2 \times (25 - 2P) = 20$
$3P - 50 + 4P = 20$
$7P = 20 + 50 = 70$
$P = 10$ N
Using this value of P in the horizontal equation:
$(2 \times 10) + Q = 25$
$Q = 25 - 20 = 5$
$Q = 5$ N
P is 10 N and Q is 5 N.

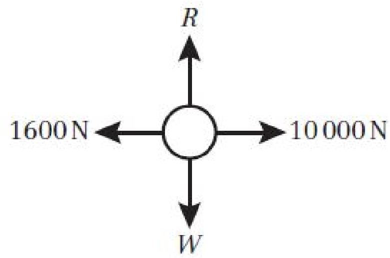
12 a i Overall horizontal force = $100 - 100 = 0$
Overall vertical force = $40 - 20 = 20$
The resultant force is 20 N upward.

ii The particle accelerates vertically upward.

12 b i Overall horizontal force = 25 – 5 = 20
Overall vertical force = 10 – 10 = 0
The resultant force is 20 N to the right.

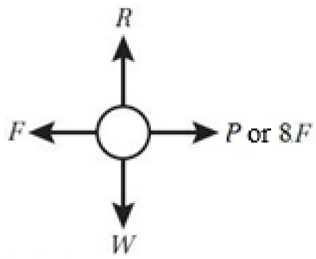
ii The particle accelerates to the right.

13 a R is the normal reaction of the road on the car.
W is the weight of the car.
The forward thrust of the car's engine acts to the right in the diagram.
The car is travelling to the right (positive direction).
The frictional forces on the car are acting to the left.

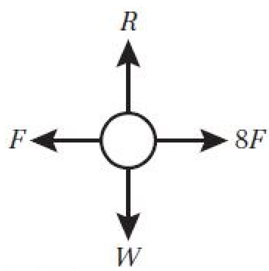


b Considering horizontal forces only:
Resultant force = 10 000 – 1600
There is no overall vertical force: R and W must be balanced, otherwise the car would lift off the road or sink into it.
The resultant force is 8400 N in the direction of travel.

14 a R is the normal reaction of the road on the car.
W is the weight of the car.
P is the driving force produced by the car's engine.
F is the resistance to the car's motion. or



b The magnitude of the driving force is eight times the magnitude of the resistance force, so



$P = 8F$

The resultant force is the difference between the forward force P and the resistance force F, so

$8F - F = 7F = 4200$

$$F = \frac{4200}{7} = 600$$

The magnitude of the resistance force is 600 N.

Forces and motion 10B

1 a $(-\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} - \mathbf{j}) = (3\mathbf{i} + 2\mathbf{j})$
The resultant force is $(3\mathbf{i} + 2\mathbf{j})$ N.

b $\begin{pmatrix}5\\3\end{pmatrix}+\begin{pmatrix}-3\\-6\end{pmatrix}=\begin{pmatrix}2\\-3\end{pmatrix}$

The resultant force is $\begin{pmatrix}2\\-3\end{pmatrix}$ N.

c $(\mathbf{i} + \mathbf{j}) + (5\mathbf{i} - 3\mathbf{j}) + (-2\mathbf{i} - \mathbf{j}) = (4\mathbf{i} - 3\mathbf{j})$
The resultant force is $(4\mathbf{i} - 3\mathbf{j})$ N.

d $\begin{pmatrix}-1\\4\end{pmatrix}+\begin{pmatrix}6\\0\end{pmatrix}+\begin{pmatrix}-2\\-7\end{pmatrix}=\begin{pmatrix}3\\-3\end{pmatrix}$

The resultant force is $\begin{pmatrix}3\\-3\end{pmatrix}$ N.

2 a
$$\begin{aligned}&\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3=0\\&\Rightarrow(2\mathbf{i}+7\mathbf{j})+(-3\mathbf{i}+\mathbf{j})+\mathbf{F}_3=0\\&\Rightarrow\mathbf{F}_3=-(2\mathbf{i}+7\mathbf{j})-(-3\mathbf{i}+\mathbf{j})\\&\qquad=-2\mathbf{i}-7\mathbf{j}+3\mathbf{i}-\mathbf{j}\\&\qquad=\mathbf{i}-8\mathbf{j}\end{aligned}$$

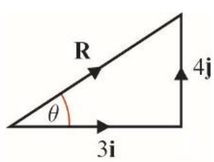
b
$$\begin{aligned}&\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3=0\\&\Rightarrow(3\mathbf{i}-4\mathbf{j})+(2\mathbf{i}+3\mathbf{j})+\mathbf{F}_3=0\\&\Rightarrow\mathbf{F}_3=-(3\mathbf{i}-4\mathbf{j})-(2\mathbf{i}+3\mathbf{j})\\&\qquad=-3\mathbf{i}+4\mathbf{j}-2\mathbf{i}-3\mathbf{j}\\&\qquad=-5\mathbf{i}+\mathbf{j}\end{aligned}$$

3 Since object is in equilibrium:

$$\begin{pmatrix}a\\2b\end{pmatrix}+\begin{pmatrix}-2a\\-b\end{pmatrix}+\begin{pmatrix}3\\-4\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$
$$\begin{pmatrix}a\\2b\end{pmatrix}+\begin{pmatrix}-2a\\-b\end{pmatrix}=\begin{pmatrix}-3\\4\end{pmatrix}$$
$$\begin{pmatrix}-a\\b\end{pmatrix}=\begin{pmatrix}-3\\4\end{pmatrix}$$

$a = 3$ and $b = 4$

4 a $(3\mathbf{i} + 4\mathbf{j})$



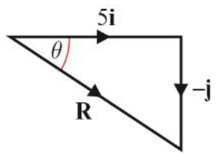
4 a i $\sqrt{3^2+4^2}=\sqrt{25}$

The resultant force is 5 N.

ii $\tan\theta=\frac{4}{3}$

The force makes an angle of 53.1° with **i**.

b (5**i** – **j**)



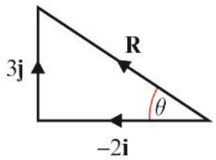
i $\sqrt{5^2+1^2}=\sqrt{26}$

The resultant force is $\sqrt{26}$ N.

ii $\tan\theta=\frac{1}{5}$

The force makes an angle of 11.3° with **i**.

c (−2**i** + 3**j**)



i $\sqrt{2^2+3^2}=\sqrt{13}$

The resultant force is $\sqrt{13}$ N.

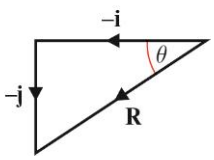
ii $\tan\theta=\frac{3}{2}$

θ = 56.3° This is the angle made with the negative **i** vector

Angle made with the positive **i** vector = 180 − θ

The force makes an angle of 123.7° with **i**.

d



i $\sqrt{1^2+1^2}=\sqrt{2}$

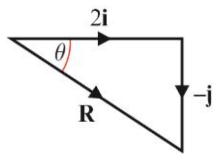
The resultant force is $\sqrt{2}$ N.

4 d ii $\tan\theta = \frac{1}{1}$

$\theta = 45°$. This is the angle made with the negative **i** vector.
The obtuse angle made with the positive **i** vector = $180 - \theta$
The force makes an angle of 135° with **i**.

5 a i $(-2\mathbf{i} + \mathbf{j}) + (5\mathbf{i} + 2\mathbf{j}) + (-\mathbf{i} - 4\mathbf{j}) = (2\mathbf{i} - \mathbf{j})$
The resultant vector is $(2\mathbf{i} - \mathbf{j})$ N.

ii $\sqrt{2^2 + 1^2} = \sqrt{5}$
The magnitude of the resultant vector is $\sqrt{5}$ N.

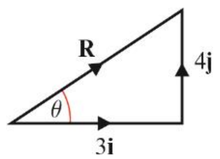


iii $\tan\theta = \frac{1}{2}$

$\theta = -26.6°$ This is the angle made from **east**, with **anticlockwise** defined as positive.
The **bearing** is the angle made from **north**, with **clockwise** defined as positive = $90 - \theta$
The force acts at a bearing of 116.6°.

b i $(-2\mathbf{i} + \mathbf{j}) + (2\mathbf{i} - 3\mathbf{j}) + (3\mathbf{i} + 6\mathbf{j}) = (3\mathbf{i} + 4\mathbf{j})$
The resultant vector is $(3\mathbf{i} + 4\mathbf{j})$ N

ii $\sqrt{3^2 + 4^2} = \sqrt{25}$
The resultant force is 5 N.



iii $\tan\theta = \frac{4}{3}$

$\theta = 53.1°$ This is the angle made from **east**, with **anticlockwise** defined as positive.
The **bearing** is the angle made from **north**, with **clockwise** defined as positive = $90 - \theta$
The force acts at a bearing of 036.9°.

6 Since the object is in equilibrium:
$(a\mathbf{i} - b\mathbf{j}) + (b\mathbf{i} + a\mathbf{j}) + (-4\mathbf{i} - 2\mathbf{j}) = 0$
Considering **i** components:
$a + b - 4 = 0$
so $b = 4 - a$ **(1)**
Considering **j** components:
$-b + a - 2 = 0$
Substituting $b = 4 - a$ from **(1)**:
$-(4 - a) + a - 2 = 0$
$2a = 2 + 4 = 6$
$a = 3$ **(2)**

6 Substituting **(2)** into **(1)**:

$b = 4 - 3 = 1$
The values of a and b are 3 and 1, respectively.

7 Since the object is in equilibrium:
$(2a\mathbf{i} + 2b\mathbf{j}) + (-5b\mathbf{i} + 3a\mathbf{j}) + (-11\mathbf{i} - 7\mathbf{j}) = 0$

Considering **i** components:
$2a - 5b - 11 = 0$ **(1)**

Considering **j** components:
$2b + 3a - 7 = 0$ **(2)**

equation **(1)** × 3 → $6a - 15b - 33 = 0$ **(3)**
equation **(2)** × 2 → $6a + 4b - 14 = 0$ **(4)**

Subtracting (**4**) from (**3**):
$-15b - 33 - 4b - (-14) = 0$
$-19b = 33 - 14$
$b = -1$
Substituting this value into equation (**1**):
$2a - 5(-1) - 11 = 0$
$2a = 11 - 5 = 6$
The values of a and b are 3 and −1, respectively.

8 a $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

$\Rightarrow (-3\mathbf{i} + 7\mathbf{j}) + (\mathbf{i} - \mathbf{j}) + (p\mathbf{i} + q\mathbf{j}) = 0$

$(-3 + 1 + p)\mathbf{i} + (7 - 1 + q)\mathbf{j} = 0$

$p = 2,\ q = -6$

b $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$

$= (-3\mathbf{i} + 7\mathbf{j}) + (\mathbf{i} - \mathbf{j})$

$= -2\mathbf{i} + 6\mathbf{j}$

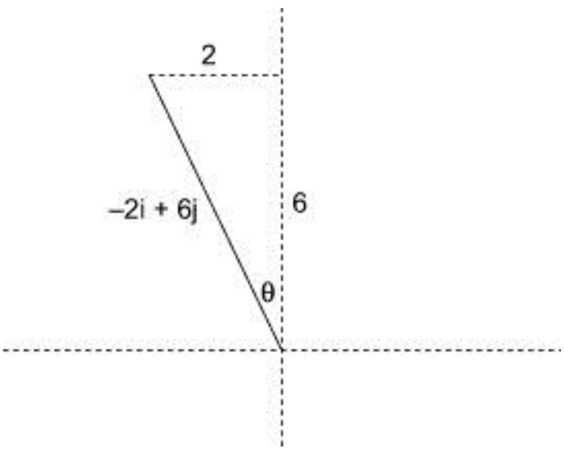
$|\mathbf{R}| = \sqrt{(-2)^2 + 6^2}$

$= \sqrt{4 + 36}$

$= \sqrt{40}$

$= 6.32$ N

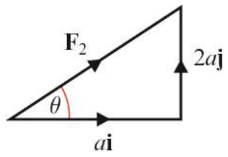
c



$\tan\theta = \dfrac{2}{6}$

$\theta = 18^\circ$

9 a $\mathbf{F}_2 = (a\mathbf{i} + 2a\mathbf{j})$



$\tan\theta = \dfrac{2a}{a} = 2$

$\mathbf{F}_2$ makes an angle of 63.4° with $\mathbf{i}$.

b $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = (3\mathbf{i} - 2\mathbf{j}) + (a\mathbf{i} + 2a\mathbf{j})$

$\mathbf{i}$ vector $= 3 + a$ $\qquad$ $\mathbf{j}$ vector $= -2 + 2a$

In the vector $(13\mathbf{i} + 10\mathbf{j})$:
$\mathbf{i}$ vector $= 13$ $\qquad$ $\mathbf{j}$ vector $= 10$

Let θ_1 = the angle of vector $\mathbf{R}$ and θ_2 = the angle of vector $(13\mathbf{i} + 10\mathbf{j})$

Since the vectors are parallel, $\theta_1 = \theta_2$ so $\tan\theta_1 = \tan\theta_2$:

$\tan\theta_1 = \dfrac{\mathbf{j}\text{ vector}}{\mathbf{i}\text{ vector}} = \dfrac{-2+2a}{3+a}$ $\qquad$ $\tan\theta_2 = \dfrac{\mathbf{j}\text{ vector}}{\mathbf{i}\text{ vector}} = \dfrac{10}{13}$

$\Rightarrow \dfrac{-2+2a}{3+a} = \dfrac{10}{13}$

$(-2 + 2a) \times 13 = (3 + a) \times 10$

$16a = 56$

$a = 3.5$

10 a Since the particle P is in equilibrium:

$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

$$\begin{pmatrix}-7\\-4\end{pmatrix} + \begin{pmatrix}4\\2\end{pmatrix} + \begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

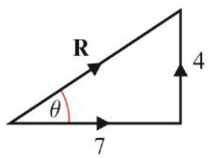
$$\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}7\\4\end{pmatrix} + \begin{pmatrix}-4\\-2\end{pmatrix}$$

$$\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}3\\2\end{pmatrix}$$

The values are $a = 3$, $b = 2$

b $\mathbf{R} = \mathbf{F}_2 + \mathbf{F}_3$

$$\mathbf{R} = \begin{pmatrix}4\\2\end{pmatrix} + \begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}7\\4\end{pmatrix}$$



10 b i $|\mathbf{R}| = \sqrt{7^2 + 4^2} = \sqrt{65}$

The magnitude of **R** is $\sqrt{65}$ N.

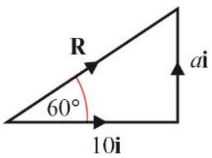
ii $\tan\theta = \dfrac{4}{7}$

$\theta = 29.7\ldots°$

R acts at 30° above the horizontal (to 2 s.f.)

Challenge

Redrawing the diagram as a closed triangle:



$\tan 60 = \dfrac{a}{10}$

$a = 10\tan 60 = 10 \times \sqrt{3}$

$\mathbf{R} = \begin{pmatrix} 10 \\ a \end{pmatrix} = \begin{pmatrix} 10 \\ 10\sqrt{3} \end{pmatrix}$

$|\mathbf{R}| = \sqrt{10^2 + \left(10\sqrt{3}\right)^2} = \sqrt{100 + 300} = \sqrt{400}$

The value of a is 17.3 (to 3 s.f.), and the magnitude of the resultant force is 20 N.

Forces and motion 10C

1 $F = ma$
$120 = 400a$
$a = 0.3$

The acceleration is $0.3\,\text{m}\,\text{s}^{-2}$

2 $W = mg$
$= 4 \times 9.8$
$= 39.2$

The weight of the particle is $39.2\,\text{N}$

3 $F = ma$
$30 = 1.2m$
$m = 25$

The mass of the object is $25\,\text{kg}$.

4 On Earth: $W = 735$ N, $g = 9.8$ m s^{-2}, $m = ?$

$W = mg$
$735 = m \times 9.8$
$m = 735 \div 9.8 = 75$ kg

On the moon: $W = 120$ N, $g = ?$, $m = 75$
$W = mg$
$120 = 75 \times g$
$g = 120 \div 75 = 1.6$

On the Moon, the acceleration due to gravity is 1.6 m s^{-2}.

5 Always resolve in the direction of acceleration.

a $R(\uparrow)$, $P - 2g = 2 \times 3$
$P = 25.6$

The magnitude of P is $25.6\,\text{N}$

b $R(\downarrow)$, $4g + 10 - P = 4 \times 2$
$49.2 - P = 8$
$P = 41.2$

The magnitude of P is $41.2\,\text{N}$

6 a $R(\downarrow)$, $mg - 10 = m \times 5$

$$9.8m - 10 = 5m$$
$$m = 2.1 \quad (2\text{ s.f.})$$

The mass of the body is 2.1 kg

b $R(\uparrow)$, $20 - mg = m \times 2$

$$20 - 9.8m = 2m$$
$$m = 1.7 \quad (2\text{ s.f.})$$

The mass of the body is 1.7 kg

7 a $R(\downarrow)$, $2g - 8 = 2a$

$$5.8 = a$$

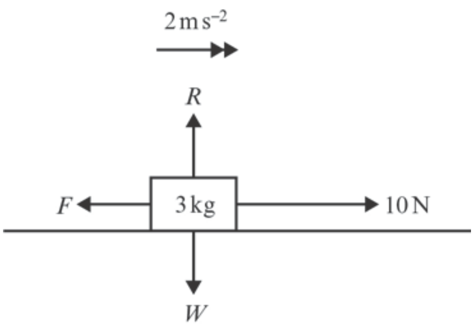
The acceleration of the body is $5.8\,\text{m s}^{-2}$

b $R(\uparrow)$, $100 - 8g = 8a$

$$2.7 = a$$

The acceleration of the body is $2.7\,\text{m s}^{-2}$

8 W and R can be ignored, as they act at right angles to the motion.



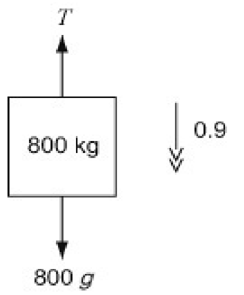
Resultant force = ma
$m = 3$ kg, $a = 2$ ms^{-2}
$R\,(\rightarrow)$, $10 - F = 3 \times 2 = 6$
$F = 10 - 6$
The force due to friction is 4 N.

9 a $u = 0$, $v = 3$, $s = 5$, $a = ?$

$$v^2 = u^2 + 2as$$
$$3^2 = 0^2 + 2a \times 5$$
$$9 = 10a$$
$$a = 0.9$$

The acceleration of the lift is $0.9\,\text{m s}^{-2}$

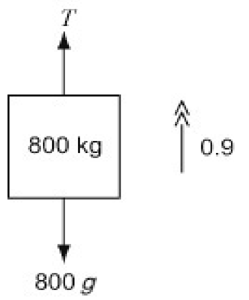
9 b



$$R(\downarrow),\quad 800g - T = 800 \times 0.9$$
$$7840 - T = 720$$
$$T = 7120$$

The tension in the cable is 7120 N.

c



$$R(\uparrow),\quad T - 800g = 800 \times 0.9$$
$$T - 7840 = 720$$
$$T = 8560$$

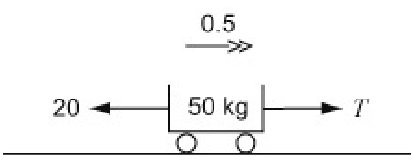
The tension in the cable is 8560 N.

10 a $u = 0,\ v = 1,\ t = 2,\ a = ?$

$$v = u + at$$
$$1 = 0 + a \times 2$$
$$a = 0.5$$

The acceleration of the trolley is $0.5\,\mathrm{m\,s^{-2}}$

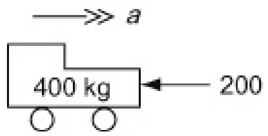
b



$$R(\rightarrow),\quad T - 20 = 50 \times 0.5$$
$$T = 45$$

The tension in the rope is 45 N.

11 a



$R(\rightarrow), \quad -200 = 400a$
$a = -0.5$

$u = 16, \ v = 0, \ a = -0.5, \ t = ?$
$v = u + at \quad (\rightarrow)$
$0 = 16 - 0.5t$
$0.5t = 16$
$t = 32$

It takes 32 s for the van to stop.

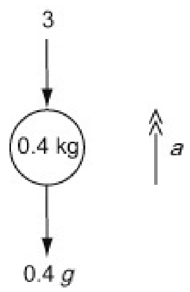
b $u = 16, \ v = 0, \ a = -0.5, \ s = ?$
$v^2 = u^2 + 2as \quad (\rightarrow)$
$0^2 = 16^2 + 2(-0.5)s$
$0 = 256 - s$
$s = 256$

The van travels 256 m before it stops.

c Air resistance is unlikely to be of constant magnitude. (It is usually a function of speed.)

