Elastic collisions in one dimension 4A

1 Use Newton's law of restitution $e = \dfrac{\text{speed of separation}}{\text{speed of approach}}$

a $e = \dfrac{4-0}{6-0} = \dfrac{2}{3}$

b $e = \dfrac{3-2}{4-2} = \dfrac{1}{2}$

c $e = \dfrac{2-(-3)}{9-(-6)} = \dfrac{5}{15} = \dfrac{1}{3}$

2 a Using conservation of linear momentum for the system $(\rightarrow)$:

$0.25 \times 6 + 0.5 \times 0 = 0.25v_1 + 0.5v_2$

Multiply this equation by 4:

$6 = v_1 + 2v_2$ **(1)**

Using Newton's law of restitution:

$\dfrac{1}{2} = \dfrac{v_2 - v_1}{6-0}$

$\Rightarrow 3 = v_2 - v_1$ **(2)**

Add equations **(1)** and **(2)**:

$9 = 3v_2$

$\Rightarrow v_2 = 3$

Substituting this value into equation **(1)** gives:

$6 = v_1 + 2 \times 3$

$\Rightarrow v_1 = 0$

After the collision, A is at rest and B moves at $3\,\text{m}\,\text{s}^{-1}$.

2 b Using conservation of linear momentum for the system $(\rightarrow)$:

$2\times4+3\times2=2v_1+3v_2$

$\Rightarrow 14=2v_1+3v_2$ **(1)**

Using Newton's law of restitution:

$0.25=\dfrac{v_2-v_1}{4-2}$

$\Rightarrow 0.5=v_2-v_1$ **(2)**

Multiply equation **(2)** by 2 and add to equation **(1)**:

$15=5v_2$

$\Rightarrow v_2=3$

Substituting this value into equation **(1)** gives:

$14=2v_1+3\times3$

$\Rightarrow v_1=\dfrac{5}{2}=2.5$

After the collision, *A* and *B* move with speeds of $2.5\,\text{m s}^{-1}$ and $3\,\text{m s}^{-1}$ respectively.

c Using conservation of linear momentum for the system $(\rightarrow)$:

$3\times8+1\times(-6)=3v_1+1v_2$

$\Rightarrow 18=3v_1+v_2$ **(1)**

Note that in deriving equation **(1)** the speed of particle *B* appears in the equation as –6 because it is directed to the left in the diagram.

Using Newton's law of restitution:

$\dfrac{1}{7}=\dfrac{v_2-v_1}{8-(-6)}$

$\Rightarrow 2=v_2-v_1$ **(2)**

Subtracting equation **(2)** from equation **(1)** gives:

$16=4v_1$

$\Rightarrow v_1=4$

Substituting this value into equation **(1)** gives:

$18=3\times4+v_2$

$\Rightarrow v_2=6$

This answer may be checked by using equation **(2)**.

After the collision, *A* and *B* move with speeds of $4\,\text{m s}^{-1}$ and $6\,\text{m s}^{-1}$ respectively.

2 d Using conservation of linear momentum for the system $(\rightarrow)$:

$$0.4\times 6+0.4\times(-6)=0.4v_1+0.4v_2$$

$$\Rightarrow 0=v_1+v_2 \qquad \textbf{(1)}$$

Using Newton's law of restitution:

$$\frac{2}{3}=\frac{v_2-v_1}{6-(-6)}=\frac{v_2-v_1}{12}$$

$$\Rightarrow v_2-v_1=8 \qquad \textbf{(2)}$$

Adding equations **(1)** and **(2)** gives:

$$2v_2=8$$

$$\Rightarrow \ v_2=4$$

Substituting this value into equation **(1)** gives:

$$v_1=-4$$

After the collision, the speeds of A and B are $4\,\text{m s}^{-1}$, and both particles change direction.