Challenge

a

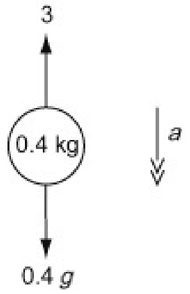


$R(\uparrow), \quad -3 - 0.4g = 0.4a$
$a = -17.3$

$u = 10, \ v = 0, \ a = -17.3, \ s = ?$
$v^2 = u^2 + 2as \quad (\uparrow)$
$0 = 10^2 + 2(-17.3)s$
$0 = 100 - 34.6s$
$s = 2.89\ldots = 2.9 \quad (2 \text{ s.f.})$

The stone rises to a height of 2.9 m above the bottom of the pond.

b



$R(\downarrow)$, $\quad 0.4g - 3 = 0.4a$

$$0.92 = 0.4a$$
$$a = 2.3$$

$u = 0,\ s = \frac{100}{34.6},\ a = 2.3,\ v = ?$

$v^2 = u^2 + 2as \quad (\downarrow)$

$v^2 = 0^2 + 2 \times 2.3 \times \frac{100}{34.6}$

$v = 3.646.. = 3.6 \quad$ (2 s.f.)

The stone hits the bottom of the pond with speed $3.6\,\mathrm{m\,s^{-1}}$

c $u = 10,\ v = 0,\ a = -17.3,\ t = ?$

$v = u + at \quad (\uparrow)$

$0 = 10 - 17.3t,$

$t_1 = \frac{10}{17.3} = 0.57803\ldots$

$u = 0,\ a = 2.3,\ s = \frac{100}{34.6},\ t = ?$

$s = ut + \frac{1}{2}at^2 \quad (\downarrow)$

$\frac{100}{34.6} = 0 + \frac{1}{2} \times 2.3t_2^2$

$$t_2^2 = \frac{2 \times 100}{2.3 \times 34.6} = 2.51319$$

$t_2 = 1.585$

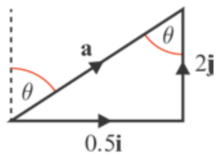
$t_1 + t_2 = 0.57803 + 1.585 = 2.16$

The total time is 2.16 s (3 s.f.)

Forces and motion 10D

1 a $F = (\mathbf{i} + 4\mathbf{j})$, $m = 2$, $\mathbf{a} = ?$
$F = m\mathbf{a}$
$(\mathbf{i} + 4\mathbf{j}) = 2\mathbf{a}$
$\mathbf{a} = \dfrac{(\mathbf{i}+4\mathbf{j})}{2}$
The acceleration of the particle is $(0.5\mathbf{i} + 2\mathbf{j})$ m s^{-2}.

b

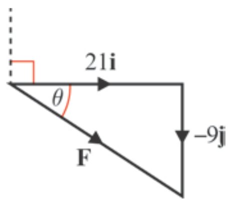


$|\mathbf{a}| = \sqrt{0.5^2 + 2^2} = \sqrt{4.25}$

The magnitude of the acceleration is 2.06 m s^{-2}.
Using Z angles (see diagram), bearing = θ
$\tan\theta = \dfrac{0.5}{2}$
$\theta = 14°$
The bearing of the acceleration is 014°.

2 $F = (4\mathbf{i} + 3\mathbf{j})$, $\mathbf{a} = (20\mathbf{i} + 15\mathbf{j})$, $m = ?$
$F = m\mathbf{a}$
$(4\mathbf{i} + 3\mathbf{j}) = m \times (20\mathbf{i} + 15\mathbf{j})$
$m = \dfrac{(4\mathbf{i}+3\mathbf{j})}{(20\mathbf{i}+15\mathbf{j})} = \dfrac{1}{5}$
The mass of the particle is 0.2 kg.

3 a $\mathbf{a} = (7\mathbf{i} - 3\mathbf{j})$, $m = 3$, $F = \mathbf{?}$
$F = m\mathbf{a}$
$= 3 \times (7\mathbf{i} - 3\mathbf{j})$
$= (21\mathbf{i} - 9\mathbf{j})$



b
$|\mathbf{F}| = \sqrt{21^2 + 9^2} = \sqrt{522}$
The force has a magnitude of 22.8 N (3 s.f.)
$\tan\theta = \dfrac{9}{21}$
$\theta = 23.19\ldots°$
But bearing = $90° + \theta$ (see diagram)
The force acts at a bearing of 113° (to the nearest degree).

4 a $\mathbf{F}_1 = (2\mathbf{i} + 7\mathbf{j})$, $\mathbf{F}_2 = (-3\mathbf{i} + \mathbf{j})$, $m = 0.25$

$F = \mathbf{F}_1 + \mathbf{F}_2 = m\mathbf{a}$

$(2\mathbf{i} + 7\mathbf{j}) + (-3\mathbf{i} + \mathbf{j}) = 0.25\mathbf{a}$

$(-\mathbf{i} + 8\mathbf{j}) = 0.25\mathbf{a}$

$$\mathbf{a} = \frac{(-\mathbf{i} + 8\mathbf{j})}{0.25}$$

The acceleration is $(-4\mathbf{i} + 32\mathbf{j})$ m s^{-2}.

b $\mathbf{F}_1 = (3\mathbf{i} - 4\mathbf{j})$, $\mathbf{F}_2 = (2\mathbf{i} + 3\mathbf{j})$, $m = 6$

$F = \mathbf{F}_1 + \mathbf{F}_2 = m\mathbf{a}$

$(3\mathbf{i} - 4\mathbf{j}) + (2\mathbf{i} + 3\mathbf{j}) = 6\mathbf{a}$

$(5\mathbf{i} - \mathbf{j}) = 6\mathbf{a}$

$$\mathbf{a} = \frac{(5\mathbf{i} - \mathbf{j})}{6}$$

The acceleration is $\left(\frac{5}{6}\mathbf{i} - \frac{1}{6}\mathbf{j}\right)$ m s^{-2}.

c $\mathbf{F}_1 = (-40\mathbf{i} - 20\mathbf{j})$, $\mathbf{F}_2 = (25\mathbf{i} + 10\mathbf{j})$, $m = 15$

$F = \mathbf{F}_1 + \mathbf{F}_2 = m\mathbf{a}$

$(-40\mathbf{i} - 20\mathbf{j}) + (25\mathbf{i} + 10\mathbf{j}) = 15\mathbf{a}$

$(-15\mathbf{i} - 10\mathbf{j}) = 15\mathbf{a}$

$$\mathbf{a} = \frac{(-15\mathbf{i} - 10\mathbf{j})}{15}$$

The acceleration is $\left(-\mathbf{i} - \frac{2}{3}\mathbf{j}\right)$ m s^{-2}.

d $\mathbf{F}_1 = 4\mathbf{j}$, $\mathbf{F}_2 = (-2\mathbf{i} + 5\mathbf{j})$, $m = 1.5$

$F = \mathbf{F}_1 + \mathbf{F}_2 = m\mathbf{a}$

$4\mathbf{j} + (-2\mathbf{i} + 5\mathbf{j}) = 1.5\mathbf{a}$

$(-2\mathbf{i} + 9\mathbf{j}) = 1.5\mathbf{a}$

$$\mathbf{a} = \frac{(-2\mathbf{i} + 9\mathbf{j})}{1.5}$$

The acceleration is $\left(-\frac{4}{3}\mathbf{i} + 6\mathbf{j}\right)$ m s^{-2}.

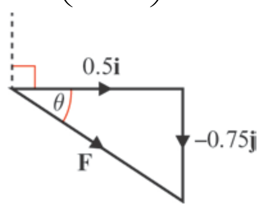
5 a Resultant force, $F = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$$F = \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ -5 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$$

$F = m\mathbf{a}$

$$8\mathbf{a} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$$

$$\mathbf{a} = \begin{pmatrix} 0.5 \\ -0.75 \end{pmatrix}$$



5 a $|\mathbf{a}| = \sqrt{0.5^2 + 0.75^2} = \sqrt{0.8125}$

$\tan\theta = \dfrac{0.75}{0.5}$

$\theta = 56°$

But bearing = 90° + θ (see diagram)

The acceleration has a magnitude of 0.901 m s^{-2} and acts at a bearing of 146°.

b $s = 20$, $u = 0$, $a = 0.901$

$s = ut + \frac{1}{2}at^2$

$20 = (0 \times t) + \left(\frac{1}{2} \times 0.901 \times t^2\right)$

$t^2 = \dfrac{20 \times 2}{0.901} = 44.39$

The particle takes 6.66 s to travel 20 m.

6 $\mathbf{R} = (2\mathbf{i} + 3\mathbf{j}) + (p\mathbf{i} + q\mathbf{j})$

Since $\mathbf{R}$ is parallel to $(-\mathbf{i} + 4\mathbf{j})$,

$\mathbf{R} = (-k\mathbf{i} + 4k\mathbf{j})$ where k is a constant

$(2\mathbf{i} + 3\mathbf{j}) + (p\mathbf{i} + q\mathbf{j}) = (-k\mathbf{i} + 4k\mathbf{j})$

Collecting **i** terms: $2 + p = -k$

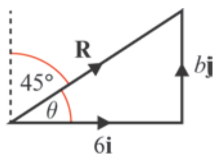
so $k = -2 - p$

Collecting **j** terms: $3 + q = 4k$

Substituting for k: $3 + q = 4(-2 - p)$

so $3 + q = -8 - 4p$

$4p + q + 11 = 0$



7 a $\theta = 90° - 45°$ (see diagram)

$\tan 45° = \dfrac{b}{6}$

$b = 6 \times \tan 45° = 6 \times 1$

The value of b is 6.

b $|\mathbf{R}| = \sqrt{6^2 + 6^2} = \sqrt{72}$

The magnitude of **R** is $6\sqrt{2}$ N (8.49 N to 3.s.f)

c $F = 6\sqrt{2}$, $m = 4$, $a = ?$

$F = ma$

$6\sqrt{2} = 4a$

The magnitude of the acceleration of the particle is $\dfrac{3\sqrt{2}}{2}$ m s^{-2} (2.12 m s^{-2} to 3 s.f.)

7 d $t = 5$, $u = 0$, $a = \frac{3\sqrt{2}}{2}$, $s = ?$

$$s = ut + \frac{1}{2}at^2$$

$$s = (0 \times 5) + \left(\frac{1}{2} \times \frac{3\sqrt{2}}{2} \times 5^2\right)$$

$$s = \frac{75\sqrt{2}}{4}$$

In the first 5 s the particle travels $\frac{75\sqrt{2}}{4}$ m (26.5 m to 3 s.f.).

8 a Since particle is in equilibrium, $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

$(-3\mathbf{i} + 7\mathbf{j}) + (\mathbf{i} - \mathbf{j}) + (p\mathbf{i} + q\mathbf{j}) = 0$
Collecting **i** terms: $-3 + 1 + p = 0$
Collecting **j** terms: $7 - 1 + q = 0$

The value of p is 2, and the value of q is −6.

b When $\mathbf{F}_2$ is removed, resultant force, $F = \mathbf{F}_1 + \mathbf{F}_3$
$F = (-3\mathbf{i} + 7\mathbf{j}) + (2\mathbf{i} - 6\mathbf{j}) = (-\mathbf{i} + \mathbf{j})$
The magnitude of this force is $\sqrt{1^2 + 1^2} = \sqrt{2}$

$s = 12$, $t = 10$, $u = 0$, $a = ?$

$$s = ut + \frac{1}{2}at^2$$

$$12 = (0 \times 20) + \left(\frac{1}{2} \times a \times 10^2\right)$$

$$12 = 50a$$

$$a = \frac{12}{50} = \frac{6}{25}$$

$$F = \sqrt{2},\ a = \frac{6}{25}$$

$$F = ma$$

$$\sqrt{2} = m \times \frac{6}{25}$$

The mass of the particle is $\frac{25\sqrt{2}}{6}$ kg.

9 Resultant force, $F = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$F = (5\mathbf{i} + 6\mathbf{j}) + (2\mathbf{i} - 2\mathbf{j}) + (-\mathbf{i} - 4\mathbf{j}) = 6\mathbf{i}$
Since this has only a single component, the magnitude of the force is 6 N.
$a = 7$
$F = ma$
$6 = m \times 7$
$m = 6 \div 7$
The mass of the particle is 0.86 kg.

10 a $\mathbf{R} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} p \\ q \end{pmatrix}$

Since **R** is parallel to $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\mathbf{R} = \begin{pmatrix} k \\ -2k \end{pmatrix}$ where k is a constant

$\begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} k \\ -2k \end{pmatrix}$

Collecting **i** terms: $2 + p = k$
Collecting **j** terms: $5 + q = -2k$
Substituting for k: $5 + q = -2(2 + p)$
so $5 + q = -4 - 2p$
$2p + q + 9 = 0$

b $p = 1$
From **a** above, $k = 2 + p$
so $k = 2 + 1 = 3$
so $\mathbf{R} = \begin{pmatrix} k \\ -2k \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$

$|\mathbf{R}| = \sqrt{3^2 + (-6)^2} = \sqrt{45}$

$a = 15\sqrt{5}$, $F = \sqrt{45}$

$F = ma$

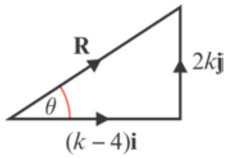
$\sqrt{45} = m \times 15\sqrt{5}$

$m = \frac{\sqrt{45}}{15\sqrt{5}} = \frac{\sqrt{9 \times 5}}{15\sqrt{5}} = \frac{3\sqrt{5}}{15\sqrt{5}} = \frac{1}{5} = 0.2$

The mass of the particle is 0.2 kg.

Challenge

Resultant force, $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$
$\mathbf{R} = -4\mathbf{i} + (k\mathbf{i} + 2k\mathbf{j})$



$F = ma$

$m = 0.5,\ a = 8\sqrt{17}$

So magnitude of the resultant force $= 0.5 \times 8\sqrt{17} = 4\sqrt{17}$

$|\mathbf{R}|^2 = (k-4)^2 + (2k)^2$

$\left(4\sqrt{17}\right)^2 = 16 \times 17 = k^2 - 8k + 16 + 4k^2$

$272 = 5k^2 - 8k + 16$

$5k^2 - 8k - 256 = 0$

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

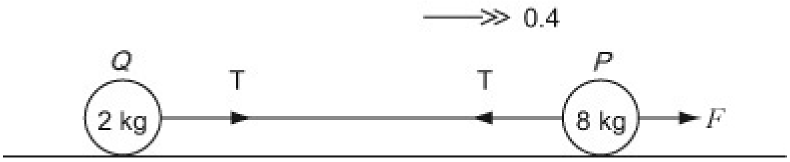
$$k = \frac{8 \pm \sqrt{8^2 - 4 \times 5 \times (-256)}}{2 \times 5} = \frac{8 \pm \sqrt{5184}}{10} = \frac{8 \pm 72}{10}$$

$k = -6.4$ or 8

Since k is given as a positive constant, the value of k is 8.

Forces and motion 10E

1



a $R(\rightarrow)$, $F = (2+8) \times 0.4$
$= 4$

Hence F is 4 N.

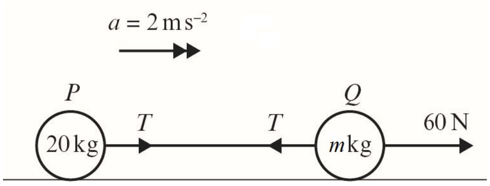
b For Q:

$R(\rightarrow)$, $T = 2 \times 0.4$
$= 0.8$

The tension in the string is 0.8 N.

c Treating the string as inextensible (i.e. it does not stretch) allows us to assume that the acceleration of both masses is the same. Treating the string as light (i.e. having no/negligible mass) allows us to assume that the tension is the same throughout the length of the string and that its mass does not need to be considered when treating the system as a whole.

2

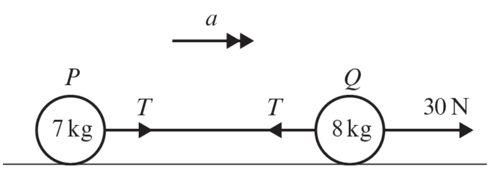


$F = ma$

a For the whole system: $F = 60$, $m = 20 + m = 10$, $a = 2$
$60 = (20 + m) \times 2$
$20 + m = 60 \div 2$
$m = 30 - 20$
The mass of Q is 10 kg.

b For P: $F = T$, $m = 20$, $a = 2$
$T = 20 \times 2$
The tension in the string is 40 N.

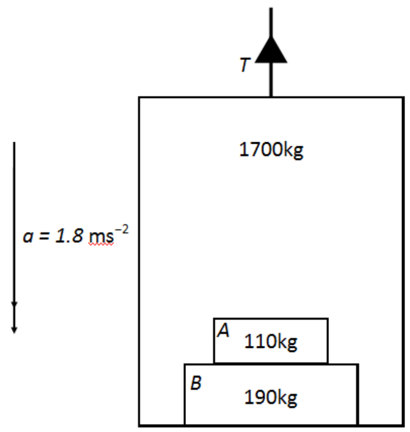
3 $F = ma$



3 **a** For the whole system: $F = 30$, $m = 8 + 7 = 15$, $a = ?$
$30 = 15a$
The acceleration of the system is 2 m s^{-2}.

b For P: $F = T$, $m = 7$, $a = 2$
$T = 7 \times 2$
The tension in the string is 14 N.

4



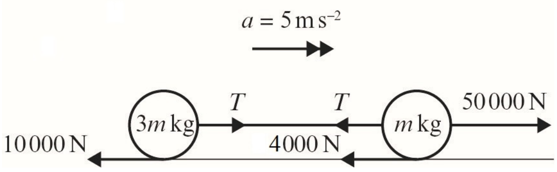
a Considering the system as a whole: total mass, m = 1700 + 110 + 190 = 2000 kg
Taking down as positive:
$F = ma = mg - T$
$2000 \times 1.8 = (2000 \times 9.8) - T$
$T = 19600 - 3600$
The tension in the cable is 16 000 N.

b **i** Force exerted on box A by box B is a normal reaction force, R_1 which acts upwards.
For box A, taking down as positive:
$110 \times 1.8 = 110g - R_1$
$R_1 = 110(g - 1.8)$
$R_1 = 110 \times 8$
Box B exerts an upwards force of 880 N on box A.

ii Let downward force exerted on lift by box B be S.
For lift alone, taking down as positive:
$1700 \times 1.8 = 1700g + S - T$
$S = T + 1700(1.8 - g)$
$S = 16\,000 - 13600 = 2400$

Alternatively (or as check), use Newton's third law of motion:
|Force exerted box B by box A| = |Force exerted on box A by box B| = 880 N
|Force exerted on lift by box B| = |Force exerted on box B by lift| = $|R_2|$
For box B, taking down as positive:
$190 \times 1.8 = 880 + 190g - R_2$
$R_2 = 880 + 190(g - 1.8)$
$R_2 = 880 + 1520 = 2400$

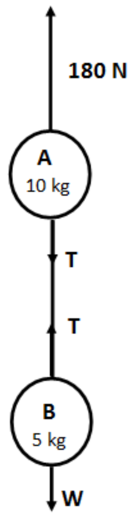
5 $F = ma$



a For the whole system:
$F = 50\,000 - 10\,000 - 4000 = 36\,000$
$a = 5$
total mass $= 3m + m = 4m$
$36\,000 = a \times$ total mass $= 4m \times 5 = 20m$
$m = 1800$
so $3m = 5400$
The mass of the lorry is 1800 kg, and that of the trailer is 5400 kg.

b For the trailer:
$F = T - 10\,000$, $m = 5400$, $a = 5$
$T - 10\,000 = 5400 \times 5 = 27\,000$
$T = 37\,000$
The tension in the tow-bar is 37 000 N.

c Treating the tow-bar as inextensible (i.e. it does not stretch) allows us to assume that the acceleration of the truck and the trailer are the same. Treating the tow-bar as light (i.e. having no/negligible mass) allows us to assume that the tension is the same throughout its length and that its mass does not need to be considered when treating the system as a whole.



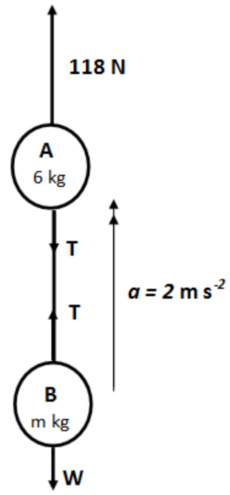
6 $F = ma$, $W = mg$
Taking upwards as positive

a For the whole system:
$180 - 15g = 15a$
$15a = 180 - (15 \times 9.8)$
$$a = \frac{180 - 147}{15} = 2.2$$
The acceleration is 2.2 m s^{-2}.

b For B:
$ma = T - W$
$5 \times 2.2 = T - (5 \times 9.8)$
$11 = T - 49$
The tension in the string is 60 N.

7 $F = ma,\ W = mg$

Taking up as positive



a For the whole system:

$118 - (6 + m)g = (6 + m) \times 2$

$118 = (6 + m)(2 + g) = (6 + m)(2 + 9.8)$

$\dfrac{118}{11.8} = 6 + m$

$10 = 6 + m$

The mass of B is 4 kg.

b For B:

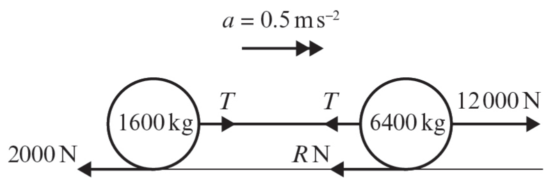
$ma = T - W$

$4 \times 2 = T - (4 \times 9.8)$

$8 = T - 39.2$

The tension in the string is 47.2 N.

8 $F = ma$



a For the whole system:

$F = 12\,000 - 2000 - R$

$m = 1600 + 6400 = 8000$

$a = 0.5$

$10\,000 - R = 8000 \times 0.5 = 4000$

The resistance to the motion of the engine is 6000 N.

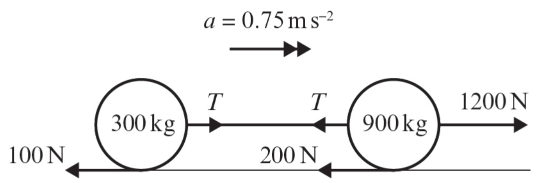
b For the carriage:

$F = T - 2000,\ m = 1600,\ a = 0.5$

$T - 2000 = 1600 \times 0.5 = 800$

The tension in the coupling is 2800 N.

9 $F = ma$



a For the whole system:

$F = 1200 - 1000 - 200 = 900$

$m = 900 + 300 = 1200$

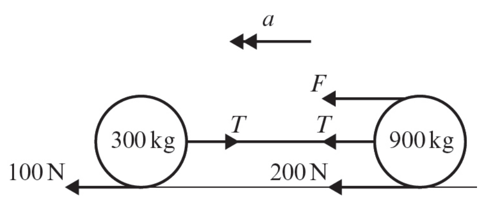
$900 = 1200a$

$a = 900 \div 1200 = 0.75$

The acceleration is $0.75\ \text{m s}^{-2}$, as required.

9 b For the trailer:
$F = T - 100$, $m = 300$, $a = 0.75$
$T - 100 = 300 \times 0.75 = 225$
The tension in the towbar is 325 N.

c



Taking ← as positive
Deceleration = α
Force on trailer = resistance to motion + thrust from tow-bar
Using $F = ma$
$100 + 100 = 300\alpha$

$$\alpha = \frac{200}{300} = \frac{2}{3}$$

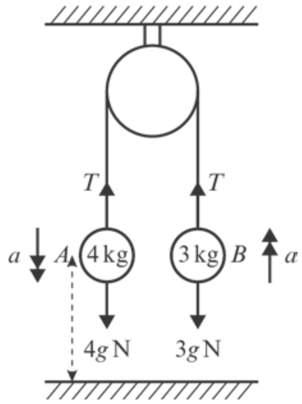
For car:
$F + 200 - 100 = 900\alpha$

$$F = \left(900 \times \frac{2}{3}\right) - 100 = 500$$

The force the brakes produce on the car is 500 N.

Forces and motion 10F

1 a



For A: $R(\downarrow)$, $4g - T = 4a$ **(1)**

For B: $R(\uparrow)$, $T - 3g = 3a$ **(2)**

$(\mathbf{1}) + (\mathbf{2}): 4g - 3g = 7a$

$\Rightarrow a = \frac{g}{7}$

Substituting into equation (**2**):

$T = 3a + 3g = \frac{3g}{7} + 3g = \frac{24g}{7}$

$= 33.6$ N (3 s.f.)

b $u = 0, a = \frac{g}{7}$, $s = 2$, m, $v = ?$

$v^2 = u^2 + 2as$

$v^2 = 0^2 + 2 \times \frac{g}{7} \times 2 = \frac{4g}{7} = 5.6$

$v = \sqrt{5.6} = 2.366...$

When A hits the ground it is travelling at 2.37 m s^{-1} (3 s.f.).

c For A: $(\downarrow)$

From part **b**, $v^2 = \frac{4g}{7}$

This represents the initial velocity of B when A hits the ground.

For B: $(\uparrow)$

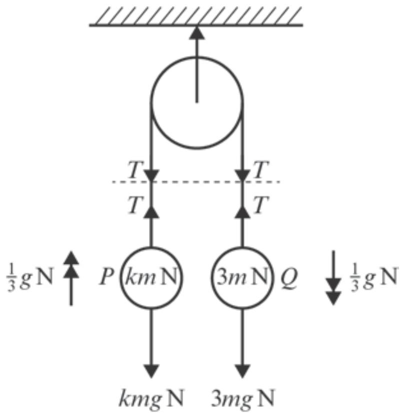
$u^2 = \frac{4g}{7}$, $v = 0$, $a = -g$, $s = ?$

$v^2 = u^2 + 2as$

$0 = \frac{4g}{7} - 2gs \Rightarrow s = \frac{2}{7}$

The height above the initial position is $2\frac{2}{7}$ m.

2



a For Q, $R(\downarrow)$: $3mg - T = 3m \times \frac{1}{3}g = mg$

$2mg = T$

The tension in the string is $2mg$ N.

b For P, $R(\uparrow)$: $T - kmg = km \times \frac{1}{3}g$

$3T - 3kmg = kmg$
$3T = 4kmg$
Substituting for T: $6mg = 4kmg$

$$k = \frac{6mg}{4mg}$$

The value of k is 1.5.

c Because the pulley is smooth, there is no friction, so the magnitude of acceleration of P = the magnitude of acceleration of Q.

d Up is positive.
While Q is descending, the distance travelled by $P = s_1$

$u = 0$, $a = \frac{1}{3}g$, $t = 1.8$, $s = s_1$

$$s = ut + \frac{1}{2}at^2$$

$$s_1 = (0 \times 1.8) + \left(\frac{1}{2} \times \frac{g}{3} \times 1.8^2\right) = \frac{3.24g}{6} = 0.54g \qquad \textbf{(1)}$$

Speed of P at this time = v_1
Using $v^2 = u^2 + 2as$

After Q hits the ground, P travels freely under gravity and rises by a further distance s_2
$v = 0$, $u = v_1$, $a = -g$, $s = s_2$
$v^2 = u^2 + 2as$
$0^2 = 0.36g^2 - 2gs_2$

$$s_2 = \frac{0.36g^2}{2g} = 0.18g \qquad \textbf{(2)}$$

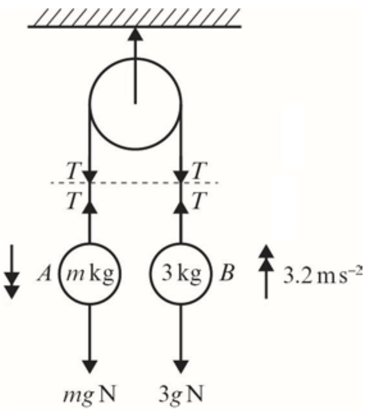
(**1**) + (**2**): Total distance travelled by P from its initial position = $s_1 + s_2$

2 d P and Q are at the same height initially, so P starts at height s_1 above the plane.
Its final position = initial position + total distance travelled

$$= s_1 + (s_1 + s_2) = 2s_1 + s_2 = 2 \times 0.54g + 0.18g = 1.26g$$

P reaches a maximum height of $1.26g$ m above the plane, as required.

3



a Since the pulley is smooth, |acceleration of A| = |acceleration of B|
For A: $s = 2.5$, $u = 0$, $t = 1.25$, a = ? (down is positive)

$$s = ut + \tfrac{1}{2}at^2$$

$$2.5 = (0 \times 1.25) + \tfrac{1}{2}a \times 1.25^2$$

$$a = \frac{2.5 \times 2}{1.25^2} = 3.2$$

The initial acceleration of B is 3.2 m s^{-2} as required.

b For B, $R(\uparrow)$: $T - 3g = 3a$
$T = 3(a + g) = 3(3.2 + 9.8) = 39$
The tension in the string is 39 N.

c For A, $R(\downarrow)$: $mg - T = ma$
$T = m(g - a) = m(9.8 - 3.2) = 6.6m$
Substituting for T:
$39 = 6.6m$

$$m = \frac{39}{6.6} = \frac{390}{66} = \frac{65}{11} \text{ as required}$$

d Because the string is inextensible, the tension on both sides of the pulley is the same.

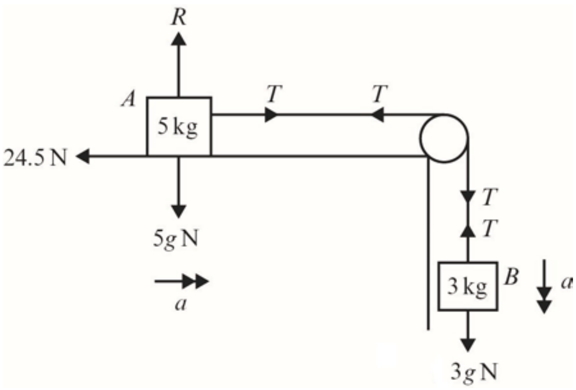
e The string will become taut again when B has risen to its maximum height and then descended to the point where A is just beginning to rise again.
If B reaches the maximum height t seconds after A hits the ground, it will also take t seconds to return to the same position as it is moving under gravity alone throughout this period. The total time of travel will be $2t$.
For B, taking up as positive, while the string is taut:
$u = 0$, $a = 1.4$, $s = 2.5$, m, $v = v_1$
$v^2 = u^2 + 2as$
$v_1^2 = 0^2 + 2 \times 3.2 \times 2.5 = 16$
Once the string is slack: $u = v_1 = 4$, $v = 0$, $a = -9.8$, t =?
$v = u + at$
$0 = 4 - 9.8t$

3 e $t = \dfrac{4}{9.8} = \dfrac{40}{98} = \dfrac{20}{49}$

At this point B descends under gravity. After a further t seconds the string once again becomes taut.

The string becomes taut again $2t = \dfrac{40}{49}$ s after A hits the ground.

4



a For A: $R(\rightarrow)$, $T - 24.5 = 5a$ **(1)**

For B: $R(\downarrow)$, $3g - T = 3a$

$29.4 - T = 3a$ **(2)**

(1) + **(2)**: $29.4 - 24.5 = 8a$

$4.9 = 8a$

$0.6125 = a$

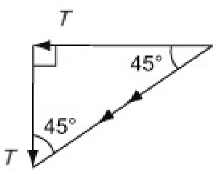
The acceleration of the system is $0.613\ \text{m s}^{-2}$ (3 s.f.)

b $T - 24.5 = 5 \times 0.6125$

$T = 27.5625$

The tension in the string is 27.6 N (3 s.f.)

c



By Pythagoras,

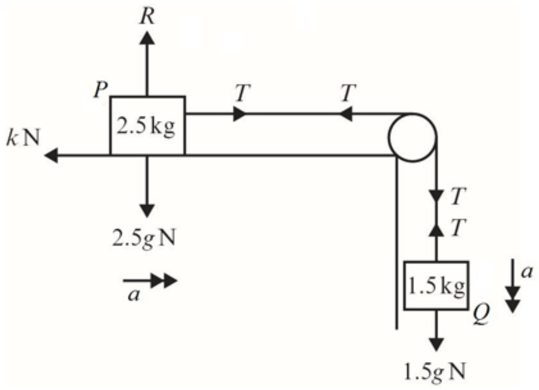
$F^2 = T^2 + T^2 = 2T^2$

$F = T\sqrt{2} = 27.5625 \times \sqrt{2}$

$= 38.979\ldots$

The magnitude of the force exerted on the pulley is 39 N (2 s.f.)

5



a **i** For Q: $s = 0.8$, $u = 0$, $t = 0.75$, $a = ?$ (down is positive)

$$s = ut + \tfrac{1}{2}at^2$$

$$0.8 = (0 \times 0.75) + \tfrac{1}{2}a \times 0.75^2$$

$$a = \frac{0.8 \times 2}{0.75^2} = 2.844...$$

The acceleration of Q is 2.84 m s^{-2} (3 s.f.)

ii For Q, $R(\downarrow)$: $1.5g - T = 1.5a$

$T = 1.5(g - a) = 1.5(9.8 - 2.84) = 10.44$

The tension in the string is 10.4 N (to 3 s.f.), as required.

iii For P, $R\ (\rightarrow)$: $T - k = 2.5a$

Substituting:

$10.4 - k = 2.5 \times 2.84$

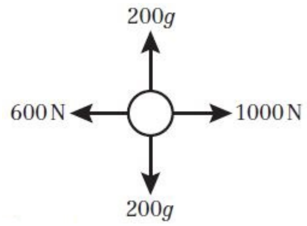
$k = 10.4 - 7.1$

The value of k is 3.3 N

b Because the string is inextensible, the tension in all parts of it is the same.

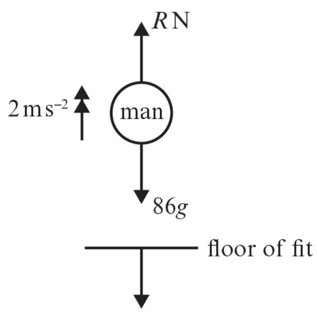
Forces and motion, Mixed exercise 10

1 a



b Vertical forces can be ignored as they are in equilibrium and at right angles to the direction of interest.
$F = ma$
$m = 200$, Resultant force, $F = 1000 - 200 - 400 = 400$
$400 = 200a$
The acceleration of the motorcycle is 2 m s^{-2}.

2



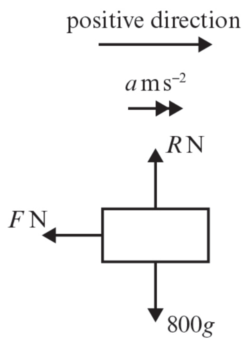
For the man

$$R(\uparrow),\quad R - 86g = 86 \times 2$$
$$R = 86 \times 9.8 + 86 \times 2$$
$$= 1014.8 \approx 1000$$

The reaction on the man on the floor is of equal magnitude to the action of the floor on the man and in the opposite direction.

The force that the man exerts on the floor of the lift is of magnitude 1000 N (2 s.f.) and acts vertically downwards.

3



3 a $u = 18,\ v = 12,\ t = 2.4,\ a = ?$

$$v = u + at$$
$$12 = 18 + 2.4a$$
$$a = \frac{12-18}{2.4} = -2.5$$

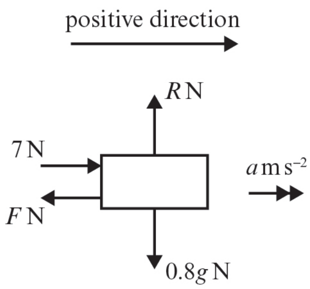
$$F = ma$$
$$-F = 800 \times -2.5 = -2000$$
$$F = 2000\text{ N}$$

b $u = 18,\ v = 12,\ t = 2.4,\ s = ?$

$$s = \left(\frac{u+v}{2}\right)t$$
$$= \left(\frac{18+12}{2}\right) \times 2.4$$
$$= 15 \times 2.4 = 36$$

The distance moved by the car is 36 m

4



a $u = 2,\ v = 4,\ s = 4.8,\ a = ?$

$$v^2 = u^2 + 2as$$
$$4^2 = 2^2 + 9.6a$$
$$a = \frac{16-4}{9.6} = 1.25$$

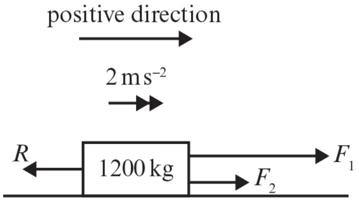
The magnitude of the acceleration of the block is 1.25 m s^{-2}

b $R(\uparrow),\quad F = ma = 0.8 \times 1.25 = 1$

$R(\rightarrow),\quad 7 - F = 6$

The magnitude of the frictional force between the block and the floor is 6 N.

5



Let R = the resistive force
Let F_1 = the driving force
Let F_2 = the resultant force

$F_2 = ma = 1200 \times 2 = 2400$

$F_1 = 3R \Rightarrow R = \frac{1}{3}F_1$

The driving force is the resultant force plus the resistive force:

$F_1 = R + F_2 = \frac{1}{3}F_1 + 2400$

$\frac{2}{3}F_1 = 2400$

$F_1 = 3600$

The magnitude of the driving force is 3600 N, as required.

6 $\mathbf{F}_1 = (3\mathbf{i} + 2\mathbf{j})$, $\mathbf{F}_2 = (4\mathbf{i} - \mathbf{j})$, $m = 0.25$

$F = \mathbf{F}_1 + \mathbf{F}_2 = ma$

$(3\mathbf{i} + 2\mathbf{j}) + (4\mathbf{i} - \mathbf{j}) = 0.25a$

$(7\mathbf{i} + \mathbf{j}) = 0.25a$

$a = \dfrac{(7\mathbf{i} + \mathbf{j})}{0.25}$

The acceleration is $(28\mathbf{i} + 4\mathbf{j})$ m s^{-2}.

7 $\mathbf{F}_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ $\mathbf{F}_2 = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ $\mathbf{F}_3 = \begin{pmatrix} a \\ -2b \end{pmatrix}$ $m = 2,\ a = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$F = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = ma$

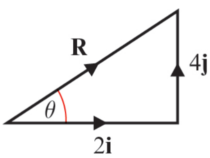
$\begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} a \\ -2b \end{pmatrix} = 2\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$

Considering $\mathbf{i}$ components: $2 + 3 + a = 6$
$a = 6 - 5$

Considering $\mathbf{j}$ components: $-1 - 1 - 2b = 4$
$-2b = 4 + 2$

The values of a and b are 1 and -3, respectively.

8



a $|R| = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$

Using $F = ma$

$2\sqrt{5} = 2a$

The acceleration of the sled is $\sqrt{5}$ m s^{-2}.

8 b $u = 0$, $t = 3$, $a = \sqrt{5}$, $s = ?$

$s = ut + \frac{1}{2}at^2$

$s = (0\times 3) + \left(\frac{1}{2}\times\sqrt{5}\times 3^2\right) = \dfrac{9\sqrt{5}}{2}$

The sled travels a distance of $\dfrac{9\sqrt{5}}{2}$ m.

9 a Since object is in equilibrium, $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

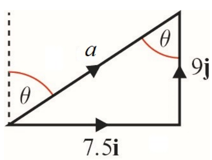
$(3a\mathbf{i} + 4b\mathbf{j}) + (5b\mathbf{i} + 2a\mathbf{j}) + (-15\mathbf{i} - 18\mathbf{j}) = 0$
Collecting **i** terms: $3a + 5b = 15$ **(1)**
Collecting **j** terms: $2a + 4b = 18$ **(2)**
Subtracting (**2**) from (**1**) gives $a + b = -3$
Therefore $b = -3 - a$

Substituting this into (**1**):
$3a + 5(-3 - a) = 15$
$3a - 15 - 5a = 15$
$-2a = 30$
$a = -15$
Substituting this into (**1**):
$3(-15) + 5b = 15$
$5b = 15 + 45 = 60$
$b = 12$

The values of a and b are -15 and 12, respectively.

b i $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$, so when $\mathbf{F}_3$ is removed, the resultant force $F = -\mathbf{F}_3$
i.e. $F = (15\mathbf{i} + 18\mathbf{j})$

$m = 2$
$F = ma$
$(15\mathbf{i} + 18\mathbf{j}) = 2a$
$a = (7.5\mathbf{i} + 9\mathbf{j})$



$|a| = \sqrt{7.5^2 + 9^2} = \sqrt{137.25}$
Using Z angles (see diagram), bearing = θ
$\tan\theta = \dfrac{7.5}{9}$
The magnitude of the acceleration is 11.7 m s^{-2} and it has a bearing of 039.8° (both to 3 s.f.).