e Noting that the particle moving in the opposite direction (i.e. to the left) has a negative velocity in the equation, using conservation of linear momentum for the system $(\rightarrow)$:

$$5\times 3+4\times(-12)=5v_1+4v_2$$

$$\Rightarrow -33=5v_1+4v_2 \qquad \textbf{(1)}$$

Using Newton's law of restitution:

$$\frac{1}{5}=\frac{v_2-v_1}{3-(-12)}=\frac{v_2-v_1}{15}$$

$$\Rightarrow \ 3=v_2-v_1 \qquad \textbf{(2)}$$

Multiply equation **(2)** by 5 and add to equation **(1)** to obtain:

$$-18=9v_2$$

$$\Rightarrow \ v_2=-2$$

Substituting this value into equation **(1)** gives:

$$-33=5v_1-8$$

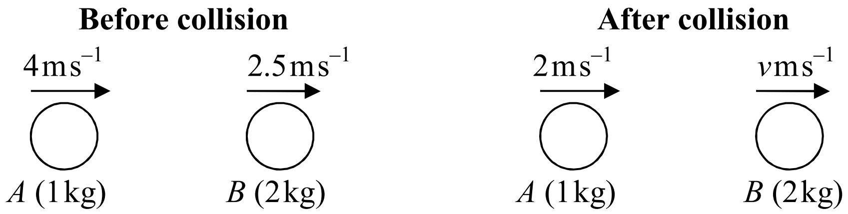
$$-25=5v_1$$

$$\Rightarrow \ v_1=-5$$

This answer may be checked by using equation **(2)**.

After the collision, the speeds of A and B are $5\,\text{m s}^{-1}$ and $2\,\text{m s}^{-1}$ respectively, and both particles move to the left, i.e. particle A changes direction in the collision.

3 a Draw a clearly labelled diagram



Using conservation of linear momentum for the system $(\rightarrow)$:

$1\times 4+2\times 2.5=1\times 2+2v$

$9=2+2v$

$2v=7$

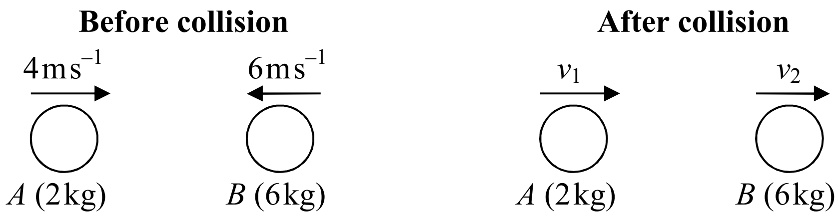
$\Rightarrow v=3.5$

Speed of B after the collision is $3.5\,\text{m}\,\text{s}^{-1}$.

b Using Newton's law of restitution:

$$e=\frac{v-2}{4-2.5}=\frac{3.5-2}{4-2.5}=\frac{1.5}{1.5}=1$$

4



Using conservation of linear momentum for the system $(\rightarrow)$:

$2\times4+6\times(-6)=2v_1+6v_2$

$\Rightarrow\ -14=v_1+3v_2$ **(1)**

Using Newton's law of restitution:

$\frac{1}{5}=\frac{v_2-v_1}{4-(-6)}=\frac{v_2-v_1}{10}$

$\Rightarrow\ 2=v_2-v_1$ **(2)**

Adding equations **(1)** and **(2)** gives:

$-12=4v_2 \Rightarrow v_2=-3$

Substituting this value into equation **(2)** gives:

$2=-3-v_1 \Rightarrow v_1=-5$

After the collision, the speeds of A and B are $5\,\text{m s}^{-1}$ and $3\,\text{m s}^{-1}$ respectively, and both particles move in the direction sphere B was moving before the impact.