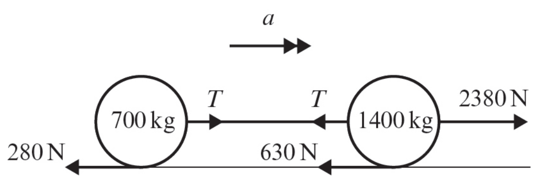
9 b ii $u = 0$, $t = 3$, $a = 11.7$, $s = ?$

$s = ut + \frac{1}{2}at^2$

$s = (0 \times 3) + \left(\frac{1}{2} \times 11.7 \times 3^2\right) = \frac{105.3}{2}$

The object travels a distance of 52.7 m (to 3 s.f.).

10



a $F = ma$

For the whole system:

$F = 2380 - 630 - 280 = 1470$

$m = 1400 + 700 = 2100$

$1470 = 2100a$

Since the tow-rope is inextensible, the acceleration of each part of the system is identical.

The acceleration of the car is 0.7 m s^{-2}.

b For the trailer:

$F = T - 280$, $m = 700$, $a = 0.7$

$T - 280 = 700 \times 0.7 = 490$

The tension in the tow-rope is 770 N.

c i For the car, after the rope breaks:

resultant force = 2380 – 630 = 1750

$m = 1400$

therefore $a = 1750 \div 1400 = 1.25$

$u = 12$

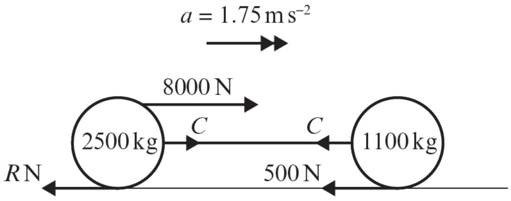
$s = ut + \frac{1}{2}at^2$

$s = (12 \times 4) + \left(\frac{1}{2} \times 1.25 \times 4^2\right) = 48 + 10$

In the first 4 s after the tow-rope breaks, the car travels 58 m.

ii Since the tow-rope is inextensible, the tension is constant throughout the length, and the acceleration of each part of the system is identical.

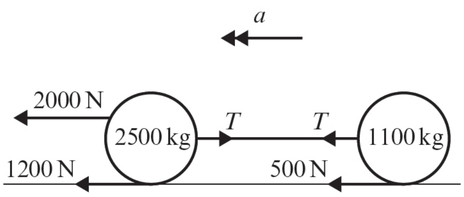
11



a $F = ma$
For the whole system:
$F = 8000 - 500 - R = 7500 - R$
$m = 2500 + 1100 = 3600$
$a = 1.75$
$7500 - R = 3600 \times 1.75 = 6300$
$R = 7500 - 6300$
The resistance to the motion of the train is 1200 N, as required.

b Considering the carriage only:
$C - 500 = 1100 \times 1.75 = 1925$
The compression force in the shunt is 2425 N.

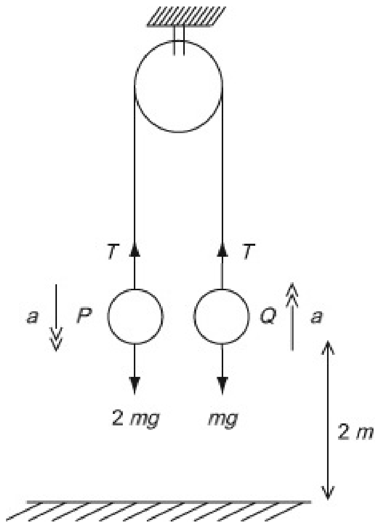
c



Taking ← as positive
Deceleration = α
Force on carriage = resistance to motion + thrust in shunt
Using $F = ma$
$500 + C = 1100\alpha$
$$\alpha = \frac{500 + C}{1100}$$
For engine:
$2000 + 1200 - C = 2500\alpha$
Substituting for α:
$$3200 - C = 2500 \times \left(\frac{500 + C}{1100}\right)$$
$$11\cancel{00}(3200 - C) = 25\cancel{00}(500 + C)$$
$$35200 - 11C = 12500 + 25C$$
$$35200 - 12500 = 11C + 25C$$
$$C = \frac{22700}{36}$$

The thrust in the shunt is 630 N (2 s.f.).

12 a



For P: $R(\downarrow)$, $2mg - T = 2ma$

For Q: $R(\uparrow)$, $T - mg = ma$

Add, $mg = 3ma$

$a = \frac{1}{3}g \text{ m s}^{-1}$

b For P:

$v^2 = u^2 + 2as$

$v^2 = 0 + 2 \times \frac{1}{3}g \times 2$

$v = \sqrt{\frac{4g}{3}}$

$= 3.6 \text{ m s}^{-1}$ (2 s.f.)

c For Q:

$R(\uparrow)$, $-mg = ma$

$a = -g$

$v^2 = u^2 + 2as$ $(\uparrow)$,

$0 = \frac{4g}{3} - 2gs$

$s = \frac{2}{3}\text{ m}$

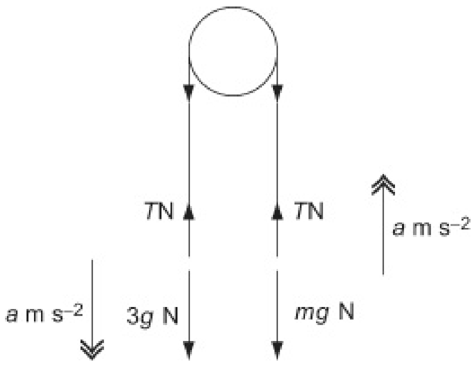
As Q is 4m above the ground when P stops moving $= 4 + s$

$\therefore$ Height above the ground $= 4\frac{2}{3}\text{ m}$

d i In an extensible string ⇒ acceleration of both masses is equal.

ii Smooth pulley ⇒ same tension in string either side of the pulley.

13 a



For the 3 kg mass

$R(\downarrow),\quad F = ma$

$3g - T = 3 \times \frac{3}{7}g$

$T = 3g - \frac{9}{7}g = \frac{12}{7}g$

The tension in the string is $\frac{12}{7}g$ N

b For the m kg mass

$R(\uparrow),\quad F = ma$

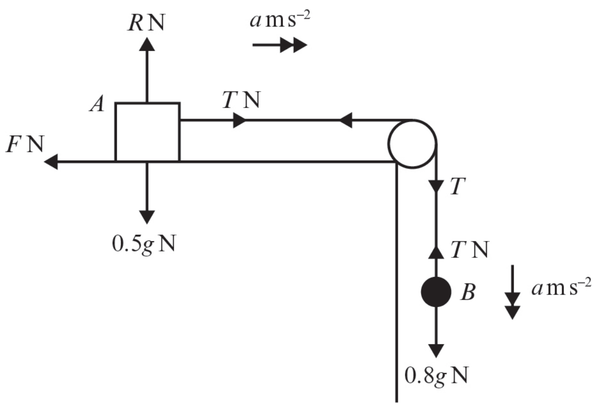
$T - mg = m \times \frac{3}{7}g$

Using the answer to **a**

$\frac{12}{7}g - mg = \frac{3}{7}mg$

$\frac{12}{7} = \frac{10}{7}m \Rightarrow m = 1.2$

14



a For B:

$u = 0,\ s = 0.4,\ t = 0.5,\ a = ?$

$s = ut + \frac{1}{2}at^2$

$0.4 = 0 + \frac{1}{2}a \times 0.5^2 = \frac{1}{8}a$

$a = 8 \times 0.4 = 3.2$

The acceleration of B is $3.2\ \mathrm{m\,s^{-2}}$

14 b For B:

$$\text{force} = ma$$

$$0.8g - T = 0.8 \times 3.2$$

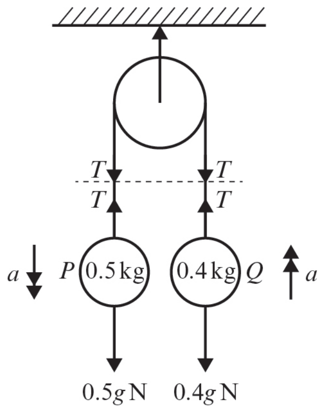
$$T = 0.8 \times 9.8 - 0.8 \times 3.2$$
$$= 5.3$$

The tension in the string is 5.3 N (2 s.f.). (As the numerical value $g = 9.8$ has been used, you should correct your answer to 2 significant figures.)

c $F = 3.7$ (2 s.f.)

d The information that the string is inextensible has been used in part **c** when the acceleration of A has been taken to be equal to the acceleration of B.

15



a **i** For P, $R(\downarrow)$: $0.5g - T = 0.5a$ **(1)**

ii For Q, $R(\uparrow)$: $T - 0.4g = 0.4a$ **(2)**

b **(1)** × 4: $2g - 4T = 2a$

(2) × 5: $5T - 2g = 2a$

Equating these:

$2g - 4T = 5T - 2g$

$9T = 4g$

The tension in the string is $\frac{4}{9}g$ N (4.35 N).

c Using equation **(1)**:

$\frac{1}{2}g - \frac{4}{9}g = \frac{1}{2}a$

$g - \frac{8}{9}g = a$

The acceleration is $\frac{1}{9}g$ m s^{-2} (1.09 m s^{-2} (3 s.f.)).

15 d When the string breaks, Q has moved up a distance s_1 and reached a speed v_1
Now Q moves under gravity (after the string breaks) initially upwards.
To reach the floor it has to travel a distance $s = 2 + s_1$

While the string is intact, up positive:

$u = 0,\ t = 0.2,\ a = \frac{g}{9},\ s_1 = ?$

$$s_1 = ut + \frac{1}{2}at^2$$
$$= (0\times 0.2) + \left(\frac{1}{2}\times\frac{g}{9}\times 0.2^2\right)$$
$$= \frac{g}{450}$$

$$v_1 = u + at$$
$$= 0 + \frac{g}{9}\times 0.2$$
$$= \frac{g}{45}$$

So, when the string breaks, Q is $2+\frac{g}{450}$ above the ground, a moving upwards with a speed of $\frac{g}{45}$.

After string breaks, Q moves under gravity. So taking down as positive, for the motion after the string breaks, we have

$u = v_1 = -\frac{g}{45},\ a = g,\ s = 2+\frac{g}{450},\ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$2+\frac{g}{450} = -\frac{g}{45}t + \frac{1}{2}gt^2$$
$$\frac{(900+g)}{450} = -\frac{g}{45}t + \frac{1}{2}gt^2$$
$$0 = \frac{1}{2}gt^2 - \frac{g}{45}t - \frac{(900+g)}{450}$$

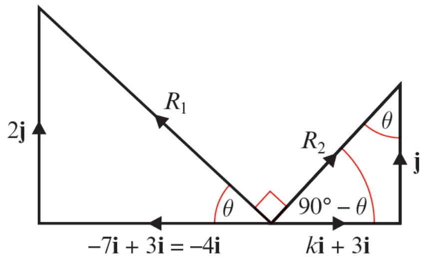
Let $g = 9.8 \Rightarrow 4.9t^2 - 0.21\dot{7}\dot{8}t - 2.02178 = 0$

$$t = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$t = \frac{-0.2178\pm\sqrt{(-0.2178)^2-(4\times 4.9\times -2.02178)}}{2\times 4.9}$$
$$= \frac{-0.218\pm\sqrt{39.674}}{9.8}$$
$$= 0.66\text{ s or } -0.621\text{ s}$$

Only the positive root is relevant: $t = 0.66$ (2 s.f.)
Q hits the floor 0.66 s after the string breaks.

Challenge

Total force on first boat: $R_1 = (-7\mathbf{i} + 2\mathbf{j}) + 3\mathbf{i} = -4\mathbf{i} + 2\mathbf{j}$
Total force on second boat: $R_2 = (k\mathbf{i} + \mathbf{j}) + 3\mathbf{i} = (k + 3)\mathbf{i} + \mathbf{j}$
Since mass is a vector quantity, the acceleration of each boat will be parallel to the resultant force acting on it, so the relationship between the components of the accelerations is as shown in the diagram below.



From R_1: $\tan\theta = \frac{2}{4} = \frac{1}{2}$

From R_2: $\tan\theta = \frac{k+3}{1} = k + 3$

Equating these: $\frac{1}{2} = k + 3$

$2k + 6 = 1$

$2k = -5$

The value of k is -2.5.

Variable acceleration 11A

1 a $s = 9(1) - 1^3 = 8$ m

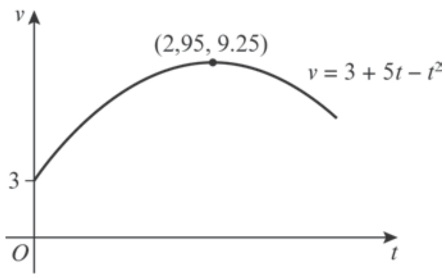
b $9t - t^3 = 0$
$t(9 - t^2) = 0$
Either $t = 0$ or $t^2 = 9$
$\Rightarrow t = 0$ or $t = \pm 3$

2 a At $t = 2$,
$s = 5(2)^2 - 2^3 = 12$
At $t = 4$,
$s = 5(4)^2 - 4^3 = 16$
Change in displacement $= 16 - 12 = 4$ m

b At $t = 3$,
$s = 5(3)^2 - 3^3 = 18$
Change in displacement in the third second $= 18 - 12 = 6$ m

3 a $v = 3 + 5(1) - 1^2 = 7$ m s^{-1}

b



At $t = 0$, $v = 3$
$v = 3$ again when $5t - t^2 = 0 \Rightarrow t = 5$

Using symmetry, turning point is when $t = 2.5$.
When $t = 2.5$,
$v = 3 + 5(2.5) - 2.5^2 = 9.25$
So in $0 \leq t \leq 4$, range of v is $3 \leq v \leq 9.25$
Greatest speed is 9.25 m s^{-1}.

c $v = 3 + 5(7) - 7^2$
$= 3 + 35 - 49$
$= -11$
When $t = 7$, the velocity of the particle is -11 m s^{-1}. This means it is moving in the opposite direction to that in which it was initially travelling.

4 a $s = 0$ when
$\frac{1}{5}(4t - t^2) = 0$
$\frac{1}{5}t(4 - t) = 0$
$\Rightarrow t = 0$ or $t = 4$
By symmetry, maximum distance is when $t = 2$.
When $t = 2$, $s = \frac{1}{5}(4(2) - 2^2)$
$= \frac{4}{5}$

4 a The maximum displacement is 0.8 m.

b When the toy car returns to P, $s = 0$

$\frac{1}{5}(4t - t^2) = 0$

$\frac{1}{5}t(4 - t) = 0$

$\Rightarrow t = 0$ or $t = 4$

The toy car returns to P after 4 s.

c The toy car travels to maximum distance and back again.

So total distance = 0.8 + 0.8 = 1.6 m

d The model is valid for $0 \leq t \leq 4$.

5 a When $t = 0$,

$v = 0 - 0 + 8 = 8$

The initial velocity is 8 m s^{-1}.

b $3t^2 - 10t + 8 = 0$

$(3t - 4)(t - 2) = 0$

The body is at rest when $t = \frac{4}{3}$ and $t = 2$.

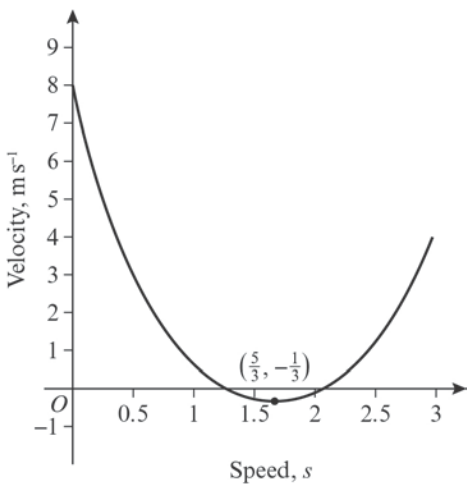
c $3t^2 - 10t + 8 = 5$

$3t^2 - 10t + 3 = 0$

$(3t - 1)(t - 3) = 0$

Velocity = 5 m s^{-1} when $t = \frac{1}{3}$ and $t = 3$.

d



Using the answer to part **b** and symmetry, the body has its maximum/minimum velocity when $t = \frac{5}{3}$ s.

When $t = \frac{5}{3}$,

$$v = 3\left(\frac{5}{3}\right)^2 - 10\left(\frac{5}{3}\right) + 8$$

$$= \frac{25}{3} - \frac{50}{3} + 8$$

$$= -\frac{25}{3} + 8$$

5 d $v = -\frac{1}{3}$

So in $0 \le t \le 2$, range of v is $-\frac{1}{3} \le v \le 8$.

Greatest speed is 8 m s^{-1}.

6 a $8t - 2t^2 = 0$

$2t(4 - t) = 0$

The particle is next at rest after 4 s.

b By symmetry, minimum/maximum velocity is when $t = 2$.

When $t = 2$,

$v = 8(2) - 2(2)^2$

$= 8$

So in $0 \le t \le 4$, greatest speed is 8 m s^{-1}.

7 $s = 3t^2 - t^3$

Model is valid until particle returns to starting point, i.e. until next point at which $s = 0$.

After this it would have a negative displacement, i.e. be beyond O.

$s = 0$ when

$3T^2 - T^3 = 0$

$T^2(3 - T) = 0$

$T \neq 0$ so $T = 3$

8 a $\frac{1}{5}\left(3t^2 - 10t + 3\right) = 0$

$3t^2 - 10t + 3 = 0$

$(3t - 1)(t - 3) = 0$

Particle is at rest when $t = \frac{1}{3}$ and $t = 3$.

b Using answer to part **a** and symmetry, the body has its maximum/minimum velocity when $t = \frac{5}{3}$.

When $t = \frac{5}{3}$,

$$v = \frac{1}{5}\left(3\left(\frac{5}{3}\right)^2 - 10\left(\frac{5}{3}\right) + 3\right)$$

$$= \frac{1}{5}\left(\frac{25}{3} - \frac{50}{3} + \frac{9}{3}\right)$$

$$= \frac{1}{5}\left(-\frac{16}{3}\right)$$

$$= -\frac{16}{15}$$

So in $0 \le t \le 3$, greatest speed is $\frac{16}{15}\text{ m s}^{-1}$.

Variable acceleration 11B

1 a $s = 4t^4 - \frac{1}{t}$

i $v = \frac{ds}{dt} = 16t^3 + \frac{1}{t^2}$

ii $a = \frac{dv}{dt} = 48t^2 - \frac{2}{t^3}$

b $x = \frac{2}{3}t^3 + \frac{1}{t^2}$

i $v = \frac{dx}{dt} = 2t^2 - \frac{2}{t^3}$

ii $a = \frac{dv}{dt} = 4t + \frac{6}{t^4}$

c $s = (3t^2 - 1)(2t + 5)$
$= 6t^3 + 15t^2 - 2t - 5$

i $v = \frac{ds}{dt} = 18t^2 + 30t - 2$

ii $a = \frac{dv}{dt} = 36t + 30$

d $x = \frac{3t^4 - 2t^3 + 5}{2t} = \frac{3t^3}{2} - t^2 + \frac{5}{2t}$

i $v = \frac{dx}{dt} = \frac{9t^2}{2} - 2t - \frac{5}{2t^2}$

ii $a = \frac{dv}{dt} = 9t - 2 + \frac{5}{t^3}$

2 a $x = 2t^3 - 8t$
$v = \frac{dx}{dt} = 6t^2 - 8$

When $t = 3$, $v = 6 \times 3^2 - 8 = 46$

The velocity of the particle when $t = 3$ is $46\ \text{m s}^{-1}$.

b $a = \frac{dv}{dt} = 12t$
When $t = 2$, $a = 12 \times 2 = 24$

The acceleration of the particle when $t = 2$ is $24\ \text{m s}^{-2}$.

3 P is at rest when $v = 0$.
$12 - t - t^2 = 0$
$(4 + t)(3 - t) = 0$
$t = -4$ or $t = 3$
$t \geq 0$, so $t = 3$

$$a = \frac{dv}{dt} = -1 - 2t$$

When $t = 3$, $a = -1 - 2 \times 3 = -7$

The acceleration of P when P is instantaneously at rest is -7 m s^{-2}, or 7 m s^{-2} in the direction of x decreasing.

4 $x = 4t^3 - 39t^2 + 120t$

$$v = \frac{dx}{dt} = 12t^2 - 78t + 120$$

P is at rest when $v = 0$.
$12t^2 - 78t + 120 = 0$
$2t^2 - 13t + 20 = 0$
$(2t - 5)(t - 4) = 0$

P is at rest when $t = 2.5$ and $t = 4$.

When $t = 2.5$, $x = 4(2.5)^3 - 39(2.5)^2 + 120(2.5) = 118.75$

When $t = 4$, $x = 4(4)^3 - 39(4)^2 + 120(4) = 112$

The distance between the two points where P is instantaneously at rest is $118.75 - 112 = 6.75$ m.

5 $v = kt - 3t^2$

a $$a = \frac{dv}{dt} = k - 6t$$

When $t = 0$, $a = 4$
$k - 6 \times 0 = 4$
$k = 4$

b P is at rest when $v = 0$.
$4t - 3t^2 = 0$
$t(4 - 3t) = 0$
P is at rest when $t = 0$ and $t = \frac{4}{3}$.

When $t = \frac{4}{3}$, $a = 4 - 6 \times \frac{4}{3} = 4 - 8 = -4$
When P is next at rest, the acceleration is -4 m s^{-2}.

6 $s = \frac{1}{4}(4t^3 - 15t^2 + 12t + 30)$

$$v = \frac{ds}{dt} = \frac{1}{4}(12t^2 - 30t + 12)$$

6 The print head is at rest when $v = 0$.

$\frac{1}{4}(12t^2 - 30t + 12) = 0$

$12t^2 - 30t + 12 = 0$

$2t^2 - 5t + 2 = 0$

$(2t - 1)(t - 2) = 0$

The print head is at rest when $t = 0.5$ and $t = 2$.

When $t = 0.5$,

$s = \frac{1}{4}(4(0.5)^3 - 15(0.5)^2 + 12(0.5) + 30)$

$= \frac{1}{4}(0.5 - 3.75 + 6 + 30)$

$= 8.1875$

When $t = 2$,

$s = \frac{1}{4}(4(2)^3 - 15(2)^2 + 12(2) + 30)$

$= \frac{1}{4}(32 - 60 + 24 + 30)$

$= 6.5$

Distance between these two points $= 8.1875 - 6.5$

$= 1.6875$ cm

$= 1.7$ cm (1 d.p.)

The distance between the points when the print head is instantaneously at rest is 1.7 cm.

Variable acceleration 11C

1 a $s = 0.4t^3 - 0.3t^2 - 1.8t + 5$

$v = \frac{ds}{dt} = 1.2t^2 - 0.6t - 1.8$

$\frac{dv}{dt} = 2.4t - 0.6$

$\frac{dv}{dt} = 0$ when $2.4t = 0.6$

$t = 0.25$

P is moving with minimum velocity at $t = 0.25$ s.

b When $t = 0.25$

$s = 0.4(0.25)^3 - 0.3(0.25)^2 - 1.8(0.25) + 5$

$= 4.54$ (3 s.f.)

When P is moving with minimum velocity, the displacement is 4.54 m.

c $v = \frac{ds}{dt} = 1.2t^2 - 0.6t - 1.8$

When $t = 0.25$, $v = 1.2 \times 0.25^2 - 0.6 \times 0.25 - 1.8$

$= -1.88$ (3 s.f.)

2 a $s = 4t^3 - t^4$

When $t = 4$,

$s = 4(4)^3 - 4^4 = 0$

The body returns to its starting position 4 s after leaving it.

b $s = 4t^3 - t^4 = s = t^3(4 - t)$

Since $t \geq 0$, t^3 is always positive.

Since $t \leq 4$, $(4 - t)$ is always positive.

So for $0 \leq t \leq 4$, s is always non-negative.

c $\frac{ds}{dt} = 12t^2 - 4t^3$

$\frac{ds}{dt} = 0$ when

$12t^2 - 4t^3 = 0$

$4t^2(3 - t) = 0$

$t = 0$ or 3

At $t = 0$, the body is at $s = 0$, so maximum displacement occurs when $t = 3$.

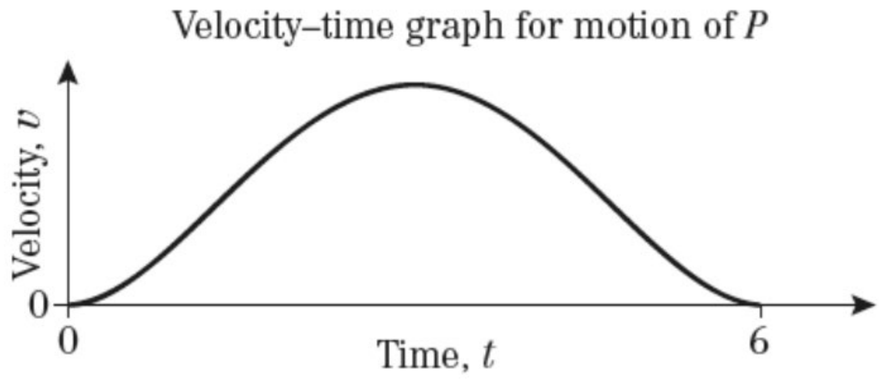
When $t = 3$, using factorised form of the equation of motion:

$s = 3^3(4 - 3) = 27$

The maximum displacement of the body from its starting point is 27 m.

3 a $v = t^2(6-t)^2$

Velocity is zero when $t = 0$ and $t = 6$.
The graph touches the time axis at $t = 0$ and $t = 6$.
Graph only shown for $0 \le t \le 6$, as this is the range over which equation is valid.



b $v = t^2(6-t)^2$
$= t^2(36 - 12t + t^2)$
$= 36t^2 - 12t^3 + t^4$

$$\frac{dv}{dt} = 72t - 36t^2 + 4t^3$$

$\frac{dv}{dt} = 0$ when
$72t - 36t^2 + 4t^3 = 0$
$4t(18 - 9t + t^2) = 0$
$4t(3-t)(6-t) = 0$

The turning points are at $t = 0$, $t = 3$ and $t = 6$.
$v = 0$ when $t = 0$ and $t = 6$, therefore the maximum velocity occurs when $t = 3$.

When $t = 3$,
$v = 3^2(6-3)^2 = 9 \times 9 = 81$

The maximum velocity is 81 m s^{-1} and the body reaches this 3 s after leaving O.

4 a $v = 2t^2 - 3t + 5$

For this particle to come to rest, v must be 0 for some positive value of t.

$2t^2 - 3t + 5 = 0$ must have real, positive roots.
$b^2 - 4ac = (-3)^2 - 4(2)(5)$
$= 9 - 40$
$= -31 < 0$

The equation therefore has no real roots, so v is never zero.

4 b $v = 2t^2 - 3t + 5$

$\frac{dv}{dt} = 4t - 3$

$\frac{dv}{dt} = 0$ when $4t = 3$

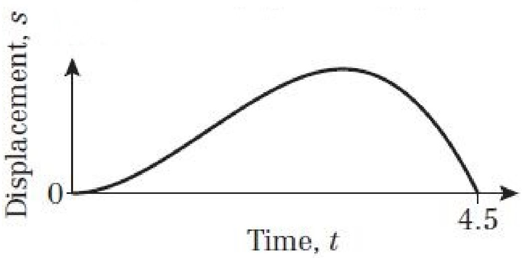
$t = 0.75$

Minimum velocity is when $t = 0.75$.
When $t = 0.75$, $v = 2(0.75)^2 - 3(0.75) + 5$
$= 1.125 - 2.25 + 5$
$= 3.875$
$= 3.88$ (3 s.f.)
The minimum velocity of the particle is 3.88 m s^{-1}.

5 a $s = \frac{9t^2}{2} - t^3$
$= t^2(4.5 - t)$

Displacement is zero when $t = 0$ and $t = 4.5$.
The graph touches the time axis at $t = 0$ and crosses it at $t = 4.5$.

Graph only shown for $0 \le t \le 4.5$, as this is range over which equation is valid.
The curve is cubic, so not symmetrical.



b For values of $t > 4.5$, s is negative. However s is a distance and can only be positive.

c $s = \frac{9t^2}{2} - t^3$

$\frac{ds}{dt} = 9t - 3t^2$

$\frac{ds}{dt} = 0$ when

$9t - 3t^2 = 0$
$3t(3 - t) = 0$

The turning points are at $t = 0$ and $t = 3$.
$s = 0$ when $t = 0$, so maximum distance occurs when $t = 3$.

When t =3, using factorised form of the equation of motion:
$s = 3^2(4.5 - 3) = 9 \times 1.5 = 13.5$

The maximum distance of P from O is 13.5 m.

5 d $v = \dfrac{ds}{dt} = 9t - 3t^2$

$a = \dfrac{dv}{dt} = 9 - 6t$

When $t = 3$,

$a = 9 - 6 \times 3 = -9$

The magnitude of the acceleration of P at the maximum distance is 9 m s^{-2}.

6 $s = 3.6t + 1.76t^2 - 0.02t^3$

$\dfrac{ds}{dt} = 3.6 + 3.52t - 0.06t^2$

Maximum distance occurs when $\dfrac{ds}{dt} = 0$.

$\dfrac{ds}{dt} = 0$ when

$3.6 + 3.52t - 0.06t^2 = 0$

$3t^2 - 176t - 180 = 0$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{176 \pm \sqrt{(-176)^2 + (4)(3)(180)}}{2 \times 3}$$
$$= \frac{176 \pm \sqrt{33\,136}}{6}$$
$$= -1.005 \text{ or } 59.67$$

$t > 0$, so maximum distance occurs when $t = 59.67$.

When $t = 59.67$, $s = 3.6(59.67) + 1.76(59.67)^2 - 0.02(59.67)^3$

$= 2230$ (3 s.f.)

The maximum distance from the start of the track is 2230 m or 2.23 km.
Since this is less than 4 km, the train never reaches the end of the track.

Variable acceleration 11D

1 a $s = \int v \, dt$

$= \int (3t^2 - 1) dt$

$= t^3 - t + c$, where c is a constant of integration.

When $t = 0$, $x = 0$:
$0 = 0 - 0 + c \Rightarrow c = 0$
$s = t^3 - t$

b $s = \int v \, dt$

$= \int \left(2t^3 - \frac{3t^2}{2}\right) dt$

$= \frac{t^4}{2} - \frac{t^3}{2} + c$, where c is a constant of integration.

When $t = 0$, $x = 0$:
$0 = 0 - 0 + c \Rightarrow c = 0$

$s = \frac{t^4}{2} - \frac{t^3}{2}$

c $s = \int v \, dt$

$= \int (2\sqrt{t} + 4t^2) dt$

$= \frac{4}{3}t^{\frac{3}{2}} + \frac{4t^3}{3} + c$, where c is a constant of integration.

When $t = 0$, $x = 0$:
$0 = 0 + 0 + c \Rightarrow c = 0$

$s = \frac{4}{3}t^{\frac{3}{2}} + \frac{4t^3}{3}$

2 a $v = \int a \, dt$

$= \int (8t - 2t^2) dt$

$= 4t^2 - \frac{2t^3}{3} + c$, where c is a constant of integration.

When $t = 0$, $v = 0$:
$0 = 0 - 0 + c \Rightarrow c = 0$

$v = 4t^2 - \frac{2t^3}{3}$

b $v = \int a \, dt$

$= \int \left(6 + \frac{t^2}{3}\right) dt$

2 b $v = 6t + \frac{t^3}{9} + c$, where c is a constant of integration.

When $t = 0$, $v = 0$:
$0 = 0 + 0 + c \Rightarrow c = 0$
$v = 6t + \frac{t^3}{9}$

3 $x = \int v\,dt$
$= \int (8 + 2t - 3t^2)\,dt$
$= 8t + t^2 - t^3 + c,$ where c is a constant of integration.

When $t = 0$, $x = 4$:
$4 = 0 + 0 - 0 + c \Rightarrow c = 4$
$x = 8t + t^2 - t^3 + 4$

When $t = 1$,
$x = 8 + 1 - 1 + 4 = 12$

The distance of P from O when $t = 1$ is 12 m.

4 a $v = \int a\,dt$
$= \int (16 - 2t)\,dt$
$= 16t - t^2 + c,$ where c is a constant of integration.

When $t = 0$, $v = 6$:
$6 = 0 - 0 + c \Rightarrow c = 6$
$v = 16t - t^2 + 6$

b $x = \int v\,dt$
$= \int (16t - t^2 + 6)\,dt$
$= 8t^2 - \frac{t^3}{3} + 6t + k$, where k is a constant of integration.

When $t = 3$, $x = 75$:
$75 = 8 \times 3^2 - \frac{3^3}{3} + 6 \times 3 + k$
$\Rightarrow k = 75 - 72 + 9 - 18 = -6$
$x = 8t^2 - \frac{t^3}{3} + 6t - 6$

4 b When $t = 0$,
$x = 0 - 0 + 0 - 6 = -6$

5 $v = 6t^2 - 51t + 90$
P is at rest when $v = 0$.
$6t^2 - 51t + 90 = 0$

$2t^2 - 17t + 30 = 0$

$(2t-5)(t-6)=0$

P is at rest when $t = 2.5$ and when $t = 6$.

$$s = \int_{2.5}^{6} (6t^2 - 51t + 90)\,dt$$
$$= \left[2t^3 - \frac{51t^2}{2} + 90t\right]_{2.5}^{6}$$
$$= \left(2\times 6^3 - \frac{51\times 6^2}{2} + 90\times 6\right) - \left(2\times 2.5^3 - \frac{51\times 2.5^2}{2} + 90\times 2.5\right)$$
$$= (432 - 918 + 540) - (31.25 - 159.375 + 225)$$
$$= -42.875\ldots$$
$$= -42.9 \text{ (3 s.f.)}$$

The negative sign indicates that the displacement is negative, but this can be ignored as distance is required.

The distance between the two points where P is at rest is 42.9 m (3 s.f.).

6 $s = \int v\,dt$

$$= \int (12 + t - 6t^2)\,dt$$
$$= 12t + \frac{t^2}{2} - 2t^3 + c \text{, where } c \text{ is a constant of integration.}$$

When $t = 0$, $s = 0$:
$0 = 0 + 0 - 0 + c \Rightarrow c = 0$
$$s = 12t + \frac{t^2}{2} - 2t^3$$

$v = 0$ when
$12 + t - 6t^2 = 0$
$(3-2t)(4+3t) = 0$
$t > 0$, so $t = 1.5$

When $t = 1.5$, $s = 12\times 1.5 + \frac{1.5^2}{2} - 2\times 1.5^3$
$= 12.375\ldots$
$= 12.4$ (3 s.f.)

The distance of P from O when $v = 0$ is 12.4 m.

7 a $v = 4t - t^2$
P is at rest when $v = 0$.
$4t - t^2 = 0$
$t(4 - t) = 0$
$t > 0$, so $t = 4$

$$x = \int v\,\mathrm{d}t$$
$$= \int (4t - t^2)\,\mathrm{d}t$$
$$= 2t^2 - \frac{t^3}{3} + c$$, where c is a constant of integration.

When $t = 0$, $x = 0$
$0 = 0 + 0 + c \Rightarrow c = 0$
$$x = 2t^2 - \frac{t^3}{3}$$

When $t = 4$, $x = 2 \times 4^2 - \frac{4^3}{3}$
$= 10\frac{2}{3}$

b When $t = 5$, $x = 2 \times 5^2 - \frac{5^3}{3}$
$= 8\frac{1}{3}$

In the interval $0 \leq t \leq 5$, P moves to a point $10\frac{2}{3}$ m from O and then returns to a point $8\frac{1}{3}$ m from O.

The total distance moved is $10\frac{2}{3} + (10\frac{2}{3} - 8\frac{1}{3}) = 13$ m.

8 $$x = \int v\,\mathrm{d}t$$
$$= \int (6t^2 - 26t + 15)\,\mathrm{d}t$$
$= 2t^3 - 13t^2 + 15t + c$, where c is a constant of integration.

When $t = 0$, $x = 0$
$0 = 0 - 0 + 0 + c \Rightarrow c = 0$
$x = 2t^3 - 13t^2 + 15t$
$= t(2t^2 - 13t + 15)$
$= t(2t - 3)(t - 5)$

When $x = 0$ and t is non-zero, $t = 1.5$ or $t = 5$

P is again at O when $t = 1.5$ and $t = 5$.

9 a $$x = \int v\,\mathrm{d}t$$
$$= \int (3t^2 - 12t + 5)\,\mathrm{d}t$$
$= t^3 - 6t^2 + 5t + c$, where c is a constant of integration.

9 a When $t = 0$, $x = 0$

$0 = 0 - 0 + 0 + c \Rightarrow c = 0$

$x = t^3 - 6t^2 + 5t$

P returns to O when $x = 0$.

$t^3 - 6t^2 + 5t = 0$

$t(t^2 - 6t + 5) = 0$

$t(t - 1)(t - 5) = 0$

P returns to O when $t = 1$ and $t = 5$.

b $v = 0$ when

$3t^2 - 12t + 5 = 0$

$$t = \frac{12 \pm \sqrt{(-12)^2 - 4(3)(5)}}{6}$$

$= 0.473, 3.52$

So P does not turn round in the interval $2 \leq t \leq 3$.

When $t = 2$,

$x = 2^3 - 6 \times 2^2 + 5 \times 2$

$= 8 - 24 + 10$

$= -6$

When $t = 3$,

$x = 3^3 - 6 \times 3^2 + 5 \times 3$

$= 27 - 54 + 15$

$= -12$

The distance travelled by P in the interval $2 \leq t \leq 3$ is 6 m.

10 $v = \int a \, dt$

$= \int (4t - 3) \, dt$

$= 2t^2 - 3t + c$, where c is a constant of integration.

When $t = 0$, $v = 4$

$4 = 0 - 0 + c \Rightarrow c = 4$

$v = 2t^2 - 3t + 4$,

When $t = T$, $v = 4$ again

$4 = 2T^2 - 3T + 4$

$2T^2 - 3T = 0$

$T(2T - 3) = 0$

$T \neq 0$, so $T = 1.5$

11 a $v = \int a \, dt$

$= \int (t - 3) \, dt$

$= \frac{t^2}{2} - 3t + c$,

11 a When $t = 0$, $v = 4$

$4 = 0 - 0 + c \Rightarrow c = 4$

$$v = \frac{t^2}{2} - 3t + 4$$

b P is at rest when $v = 0$.

$$\frac{t^2}{2} - 3t + 4 = 0$$

$$t^2 - 6t + 8 = 0$$

$$(t - 2)(t - 4) = 0$$

$t = 2$ or $t = 4$

P is at rest when $t = 2$ and $t = 4$.

c
$$\begin{aligned} s &= \int_2^4 \left(\frac{t^2}{2} - 3t + 4\right) dt \\ &= \left[\frac{t^3}{6} - \frac{3t^2}{2} + 4t\right]_2^4 \\ &= \left(\frac{4^3}{6} - \frac{3\times 4^2}{2} + 4\times 4\right) - \left(\frac{2^3}{6} - \frac{3\times 2^2}{2} + 4\times 2\right) \\ &= \left(\frac{32}{3} - 24 + 16\right) - \left(\frac{4}{3} - 6 + 8\right) \\ &= \frac{8}{3} - \frac{10}{3} \\ &= -\tfrac{2}{3} \end{aligned}$$

The negative sign indicates that the displacement is negative, but this can be ignored as distance is required. The distance between the two points where P is at rest is $\tfrac{2}{3}$ m.

12
$$\begin{aligned} v &= \int a\,dt \\ &= \int (6t + 2)\,dt \end{aligned}$$

$= 3t^2 + 2t + c$, where c is a constant of integration.

$$\begin{aligned} s &= \int v\,dt \\ &= \int (3t^2 + 2t + c)\,dt \end{aligned}$$

$= t^3 + t^2 + ct + k$, where k is a constant of integration.

When $t = 2$, $s = 10$

$10 = 2^3 + 2^2 + 2c + k$

$2c + k = -2$ **(1)**

When $t = 3$, $s = 38$

$38 = 3^3 + 3^2 + 3c + k$

$3c + k = 2$ **(2)**

(2) − **(1)**:

$c = 4$

12 Substituting $c = 4$ into (**1**):
$2 \times 4 + k = -2$
$k = -10$

So the equations are:
$v = 3t^2 + 2t + 4$
$s = t^3 + t^2 + 4t - 10$

a When $t = 4$
$s = 4^3 + 4^2 + 4 \times 4 - 10$
$= 64 + 16 + 16 - 10$
$= 86$

When $t = 4$ s the displacement is 86 m.

b When $t = 4$
$v = 3 \times 4^2 + 2 \times 4 + 4$
$= 48 + 8 + 4$
$= 60$

When $t = 4$ s the velocity is 60 m s^{-1}.

Challenge

At $t = k$, the velocity given by both equations is identical, so:

$$\frac{k^2}{2}+2 = 10+\frac{k}{3}-\frac{k^2}{12}$$

$$6k^2 + 24 = 120 + 4k - k^2$$

$$7k^2 - 4k - 96 = 0$$

$$k = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$= \frac{4\pm\sqrt{4^2+4\times7\times96}}{2\times7}$$

$$= \frac{4\pm52}{14}$$

$k > 0$, so $k = 4$

For first part of the motion, up to $t = 4$, $s = s_1$

$$s_1 = \int_0^4\left(\frac{t^2}{2}+2\right)\mathrm{d}t$$

$$= \left[\frac{t^3}{6}+2t\right]_0^4$$

$$= \left(\frac{4^3}{6}+2\times4\right)-\left(\frac{0^3}{6}+0\times4\right)$$

$$= \frac{56}{3}$$

For second part of the motion, from $t = 4$ to $t = 10$, $s = s_2$

$$s_2 = \int_4^{10}\left(10+\frac{t}{3}-\frac{t^2}{12}\right)\mathrm{d}t$$

$$= \left[10t+\frac{t^2}{6}-\frac{t^3}{36}\right]_4^{10}$$

$$= \left(10\times10+\frac{10^2}{6}-\frac{10^3}{36}\right)-\left(10\times4+\frac{4^2}{6}-\frac{4^3}{36}\right)$$

$$= \frac{800}{9}-\frac{368}{9}$$

$$= 48$$

$$\text{Total distance} = s_1 + s_2$$

$$= \frac{56}{3} + 48$$

$$= \frac{200}{3}$$

The total distance travelled by the arm is $\frac{200}{3}$ m.

Variable acceleration 11E

1 $v = \int a\mathrm{d}t$

$= at + c$, where c is a constant of integration.

When $t = 0$, $v = 0$

$0 = a \times 0 + c \Rightarrow c = 0$

$v = at$

$s = \int v\mathrm{d}t$

$= \int at\mathrm{d}t$

$= \frac{1}{2}at^2 + k$, where k is a constant of integration.

When $t = 0$, $s = x$

$x = \frac{1}{2} \times a \times 0^2 + k \;\Rightarrow k = x$

$s = \frac{1}{2}at^2 + x$

2 a $v = \int a\mathrm{d}t$

$= \int 5\mathrm{d}t$

$= 5t + c$, where c is a constant of integration.

When $t = 0$, $v = 12$

$12 = 0 + c \Rightarrow c = 12$

$v = 12 + 5t$

b $s = \int v\mathrm{d}t$

$= \int (12 + 5t)\mathrm{d}t$

$= 12t + \frac{5}{2}t^2 + k$, where k is a constant of integration.

When $t = 0$, $s = 7$

$7 = 0 + 0 + k \Rightarrow k = 7$

$s = 12t + \frac{5}{2}at^2 + 7$

$= 12t + 2.5t^2 + 7$

3 $s = ut + \frac{1}{2}at^2$

$v = \frac{\mathrm{d}s}{\mathrm{d}t} = u + at$

$a = \frac{\mathrm{d}v}{\mathrm{d}t} = a$

So acceleration is constant.

4 A $s = 2t^2 - t^3$

$v = \frac{\mathrm{d}s}{\mathrm{d}t} = 4t - 3t^2$

4 A $a = \frac{dv}{dt} = 4 - 6t$

Not constant

B $s = 4t + 7$
$v = \frac{ds}{dt} = 4$
$a = \frac{dv}{dt} = 0$

No acceleration

C $s = \frac{t^2}{4}$
$v = \frac{ds}{dt} = \frac{t}{2}$
$a = \frac{dv}{dt} = \frac{1}{2}$

Constant acceleration

D $s = 3t - \frac{2}{t^2}$
$v = \frac{ds}{dt} = 3 + \frac{4}{t^3}$
$a = \frac{dv}{dt} = -\frac{12}{t^4}$

Not constant

E $s = 6$
$v = \frac{ds}{dt} = 0$

Particle stationary

5 **a** $v = u + at$
$u = 5, v = 13, t = 2$
$13 = 5 + 2a$
$a = \frac{13-5}{2} = 4$

The acceleration of the particle is 4 m s^{-2}.

b $v = \int a\,dt$
$= \int 4\,dt$
$= 4t + c$, where c is a constant of integration.

5 b When $t = 0$, $v = 5$
$5 = 0 + c \Rightarrow c = 5$
$v = 4t + 5$
$s = \int v\,\mathrm{d}t$
$= \int (4t + 5)\,\mathrm{d}t$
$= 2t^2 + 5t + k$, where k is a constant of integration.

When $t = 0$, $s = 0$
$0 = 0 + 0 + k \Rightarrow k = 0$
$s = 2t^2 + 5t$

This is an equation of the required form with $p = 2$, $q = 5$ and $r = 0$.

6 a $s = 25t - 0.2t^2$
When $t = 40$, $s = 25 \times 40 - 0.2 \times 40^2$
$= 680$

The distance AB is 680 m.

b $v = \frac{\mathrm{d}s}{\mathrm{d}t} = 25 - 0.4t$

$a = \frac{\mathrm{d}v}{\mathrm{d}t} = -0.4$

The train has a constant acceleration (of -0.4 m s^{-2}).

c Taking the direction in which the train travels to be positive:
For the bird: $a = -0.6$, $u = -7$, initial displacement = 680
$v_B = \int a\,\mathrm{d}t$
$= \int -0.6\,\mathrm{d}t$
$= -0.6t + c$, where c is a constant of integration.

When $t = 0$, $v_B = -7$
$-7 = 0 + c \Rightarrow c = -7$
$v = -0.6t - 7$
$s_B = \int v_B\,\mathrm{d}t$
$= \int (-0.6t - 7)\,\mathrm{d}t$
$= -0.3t^2 - 7t + k$, where k is a constant of integration.

When $t = 0$, $s_B = 680$
$680 = 0 - 0 + k \Rightarrow k = 680$
$s_B = -0.3t^2 - 7t + 680$

When the bird is directly above the train, the displacement of both train and bird are the same.

$25t - 0.2t^2 = -0.3t^2 - 7t + 680$
$0.1t^2 + 32t - 680 = 0$
$t^2 + 320t - 6800 = 0$
$(t - 20)(t + 340) = 0$

6 c $t > 0$, so $t = 20$

When $t = 20$,
$s = 25 \times 20 - 0.2 \times 20^2$
$= 420$

The bird is directly above the train 420 m from *A*.

Variable acceleration, Mixed Exercise 11

1 a $v = 15 - 3t$

P is at rest when $v = 0$.

$0 = 15 - 3t$

$t = 5$

b $$s = \int_0^5 v\,\mathrm{d}t$$
$$= \int_0^5 (15 - 3t)\,\mathrm{d}t$$
$$= \left[15t - \frac{3t^2}{2}\right]_0^5$$
$$= \left(15 \times 5 - \frac{3 \times 5^2}{2}\right) - 0$$
$$= 37.5$$

The distance travelled by P is 37.5 m.

2 a $v = 6t + \frac{1}{2}t^3$

$$a = \frac{\mathrm{d}v}{\mathrm{d}t}$$
$$= 6 + \tfrac{3}{2}t^2$$

When $t = 4$, $a = 6 + \frac{3}{2} \times 4^2$

$= 30$

The acceleration of P when $t = 4$ is 30 m s^{-2}.

b $$x = \int v\,\mathrm{d}t$$
$$= \int (6t + \tfrac{1}{2}t^3)\,\mathrm{d}t$$
$= 3t^2 + \dfrac{t^4}{8} + k$, where k is a constant of integration.

When $t = 0$, $x = -5$

$-5 = 0 + 0 + k \Rightarrow k = -5$

$$x = 3t^2 + \frac{t^4}{8} - 5$$

When $t = 4$, $x = 3 \times 4^2 + \dfrac{4^4}{8} - 5$

$= 75$

$OP = 75$ m

3 a $v = \int a\,\mathrm{d}t$

$= \int (2 - 2t)\,\mathrm{d}t$

$= 2t - t^2 + c$, where c is a constant of integration.

When $t = 0$, $v = 8$

$8 = 0 - 0 + c \Rightarrow c = 8$

$v = 2t - t^2 + 8$

Let s m be the displacement from A at time t seconds.

$s = \int v\,\mathrm{d}t$

$= \int (2t - t^2 + 8)\,\mathrm{d}t$

$= t^2 - \frac{t^3}{3} + 8t + k$, where k is a constant of integration.

When $t = 0$, $s = 0$

$0 = 0 - 0 + 0 + k \Rightarrow k = 0$

Displacement of P from A at time t seconds $= t^2 - \frac{t^3}{3} + 8t$

b The greatest positive displacement of P occurs when $\frac{\mathrm{d}s}{\mathrm{d}t} = v = 0$:

$2t - t^2 + 8 = 0$

$t^2 - 2t - 8 = 0$

$(t + 2)(t - 4) = 0$

$t > 0$, so $t = 4$

When $t = 4$, $s = 4^2 - \frac{4^3}{3} + 8 \times 4$

$= 26\tfrac{2}{3} < 30$

Hence, P does not reach B.

c P returns to A when $s = 0$.

$t^2 - \frac{t^3}{3} + 8t = 0$

$t^3 - 3t^2 - 24t = 0$

$t(t^2 - 3t - 24) = 0$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-24)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{105}}{2}$$

$t > 0$, so $t = 6.62$

P returns to A when $t = 6.62$.

3 d Distance between two instants when P passes through A = 2 × maximum distance found in part **b**

$$= 2 \times \tfrac{80}{3}$$
$$= \tfrac{160}{3}.$$

Total distance travelled by P between the two instants when it passes through A is $\frac{160}{3}$ m.

4 a $a = \dfrac{dv}{dt}$ so speed has maximum value when $a = 0$.

$0 = 8 - 2t^2$
$2t^2 = 8$
$t^2 = 4$
$t > 0$, so $t = 2$

$$v = \int a\,dt$$
$$= \int (8 - 2t^2)\,dt$$
$$= 8t - \frac{2t^3}{3} + c$$, where c is a constant of integration.

When $t = 0$, $v = 0$
$0 = 0 - 0 + c \Rightarrow c = 0$

$$v = 8t - \frac{2t^3}{3}$$

When $t = 2$, $v = (8 \times 2) - \dfrac{2 \times 2^3}{3}$

$$= 16 - \frac{16}{3}$$
$$= \tfrac{32}{3}$$

The greatest positive speed of the particle is $\frac{32}{3}$ m s^{-1}.

b $s = \int v\,dt$

$$= \int \left(8t - \frac{2t^3}{3}\right) dt$$
$$= 4t^2 - \frac{t^4}{6} + k$$, where k is a constant of integration.

When $t = 0$, $s = 0$
$0 = 0 - 0 + k \Rightarrow k = 0$

$$s = 4t^2 - \frac{t^4}{6}$$

When $t = 2$, $s = 4 \times 2^2 - \dfrac{2^4}{6}$

$$= 16 - \frac{16}{6}$$
$$= \tfrac{40}{3}$$

The distance covered by the particle during the first two seconds is $\frac{40}{3}$ m.

5 **a** $s = -t^3 + 11t^2 - 24t$

$v = \frac{ds}{dt}$

$= -3t^2 + 22t - 24 \text{ m s}^{-1}$

b P is at rest when $v = 0$.

$-3t^2 + 22t - 24 = 0$

$3t^2 - 22t + 24 = 0$

$(3t - 4)(t - 6) = 0$

$t = \frac{4}{3}$ or $t = 6$

P is at rest when $t = \frac{4}{3}$ and $t = 6$.

c $a = \frac{dv}{dt}$

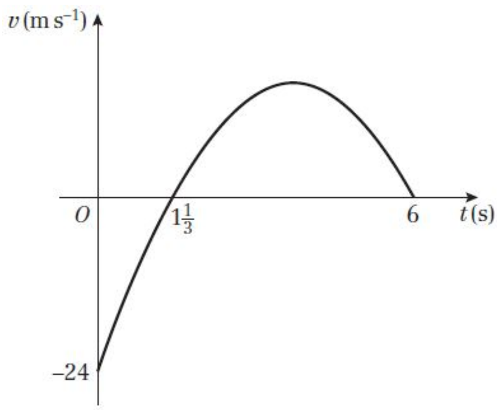
$= -6t + 22$

$a = 0$ when $0 = -6t + 22$

$t = \frac{11}{3}$

The acceleration is zero when $t = \frac{11}{3}$.

d



e The speed of P is 16 when $v = 16$ and $v = -16$.

When $v = 16$, $-3t^2 + 22t - 24 = 16$

$3t^2 - 22t + 40 = 0$

$(3t - 10)(t - 4) = 0$

$t = \frac{10}{3}$ or $t = 4$

When $v = -16$, $-3t^2 + 22t - 24 = -16$

$3t^2 - 22t + 8 = 0$

$$t = \frac{22 \pm \sqrt{22^2 - 4 \times 3 \times 8}}{2 \times 3}$$

$$= \frac{22 \pm \sqrt{388}}{6}$$

$= 0.384$ or 6.95

From the graph in part **d**, the required values are $0 \leq t < 0.384$, $\frac{10}{3} < t < 4$.

6 a The body is at rest when $v = 0$.

$3t^2 - 11t + 10 = 0$

$(3t - 5)(t - 2) = 0$

$t = \frac{5}{3}$ or $t = 2$

The body is at rest when $t = \frac{5}{3}$ and $t = 2$.

b $a = \dfrac{dv}{dt}$

$= 6t - 11$

When $t = 4$, $a = (6 \times 4) - 11$

$= 24 - 11$

$= 13$

When $t = 4$, the acceleration is $13\ \text{m s}^{-2}$.

c From part **a**, the body changes direction when $t = \frac{5}{3}$ and $t = 2$.

s_1 = displacement for $0 \leq t \leq \frac{5}{3}$

s_2 = displacement for $\frac{5}{3} \leq t \leq 2$

s_3 = displacement for $2 \leq t \leq 4$

$$s_1 = \int_0^{\frac{5}{3}} (3t^2 - 11t + 10)\,dt$$

$$= \left[t^3 - \frac{11t^2}{2} + 10t\right]_0^{\frac{5}{3}}$$

$$= \left(\left(\frac{5}{3}\right)^3 - \frac{11 \times \left(\frac{5}{3}\right)^2}{2} + 10 \times \frac{5}{3}\right) - 0$$

$$= \frac{125}{27} - \frac{275}{18} + \frac{50}{3}$$

$$= \frac{325}{54}$$

$$s_2 = \int_{\frac{5}{3}}^{2} (3t^2 - 11t + 10)\,dt$$

$$= \left[t^3 - \frac{11t^2}{2} + 10t\right]_{\frac{5}{3}}^{2}$$

$$= \left(2^3 - \frac{11 \times 2^2}{2} + 10 \times 2\right) - \frac{325}{54}$$

$$= 6 - \frac{325}{54}$$

$$= -\frac{1}{54}$$

$$s_3 = \int_2^4 (3t^2 - 11t + 10)\,dt$$

6 c $s_3 = \left[t^3 - \frac{11t^2}{2} + 10t\right]_2^4$

$= 4^3 - \frac{11 \times 4^2}{2} + 10 \times 4 - 6$

$= 64 - 88 + 40 - 6$

$= 10$

Sign to be ignored when calculating distance:

Total distance $= s_1 + s_2 + s_3$

$= \frac{325}{54} + \frac{1}{54} + 10$

$= \frac{433}{27}$

The total distance travelled is 16.0 m (1 d.p.).

7 a $v = \int a\,\mathrm{d}t$

$= \int (2t^3 - 8t)\,\mathrm{d}t$

$= \frac{t^4}{2} - 4t^2 + c$, where c is a constant of integration.

When $t = 0$, $v = 6$

$6 = 0 - 0 + c \Rightarrow c = 6$

$6 = \frac{0^4}{2} - (4 \times 0^2) + c$

$v = \frac{t^4}{2} - 4t^2 + 6$

b $s = \int v\,\mathrm{d}t$

$= \int \left(\frac{t^4}{2} - 4t^2 + 6\right)\mathrm{d}t$

$= \frac{t^5}{10} - \frac{4t^3}{3} + 6t + k$, where k is a constant of integration.

When $t = 0$, $s = 0$

$0 = 0 - 0 + 0 + k \Rightarrow k = 0$

$s = \frac{t^5}{10} - \frac{4t^3}{3} + 6t$

c Particle is at rest when $v = 0$.

$\frac{t^4}{2} - 4t^2 + 6 = 0$

$t^4 - 8t^2 + 12 = 0$

$(t^2 - 2)(t^2 - 6) = 0$

$t \geq 0$, so $t = \sqrt{2}$ or $t = \sqrt{6}$

The particle is at rest when $t = \sqrt{2}$ and $t = \sqrt{6}$.

8 $x = \dfrac{t^4 - 12t^3 + 28t^2 + 400}{50}$

$\dfrac{dx}{dt} = \dfrac{4t^3 - 36t^2 + 56t}{50}$

Maxima and minima occur when $\dfrac{dx}{dt} = 0$.

$\dfrac{4t^3 - 36t^2 + 56t}{50} = 0$

$t^3 - 9t^2 + 14t = 0$

$t(t^2 - 9t + 14) = 0$

$t(t - 2)(t - 7) = 0$

So turning points are at $t = 0$, $t = 2$ and $t = 7$.

From the sketch graph, the drone is at a greater height when $t = 2$ than when $t = 0$, and $t = 7$ corresponds to the minimum height over the given interval.

When $t = 2$, $x = \dfrac{2^4 - (12 \times 2^3) + (28 \times 2^2) + 400}{50}$

$= \dfrac{16 - 96 + 112 + 400}{50}$

$= 8.64$

When $t = 7$, $x = \dfrac{7^4 - (12 \times 7^3) + (28 \times 7^2) + 400}{50}$

$= \dfrac{2401 - 4116 + 1372 + 400}{50}$

$= 1.14$

The maximum height reached by the drone is 8.64 m, and the minimum height is 1.14 m.

9 When $t = 0$, $v = u = 800$, $s = 1500$

When $t = 25$, $v = 0$

Using $v = u + at$:

$0 = 800 + 25a$

$a = \dfrac{-800}{25} = -32$

$v = \int a\,dt$

$= \int -32\,dt$

$= -32t + c$, where c is a constant of integration.

When $t = 0$, $v = 800$

$800 = 0 + c \Rightarrow c = 800$

$v = 800 - 32t$

$s = \int v\,dt$

$= \int (800 - 32t)\,dt$

9 $s = 800t - 16t^2 + k$, where k is a constant of integration.

When $t = 0$, $s = 1500$
$1500 = 0 - 0 + k \Rightarrow k = 1500$
$s = 800t - 16t^2 + 1500$
So $a = 1500$, $b = 800$, $c = -16$

10 a $v = \int a\,dt$
$= \int (20 - 6t)\,dt$
$= 20t - 3t^2 + c$, where c is a constant of integration.

When $t = 0$, $v = 7$
$7 = 0 - 0 + c \Rightarrow c = 7$
$v = 20t - 3t^2 + 7$
$= 7 + 20t - 3t^2$

b When $t = 7$, $v = 7 + 20 \times 7 - 3 \times 7^2$
$= 7 + 140 - 147$
$= 0$

P's maximum speed in the interval $0 \leq t \leq 7$ is when $\frac{dv}{dt} = 0$

$\frac{dv}{dt} = 20 - 6t$
$0 = 20 - 6t$
$t = \frac{10}{3}$

When $t = \frac{10}{3}$, $v = 7 + 20 \times \frac{10}{3} - 3 \times \left(\frac{10}{3}\right)^2$
$= \frac{121}{3}$

The greatest speed of P in the interval $0 \leq t \leq 7$ is $40\frac{1}{3}$ m s^{-1}.

c $s = \int_0^7 (7 + 20t - 3t^2)\,dt$
$= \left[7t + 10t^2 - t^3\right]_0^7$
$= 7 \times 7 + 10 \times 7^2 - 7^3 - 0$
$= 196$

The distance travelled by P in the interval $0 \leq t \leq 7$ is 196 m.

11 $a \propto (7 - t^2)$
So $a = k(7 - t^2)$
$= 7k - kt^2$

$v = \int a\,\mathrm{d}t$
$= \int (7k - kt^2)\,\mathrm{d}t$
$= 7kt - \frac{kt^3}{3} + c$, where c is a constant of integration.
When $t = 0$, $v = 0$
$0 = 0 - 0 + c \Rightarrow c = 0$
$v = 7kt - \frac{kt^3}{3}$

When $t = 3$, $v = 6$
$6 = 21k - 9k$
$12k = 6$
$k = \frac{1}{2}$
$v = \frac{7t}{2} - \frac{t^3}{6}$

$s = \int v\,\mathrm{d}t$
$= \int \left(\frac{7t}{2} - \frac{t^3}{6}\right)\mathrm{d}t$
$= \frac{7t^2}{4} - \frac{t^4}{24} + d$, where d is a constant of integration.

When $t = 0$, $v = 0$
$0 = 0 - 0 + d \Rightarrow d = 0$
$s = \frac{7t^2}{4} - \frac{t^4}{24}$
$= \frac{1}{24}t^2(42 - t^2)$

12 a Time cannot be negative so $t \geq 0$.
When $t = 0$, $s = 0$

When $t = 5$, $s = 5^4 - 10 \times 5^3 + 25 \times 5^2$
$= 625 - 1250 + 625$
$= 0$

So when $t = 5$, the mouse is again at a distance of zero from the hole: it has returned.

12 b $s = t^4 - 10t^3 + 25t^2$

When mouse is at the greatest distance, $\frac{ds}{dt} = 0$

$\frac{ds}{dt} = 4t^3 - 30t^2 + 50t$

When $\frac{ds}{dt} = 0$, $4t^3 - 30t^2 + 50t = 0$

$$2t(2t^2 - 15t + 25) = 0$$
$$2t(2t - 5)(t - 5) = 0$$

$s = 0$ when $t = 0$ and $t = 5$, so maximum is when $t = 2.5$.

When $t = 2.5$, $s = 2.5^4 - 10 \times 2.5^3 + 25 \times 2.5^2$
$= 39.1$

The greatest distance of the mouse from the hole is 39.1 m.

13 a Any two from:

As the shuttle rises, it burns large amounts of fuel, reducing mass and therefore allowing the same force to produce greater acceleration. (Hence positive terms in the equation.)

While the shuttle remains in the atmosphere, the air resistance forces on it will be changing in a complex way: the increasing speed will cause them to increase, but reduced density of the atmosphere at greater heights will reduce their effect.

At greater heights, the gravitational pull of the Earth is less, which increases the resultant force on the shuttle and increases the acceleration. (In practice, this effect is small compared to that of the mass reduction.)

As the fuel from each tank in the booster rockets is used up, they may become less efficient, reducing the thrust they produce. (The fuel feed mechanisms are designed to prevent this and ensure smooth transitions between each stage, but any astronaut can tell you that there is no such thing as a smooth journey into space!)

b $v = \int a\,dt$

$= \int ((6.7\times10^{-7})t^3 - (3.98\times10^{-4})t^2 + 0.105t + 0.859)\,dt$

$= (1.68 \times 10^{-7})t^4 - (1.33 \times 10^{-4})t^3 + 0.0525t^2 + 0.859t + c$, where c is a constant of integration.

When $t = 124$, $v = 974$

$974 = (1.68 \times 10^{-7})(124)^4 - (1.33 \times 10^{-4})(124)^3 + 0.0525(124)^2 + 0.859(124) + c$,

$\Rightarrow c = 266$

$v = (1.68 \times 10^{-7})t^4 - (1.33 \times 10^{-4})t^3 + 0.0525t^2 + 0.859t + 274$,

c When $t = 446$, $v = (1.68 \times 10^{-7})(446)^4 - (1.33 \times 10^{-4})(446)^3 + 0.0525(446)^2 + 0.859(446) + 274$
$= 5950$

When $t = 446$s, the velocity of the space shuttle is 5950 m s^{-1} (5.95 km s^{-1}).

13 d For this section of the flight:
$a = 28.6$, $u = 5950$, $v = 7850 \text{ m s}^{-1}$
$v = u + at$

$7850 = 5950 + 28.6t$

$$t = \frac{7850 - 5950}{28.6}$$
$$= 66.4$$

Total time to reach escape velocity = 446 + 66.4
= 510 (2 s.f.)

The shuttle cuts its main engines 510 s after launch.

Challenge

1 $v = \int a\,\mathrm{d}t$

$= \int (3t^2 - 18t + 20)\,\mathrm{d}t$

$= t^3 - 9t^2 + 20t + c$, where c is a constant of integration.

When $t = 0$, $v = 0$

$0 = 0 - 0 + 0 + c \Rightarrow c = 0$

$v = t^3 - 9t^2 + 20t$

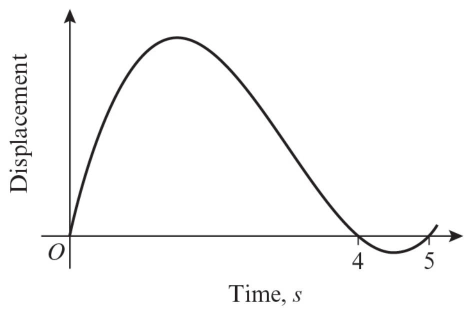
Checking for crossing points to find if the velocity becomes negative during first 5 s:

$t^3 - 9t^2 + 20t = 0$

$t(t^2 - 9t + 20) = 0$

$t(t - 4)(t - 5) = 0$

This means that displacement is positive for the first four seconds and negative in the fifth second (see sketch graph), so need to find distances separately.



s_1 = distance travelled in first 4 s

s_2 = distance travelled in fifth second

$$s_1 = \int_0^4 (t^3 - 9t^2 + 20t)\mathrm{d}t$$

$$= \left[\frac{t^4}{4} - 3t^3 + 10t^2\right]_0^4$$

$$= \left(\frac{4^4}{4} - 3\times 4^3 + 10\times 4^2\right) - 0$$

$= 64 - 192 + 160$

$= 32$

$$s_2 = \int_4^5 (t^3 - 9t^2 + 20t)\mathrm{d}t$$

$$= \left[\frac{t^4}{4} - 3t^3 + 10t^2\right]_4^5$$

$$= \left(\frac{5^4}{4} - 3\times 5^3 + 10\times 5^2\right) - 32$$

$$= \frac{625}{4} - 375 + 250 - 32$$

$= -0.75$

1 Total distance = 32 + 0.75 = 32.75
The particle covers 32.75 m in the first 5 s of its motion.

2 $v = \int a\,\mathrm{d}t$
$= \int (6t + 2)\,\mathrm{d}t$
$= 3t^2 + 2t + c$, where c is a constant of integration.

Assuming that velocity does not change direction during this time, distance travelled between $t = 3$ and $t = 4$

$v = \int_3^4 (3t^2 + 2t + c)\mathrm{d}t$
$= \left[t^3 + t^2 + ct\right]_3^4$
$= (4^3 + 4^2 + 4c) - (3^3 + 3^2 + 3c)$
$= 64 + 16 + 4c - 27 - 9 - 3c$
$= 44 + c$
So $50 = 44 + c \Rightarrow c = 6$
$v = 3t^2 + 2t + 6$

When $t = 5$, $v = 3 \times 5^2 + 2 \times 5 + 6$
$= 75 + 10 + 6$
$= 91$

At 5 s, the velocity is 91 m s^{-1}.

Review Exercise 2

1 a For the first 3 s the cyclist is moving with constant acceleration.

b For the remaining 4 s the cyclist is moving with constant speed.

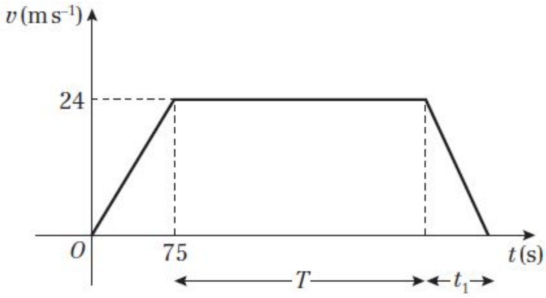
c area = trapezium + rectangle

$s = \frac{1}{2}(2+5)\times 3 + 5\times 4$

$= 10.5 + 20 = 30.5$

The distance travelled by the cyclist is 30.5 m

2 a



b Let time for which the train decelerates be t_1 s.

While decelerating

area = $\frac{1}{2}$ base × height

$600 = \frac{1}{2}t_1 \times 24$

$t_1 = \frac{1200}{24} = 50$

Acceleration is represented by the gradient.

$a = -\frac{24}{t_1} = -\frac{24}{50} = -0.48$

The deceleration is $0.48\,\text{m s}^{-2}$

c For the whole journey

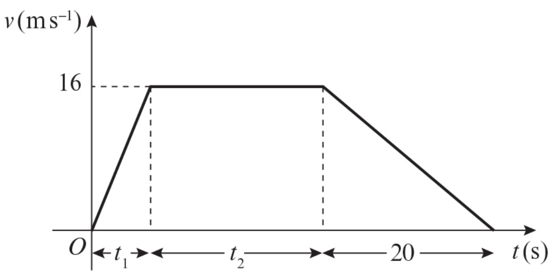
$s = \frac{1}{2}(a+b)h$

$7500 = \frac{1}{2}(T + T + 125)\times 24$

$T = \dfrac{7500 - 1500}{24} = 250$

d Total time is $(75 + T + t_1)\,\text{s} = (75 + 250 + 50)\,\text{s} = 375$ s

3 a



3 a Let the time for which the train accelerates be t_1 s and the time for which it travels at a constant speed be t_2 s.

During acceleration
$v = u + at$

$16 = 0 + 0.4t_1 \Rightarrow t_1 = \frac{16}{0.4} = 40$

At constant speed

$2000 = 16 \times t_2 \Rightarrow t_2 = \frac{2000}{16} = 125$

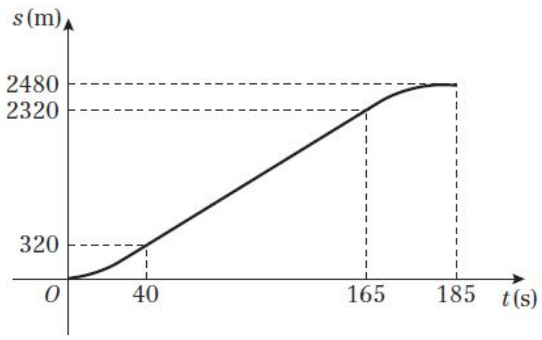
The total time is $(t_1 + t_2 + 20)\,\text{s} = (40 + 125 + 20)\,\text{s} = 185\,\text{s}$

b $s = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(125+185)\times 16 = 2\,480$

$AB = 2\,480\,\text{m}$

c



4 a Taking the upwards direction as positive.

$s = 40,\ v = 0,\ a = -9.8,\ u = ?$

$v^2 = u^2 + 2as$

$0^2 = u^2 - 2\times 9.8 \times 40$

$u^2 = 784 \Rightarrow u = 28$

The speed of projection is 28 m s^{-1}

b $s = 0,\ u = 28,\ a = -9.8,\ t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = 28t - 4.9t^2 = t(28 - 4.9t)$

$t = 0,\ t = \frac{28}{4.9} = 5.714...$

The time taken to return to A is 5.7 s (2 s.f.)

5 Find the speed of projection.
Taking the upwards direction as positive.

$v = 0,\ t = 3,\ a = -9.8,\ u = ?$

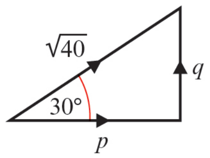
5 $v = u + at$
$0 = u - 9.8 \times 3 \Rightarrow u = 29.4$

$s = 39.2,\ u = 29.4,\ a = -9.8,\ t = ?$
$s = ut + \frac{1}{2}at^2$
$39.2 = 29.4t - 4.9t^2$
$4.9t^2 - 29.4t + 39.2 = 0$

Dividing all terms by 4.9
$t^2 - 6t + 8 = 0$
$(t-2)(t-4) = 0$
$t = 2, 4$

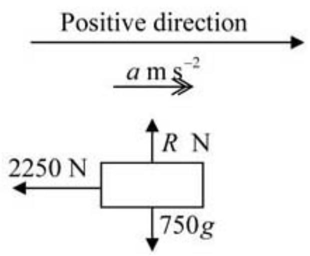
The ball is 39.2 m above its point of projection when $t = 2$ or when $t = 4$

6



$\tan 30 = \frac{1}{\sqrt{3}} = \frac{q}{p}$
$\therefore q = \frac{p}{\sqrt{3}}$
$40 = p^2 + q^2$
$p^2 + \frac{p^2}{3} = 40$
$\frac{4}{3}p^2 = 40$
$p^2 = 30$
$\therefore q^2 = 40 - 30 = 10$
The values are: $p = \sqrt{30}$ and $q = \sqrt{10}$.

7



$F = ma$
$R(\rightarrow)\ \ -2250 = 750a$
$a = -\frac{2250}{750} = -3$

7 $u = 25, v = 15, a = -3, s = ?$

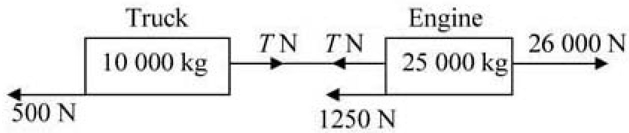
$v^2 = u^2 + 2as$

$15^2 = 25^2 - 6s$

$s = \frac{25^2 - 15^2}{6} = \frac{400}{6} = 66\tfrac{2}{3}$

The distance travelled by the car as its speed is reduced is $66\tfrac{2}{3}$ m.

8



a The resistance on the engine is $25 \times 50 = 1250$ N
The resistance on the truck is $10 \times 50 = 500$ N
For the whole system, the engine and truck

$R(\rightarrow) \qquad F = ma$

$26\,000 - 1250 - 500 = 35\,000a$

$a = \frac{26\,000 - 1250 - 500}{35\,000} = \frac{97}{140} = 0.6928...$

The acceleration of the engine and truck is $0.693\,\text{m s}^{-2}$ (3 s.f.)

b For the truck alone

$F = ma$

$T - 500 = 10\,000a$

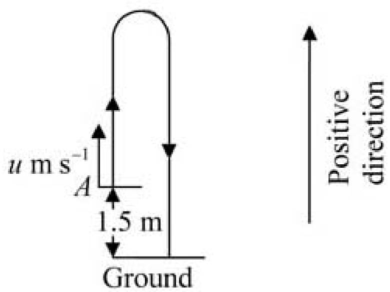
$T = 500 + 10\,000 \times 0.6928... = 7428.57...$

The tension in the coupling is 7430 N (3 s.f.)

c i Treating the engine and truck as particles allows us to assume the weight acts from the centre of mass of each object, and ignore wind resistance and rotational forces.

ii By assuming the coupling is a light horizontal rod, we treat it as if it had no mass and therefore can assume it not only stays straight but that it has no weight and the tension is constant along the entire length.

9



9 a From A to the greatest height, taking upwards as positive.

$v = 0, \; a = -9.8, \; s = 25.6, \; u = ?$

$v^2 = u^2 + 2as$

$0^2 = u^2 + 2 \times (-9.8) \times 25.6$

$u^2 = 2 \times 9.8 \times 25.6 = 501.76$

$u = \sqrt{501.76} = 22.4$, as required.

b $u = 22.4, \; s = -1.5, \; a = -9.8, \; t = T$

$s = ut + \frac{1}{2}at^2$

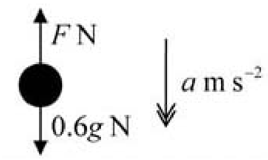
$-1.5 = 22.4T + \frac{1}{2}(-9.8)T^2$

$4.9T^2 - 22.4T - 1.5 = 0$

$$T = \frac{22.4 + \sqrt{(-22.4)^2 - 4 \times 4.9 \times -1.5}}{2 \times 4.9}$$

$= 4.637... = 4.64$ (3 s.f.)

c



To find the speed of the ball as it reaches the ground.

$u = 22.4, \; s = -1.5, \; a = -9.8, \; v = ?$

$v^2 = u^2 + 2as = 22.4^2 + 2 \times (-9.8) \times (-1.5) = 531.16$

To find the deceleration as the ball sinks into the ground.

$u^2 = 531.16, \; v = 0, \; s = 0.025, \; a = ?$

$v^2 = u^2 + 2as \Rightarrow 0^2 = 531.16 + 2 \times a \times 0.025$

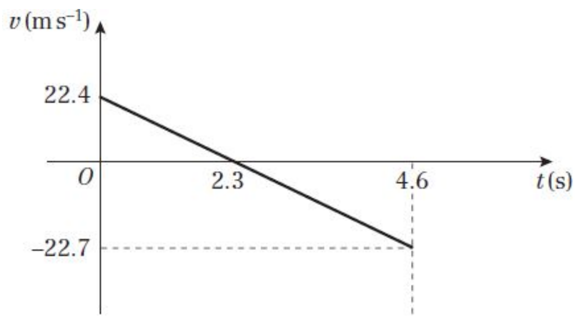
$a = -\frac{531.16}{0.05} = -10\,623.2$

$F = ma$

$0.6g - F = 0.6 \times (-10\,623.2)$

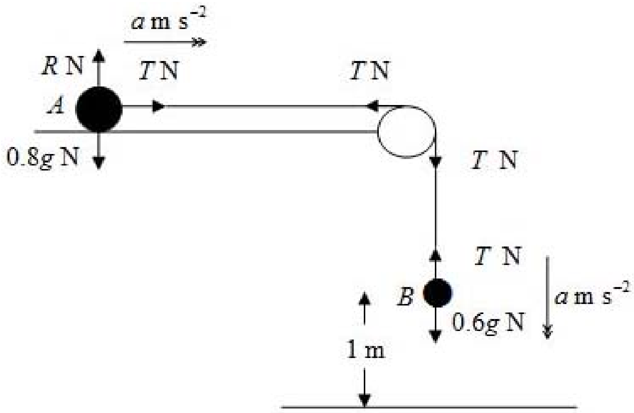
$F = 0.6g + 0.6 \times 10\,623.2 = 6380$ (3 s.f.)

d



9 e Consider air resistance during motion under gravity.

10



a For A

$R(\rightarrow)$ $\quad T = 0.8a \quad$ **(1)**

For B

$R(\downarrow)$ $\quad 0.6g - T = 0.6a \quad$ **(2)**

(1) + **(2)**

$0.6g = 1.4a$

$$a = \frac{0.6 \times 9.8}{1.4} = 4.2$$

The acceleration of A is 4.2 m s^{-2}

b Substitute $a = 4.2$ into (1)

$T = 0.8 \times 4.2 = 3.36$

The tension in the string is 3.4 N (2 s.f.)

c $u = 0,\ a = 4.2,\ s = 1,\ v = ?$

$v^2 = u^2 + 2as$

$\quad = 0^2 + 2 \times 4.2 \times 1 = 8.4$

$v = \sqrt{8.4} = 2.898...$

The speed of B when it hits the ground is 2.9 m s^{-1} (2 s.f.)

d $u = 0,\ a = 4.2,\ s = 1,\ t = ?$

$s = ut + \frac{1}{2}at^2$

$1 = 0 + 2.1t^2$

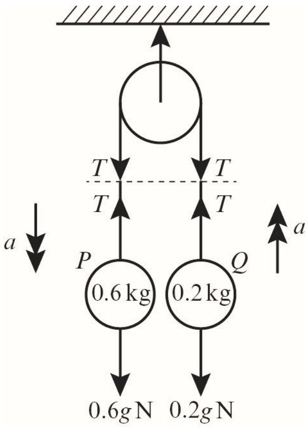
$t^2 = \frac{1}{2.1} \Rightarrow t = 0.690...$

The time taken for B to reach the ground is 0.69 s (2 s.f.)

e i Describing the string as 'light' means it has no mass (and therefore no weight).

10 e ii This fact allows us to assume that the tension is constant in all parts of the string.

11



a $F = ma$

For $P \downarrow$ positive: $0.6a = 0.6g - T$ **(1)**

For $Q \uparrow$ positive: $0.2a = T - 0.2g$ **(2)**

$3 \times$ (**2**): $0.6a = 3T - 0.6g$

Subtracting (**1**) from this: $0 = 4T - 1.2g$

$4T = 1.2g = 1.2 \times 9.8$

The tension in the string is 2.9 N (2 s.f.)

b (1) + (2): $0.8a = 0.4g$

$$a = \frac{g}{2} = \frac{9.8}{2}$$

The acceleration is 4.9 m s^{-2}.

c For P, before string breaks, taking up as positive:

$s = ?$, $u = 0$, $a = 4.9$, $t = 0.4$

$$s = ut + \tfrac{1}{2}at^2$$

$$s = (0 \times 0.4) + \tfrac{1}{2}(4.9 \times 0.4^2)$$

$$= \tfrac{1}{2}(0.784)$$

$$= 0.392\,\text{m}$$

The total distance P has to fall is therefore $1 - 0.392 = 0.608$ m.

$v = u + at$

$v = 0 + (0.4 \times 4.9) = 1.96$ m s^{-1}

Before the string breaks, P is moving downwards at 1.96 m s^{-1}. After string breaks, taking down as positive, P moves under gravity.

$s = 0.608$, $u = 1.96$, $a = 9.8$, $t = ?$

$$s = ut + \tfrac{1}{2}at^2$$

$$0.608 = 1.96t + \tfrac{1}{2}(9.8 \times t^2)$$

$$0 = 4.9t^2 - 1.96t - 0.608$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

11 c $t = \dfrac{-1.96 \pm \sqrt{(-1.96)^2 - (4 \times 4.9 \times -0.608)}}{2 \times 4.9}$

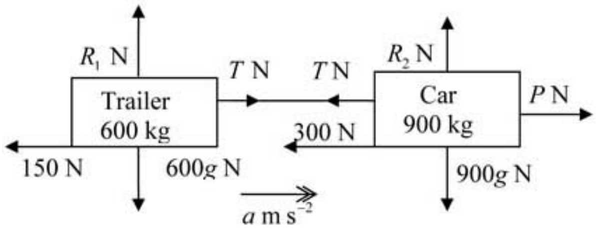
$= \dfrac{-1.96 \pm \sqrt{15.76}}{9.8}$

$= 0.205\text{ s or } -0.605\text{ s (3 d. p.)}$

Only positive answers are relevant in this context. $\therefore$ P hits the floor 0.21 s (2 s.f.) after the string breaks.

d This fact allows us to assume that the tension is constant in all parts of the string and that the acceleration of the two particles is the same.

12



a i For the whole system:

$$F = ma$$

$$R(\rightarrow) \qquad P - 300 - 150 = 1500 \times 0.4$$

$$P = 1050$$

The tractive force exerted by the engine of the car is 1050 N.

ii For the trailer alone:

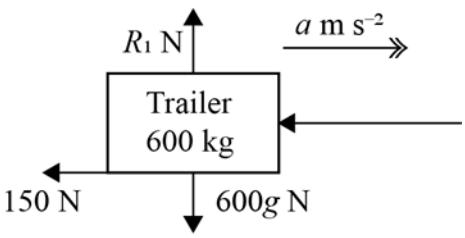
$$F = ma$$

$$R(\rightarrow) \qquad T - 150 = 600 \times 0.4$$

$$T = 390$$

The tension in the tow bar is 390 N.

b For the trailer alone:



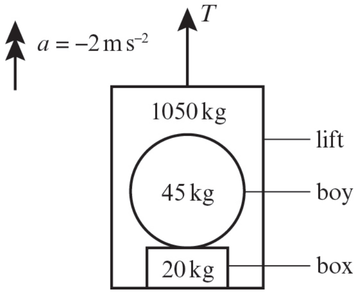
$$F = ma$$

$$R(\rightarrow) \qquad 1650 + 150 = 600a$$

$$a = -\tfrac{1800}{600} = -3$$

The greatest possible deceleration of the car is 3 m s^{-2}

13

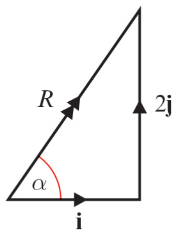


a $F = ma$
Taking up as positive:
$(1050 + 45 + 20) \times -2 = T - (1050 + 45 + 20)g$
$T = 1115(g - 2)$
$T = 1115 \times 7.8$
The tension in the cable is 8697 N.

b From Newton's third law of motion:
|Force exerted on boy by box| = |Force exerted on box by boy| = $|R_1|$
For the boy, taking up as positive:
$45 \times -2 = R_1 - 45g$
$R_1 = 45(g - 2)$
$R_1 = 45 \times 7.8$
The boy exerts a force of 351 N on the box.

c From Newton's third law of motion:
|Force exerted on box by lift| = |Force exerted on lift by box| = $|R_2|$
For the box, taking up as positive:
$20 \times -2 = R_2 - 20g - 351$
$R_2 = 351 + 20(g - 2)$
$R_2 = 351 + (20 \times 7.8) = 351 + 156$
The box exerts a force of 507 N on the lift.

14 a



Let the required angle be α.
Then $\tan\alpha = 2$
$\therefore \alpha = 63^\circ\,(2\text{ s.f.})$

b As $\mathbf{F}_1 + \mathbf{F}_2 = \mathbf{R}$
$(2\mathbf{i} + 3\mathbf{j}) + (\lambda\mathbf{i} + \mu\mathbf{j}) = k(\mathbf{i} + 2\mathbf{j})$

where k is a constant.
$\therefore 2 + \lambda = k$ and $3 + \mu = 2k$ *

Eliminate k from these two equations.

14 b Then $2(2+\lambda)=3+\mu$

$\therefore 2\lambda-\mu+1=0$

c If $\mathbf{F}_2$ is parallel to $\mathbf{j}$ then $\lambda=0$

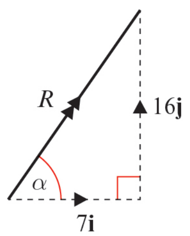
Substituting $\lambda=0$ into * gives

$\mu=1$ and $k=2$

$\therefore \mathbf{R}=2\mathbf{i}+4\mathbf{j}$

$$\therefore |\mathbf{R}|=\sqrt{2^2+4^2}$$
$$=\sqrt{20}$$
$$=4.47 \text{ (3 s.f.)}$$

15



a The magnitude of

$$\mathbf{R}=\sqrt{7^2+16^2}$$
$$=17.5 \text{ (1 d.p.)}$$

b $\tan\alpha=\frac{16}{7}$

$$\alpha=\tan^{-1}\left(\tfrac{16}{7}\right)$$
$$=66^\circ \text{ (nearest degree)}$$

c Let $\mathbf{P}=\lambda(\mathbf{i}+4\mathbf{j})$ and $\mathbf{Q}=\mu(\mathbf{i}+\mathbf{j})$

As $\mathbf{P}+\mathbf{Q}=\mathbf{R}$

$\therefore \lambda(\mathbf{i}+4\mathbf{j})+\mu(\mathbf{i}+\mathbf{j})=(7\mathbf{i}+16\mathbf{j})$

Equating $\mathbf{i}$ components

$\lambda+\mu=7 \qquad \textbf{(1)}$

Equating $\mathbf{j}$ components

$4\lambda+\mu=16 \qquad \textbf{(2)}$

Subtract $\textbf{(2)}-\textbf{(1)}$

$3\lambda=9$

$\therefore \lambda=3$

15 Substitute into equation (**1**)

$\therefore 3+\mu=7$

$\therefore \mu=4$

$\therefore \mathbf{P}=3(\mathbf{i}+4\mathbf{j})=3\mathbf{i}+12\mathbf{j}$ and $\mathbf{Q}=4(\mathbf{i}+\mathbf{j})=4\mathbf{i}+4\mathbf{j}$

16 $a=5-2t$

$v=\int a\,\mathrm{d}t=\int(5-2t)\,\mathrm{d}t$

$=5t-t^2+C$

When $t=0,\ v=6$

$6=0-0+C\Rightarrow C=6$

Hence

$v=6+5t-t^2$

When P is at rest

$0=6+5t-t^2$

$t^2-5t-6=(t-6)(t+1)=0$

$t=6,-1$

$t>0$

$\therefore t=6$

P is at rest at $t = 6$ s

17 $v=6t-2t^2$

a Maximum value of velocity occurs when $a = 0$

$a=\dfrac{dv}{dt}=6-4t$

Maximum velocity occurs at $t=\dfrac{3}{2}$ s

$v=\left(6\times\dfrac{3}{2}\right)-2\left(\dfrac{3}{2}\right)^2$

$v=9-\dfrac{9}{2}=\dfrac{9}{2}$

The maximum velocity is 4.5 m s^{-1}.

b When P returns to O, $s = 0$

$s=\int v\,dt=\int 6t-2t^2\,dt$

$s=3t^2-\dfrac{2}{3}t^3+c$

At $t = 0$, $s = 0$ so $c = 0$

17 $0 = t^2\left(3 - \frac{2}{3}t\right)$

$t = 0$ or $\frac{2}{3}t = 3$

P returns to O after 4.5 s.

18 $v = 3t^2 - 8t + 5$

a When the particle is at rest, $v = 0$

$0 = 3t^2 - 8t + 5$

$0 = 3\left(t^2 - \frac{8}{3}t + \frac{5}{3}\right)$

$0 = 3\left(t - \frac{3}{3}\right)\left(t - \frac{5}{3}\right)$

(or by using quadratic equation formula)

P is at rest at 1 s and $\frac{5}{3}$ s.

b $a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 - 8t + 5)$

$a = 6t - 8$

$t = 4$

$a = (6 \times 4) - 8$

After 4 s, the acceleration of P is 16 m s^{-2}.

c Distance travelled in third second $= s_3$

$s_3 = \int_2^3 v\,dt = \int_2^3 3t^2 - 8t + 5\,dt$

$s_3 = \left[t^3 - 4t^2 + 5t\right]_2^3$

$s_3 = \left[27 - 36 + 15\right] - \left[8 - 16 + 10\right]$

$s_3 = 6 - 2$

The distance travelled in the third second is 4 m.

19 $v = 6t - 2t^{\frac{3}{2}}$

a $a = \frac{dv}{dt}$

$a = 6 - 3t^{\frac{1}{2}}$

b At $t = 0$, $s = 0$

$s = \int v\,dt = \int 6t - 2t^{\frac{3}{2}}\,dt$

$s = 3t^2 - \frac{4}{5}t^{\frac{5}{2}} + c$

At $t = 0$, $s = 0$, $c = 0$

So $s = 3t^2 - \frac{4}{5}t^{\frac{5}{2}}$

Challenge

1 $t_1 + t_2 + t_3 = 7 \times 60 = 420$
$3t_1 = 4t_3$
$t_3 = 0.75t_1$

Considering time t_1

$$s = \left(\frac{u+v}{2}\right)t_1$$

$$1750 = \left(\frac{0+v}{2}\right)t_1$$

$$v = \frac{3500}{t_1}$$

Considering time t_2
$s_2 = vt_2$

$$17\,500 = \frac{3500}{t_1}t_2$$

$$t_2 = 5t_1$$

Considering total time:
$t_1 + 5t_1 + 0.75\ t_1 = 420$

$$t_1 = \frac{420}{6.75} = 62.22 \text{ s}$$

$\therefore t_2 = 311.11$ s
& $t_3 = 46.67$ s

Distance travelled during time t_3 is s_3

$$s_3 = \left(\frac{u+v}{2}\right)t_3$$

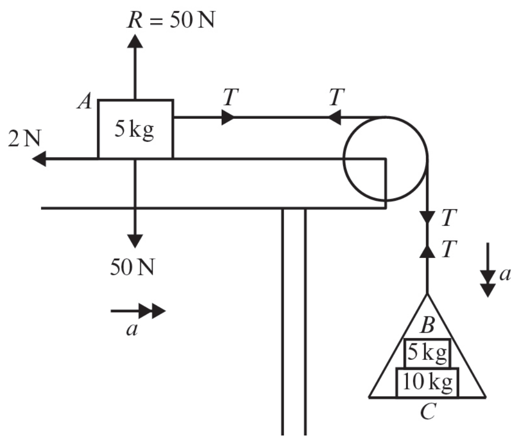
$$u = \frac{3500}{t_1} = \frac{3500}{62.22} = 56.25,\ v = 0,\ t = 46.67$$

$$s_3 = \left(\frac{56.25+0}{2}\right)46.67$$

$$s_3 = 28.125 \times 46.67 = 1312.6$$

Total distance = 1750 + 17 500 + 1312.6
The distance between the two stations is 20.6 km (3 s.f.).

2



a Considering A, → positive: $T - 2 = 5a$
Considering entire pan, ↓ positive: $(5 + 10)g - T = 15a$
So $150 - T = 15a$

Adding these gives:
$148 = 20a$
The acceleration of the pan is 7.4 m s^{-2}.

b Substituting this value into the first equation gives:
$T - 2 = 5 \times 7.4 = 37$
The tension in the string is 39 N.

c Block C exerts a normal reaction force R on block B.
Considering block B only, ↓ positive:
$5g - R = 5a$
$50 - R = 37$
Block C exerts a force of 13 N on block B.

d The force the string exerts on the pulley has two perpendicular components, each of magnitude T.
The magnitude of the total force, F, is therefore given by:
$F^2 = T^2 + T^2$
$F = \sqrt{39^2 + 39^2} = \sqrt{3042}$
The string exerts a force of magnitude 55 N (2 s.f.) on the pulley.

e The fact that the string is inextensible allows us to assume that the tension is constant in all parts of the string and that the acceleration of Block A and the scale pan are the same.

Exam–style Practice paper

Section A: Statistics

1 a $0.2 + y + 0.3 + 0.35 = 1$
$y = 1 - 0.85 = 0.15$

b P(B and M) = 0.15
$P(B) \times P(M) = (0.2 + 0.15) \times (0.3 + 0.15) = 0.35 \times 0.45 = 0.1575$
P(B and M) $\neq$ P(B) $\times$ P(M), so 'likes bananas' and 'likes mangoes' are not independent events.

2 a t is a continuous variable, because it is a measured variable which can take any value.

b mean $= \dfrac{\sum t}{n} = \dfrac{140.1}{10} = 14.01$

standard deviation $= \sqrt{\dfrac{\sum t^2}{n} - \left(\dfrac{\sum t}{n}\right)^2} = \sqrt{\dfrac{1981.33}{10} - \left(\dfrac{140.1}{10}\right)^2} = 1.36$ (to 3 s.f.)

c 15.8 °C is higher than the current mean so the mean would increase.

d Clare could take a random sample of days from the whole of September for the different locations in the UK.

3 a $0.1 + 0.2 + 0.15 + p + 0.1 + 0.25 = 1$
$p = 1 - 0.8 = 0.2$

b $P(2 \leq X \leq 5) = 1 - P(X = 1) - P(X = 6)$
$= 1 - 0.1 - 0.25 = 0.65$

c i P(odd) = 0.1 + 0.15 + 0.1 = 0.35

P(odd exactly twice) $= \binom{10}{2} 0.35^2 0.65^8$
$= 0.1757$ (to 4 d.p.)

ii P(odd more than 6 times) $= \binom{10}{7} 0.35^7 0.65^3 + \binom{10}{8} 0.35^8 0.65^2 + \binom{10}{9} 0.35^9 0.65^1 + 0.35^{10}$
$= 0.0260$ (to 4 d.p.)

4 a The test statistic is the number of plates that are flawed.
$H_0: p = 0.3$, $H_1: p < 0.3$

b $X \sim B(20, 0.3)$
$P(X \leq 2) = 0.0355 < 0.05$
$P(X \leq 3) = 0.1071 > 0.05$
The critical region is $X \leq 2$

c The actual significance level is 0.0355 = 3.55%

d 1 falls into the critical region, therefore there is evidence to support the claim.

5 a The increase in energy released is 3.1 Joules for each degree of temperature.

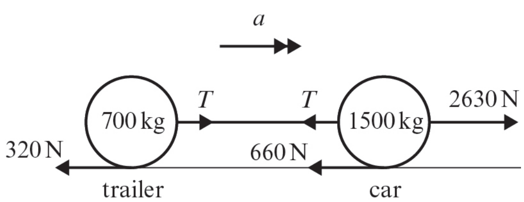
5 **b** This value of h is a long way from the range of the experimental data: hence the extrapolation is excessive and the predicted value of e would be too unreliable.

c It is not sensible. The regression line predicts a value of e given h, not the other way round.

6
$$P(4.6 \leq h \leq 6.1) = \frac{0.4\times10+0.2\times45+0.2\times60+0.2\times80+0.4\times25+0.1\times10}{0.5\times5+0.5\times10+0.2\times45+0.2\times60+0.2\times80+0.4\times25+0.5\times10}$$
$$= \frac{4+9+12+16+10+1}{2.5+5+9+12+16+10+5}$$
$$= \frac{52}{59.5}$$
$= 0.87$ (to 2 d.p.)

Section B: Mechanics

7



$F = ma$

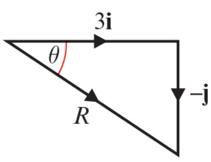
a For the whole system:
$F = 2630 - 660 - 320 = 1650$
$m = 1500 + 700 = 2200$
$1650 = 2200a$
The acceleration of the car is 0.75 m s^{-2}

b For the trailer:
$F = T - 320$, $m = 700$, $a = 0.75$
$T - 320 = 700 \times 0.75 = 525$
$T = 525 + 320$
The tension in the tow-rope is 845 N.

c Since the tow-rope is inextensible, the acceleration of each part of the system is identical and the tension in it is constant throughout.

8 **a** Resultant force, $F = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$
$F = (3\mathbf{i} - 6\mathbf{j}) + (4\mathbf{i} + 5\mathbf{j}) + (2\mathbf{i} - 2\mathbf{j}) = (9\mathbf{i} - 3\mathbf{j})$
$m = 3$, $\mathbf{a} = ?$
$F = m\mathbf{a}$
$(9\mathbf{i} - 3\mathbf{j}) = 3\mathbf{a}$
The acceleration of the particle is $(3\mathbf{i} - \mathbf{j})$ m s^{-2}

b



$\tan\theta = \frac{1}{3}$

8 b The acceleration acts at an angle of 18.4° below **i**.

c $|\mathbf{a}| = \sqrt{1^2 + 3^2} = \sqrt{10}$

The magnitude of the acceleration is $\sqrt{10}$ m s^{-2}

9 a Taking up as positive:

$s = 0$, $a = -9.8$, $t = 5$, $u = ?$

$s = ut + \frac{1}{2}at^2$

$0 = (u \times 5) + \frac{1}{2}(-9.8 \times 5^2) = 5u - 122.5$

$u = \dfrac{122.5}{5} = 24.5$

The ball is projected at a speed of 24.5 m s^{-1}

b $u = 24.5$, $a = -9.8$, $v = 0$, $s = ?$

$v^2 = u^2 + 2as$

$0 = 24.5^2 + (2 \times (-9.8s))$

$s = \dfrac{24.5^2}{2 \times 9.8} = \dfrac{600.25}{19.6} = 30.625$

The ball reaches a height of 30.6 m above P.

c $s = 15$, $u = 24.5$, $a = -9.8$, $t = ?$

$s = ut + \frac{1}{2}at^2$

$15 = 24.5t + \frac{1}{2}(-9.8 \times t^2)$

$4.9t^2 - 24.5t + 15 = 0$

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \dfrac{24.5 \pm \sqrt{24.5^2 - (4 \times 4.9 \times 15)}}{2 \times 4.9}$

$= \dfrac{24.5 \pm \sqrt{306.25}}{9.8}$

$= \dfrac{24.5 \pm 17.5}{9.8}$

$= \dfrac{42}{9.8}$ or $\dfrac{7}{9.8}$

The ball is at a height of 15 m above P at 0.714 s and 4.29 s after leaving P.

10 $v = 3 + 9t^2 - 4t^3$

When the particle is moving at maximum velocity, $a = \dfrac{dv}{dt} = 0$

$0 = \dfrac{d\left(3 + 9t^2 - 4t^3\right)}{dt}$

$= 18t - 12t^2$

$= 6t(3 - 2t)$

10 At $t = 0$, the particle moves at minimum velocity (see graph).
The particle has maximum velocity at $t = \frac{3}{2}$ seconds.

$$s = \int v\,\mathrm{d}t = \int_0^{\frac{3}{2}} \left(3 + 9t^2 - 4t^3\right)\mathrm{d}t$$

$$= \left[3t + \frac{9t^3}{3} - \frac{4t^4}{4}\right]_0^{\frac{3}{2}} = \left[3t + 3t^3 - t^4\right]_0^{\frac{3}{2}}$$

For $t = 0$, all terms are zero, so this becomes:

$$s = 3 \times \left(\frac{3}{2}\right) + 3 \times \left(\frac{3}{2}\right)^3 - \left(\frac{3}{2}\right)^4$$

$$= \frac{9}{2} + \frac{81}{8} - \frac{81}{16} = \frac{153}{16}$$

The particle is moving at maximum velocity when it is $\frac{153}{16}$ m from O.