The impulse of sphere B on sphere A = change in momentum of sphere A

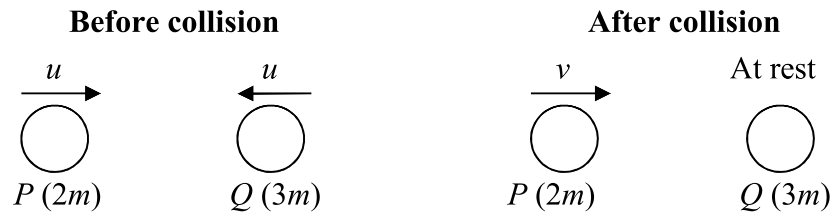
$= 2\times(-5)-2\times4=-18\,\text{N s}$

The impulse of sphere A on sphere B = change in momentum of sphere B

$= 6\times(-3)-6\times(-6)=18\,\text{N s}$

Spheres A and B experience equal and opposite impulses of magnitude $18\,\text{N s}$.

5



Using conservation of linear momentum for the system $(\rightarrow)$:

$2mu - 3mu = 2mv + 3m \times 0$

$-mu = 2mv$

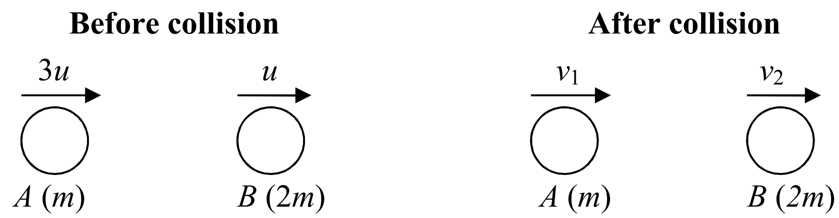
$\Rightarrow v = -\frac{u}{2}$

After the collision, particle P changes direction and has a speed of $0.5u\,\text{m}\,\text{s}^{-1}$

Using Newton's law of restitution:

$e = \frac{0 - v}{u - (-u)} = \frac{\frac{u}{2}}{2u} = \frac{1}{4}$

6



Using conservation of linear momentum for the system $(\rightarrow)$:

$m \times 3u + 2m \times u = mv_1 + 2mv_2$

$\Rightarrow \; v_1 + 2v_2 = 5u$ (cancelling out the common factor m) (1)

Using Newton's law of restitution:

$e = \frac{v_2 - v_1}{3u - u} = \frac{v_2 - v_1}{2u}$

$\Rightarrow v_2 - v_1 = 2ue$ (2)

Adding equations **(1)** and **(2)** gives:

$3v_2 = u(5 + 2e) \;\Rightarrow\; v_2 = \frac{u}{3}(5 + 2e)$

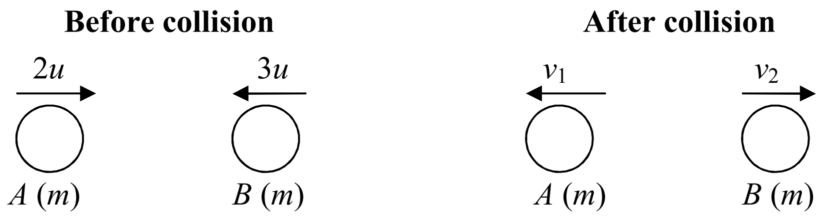
Substituting into equation **(1)** gives:

$\frac{u}{3}(5 + 2e) - v_1 = 2ue$

$3v_1 = 5u + 2ue - 6ue = u(5 - 4e)$

$\Rightarrow v_1 = \frac{u}{3}(5 - 4e)$

7



Using conservation of linear momentum for the system $(\rightarrow)$:

$m \times 2u + m \times (-3u) = m(-v_1) + mv_2$

$\Rightarrow \; v_2 - v_1 = -u$ (cancelling out the common factor *m*) **(1)**

Using Newton's law of restitution:

$e = \dfrac{v_2 - (-v_1)}{2u + 3u} = \dfrac{v_2 + v_1}{5u}$

$\Rightarrow v_2 + v_1 = 5ue$ **(2)**

Adding equations **(1)** and **(2)** gives:

$2v_2 = -u + 5eu \; \Rightarrow \; v_2 = \dfrac{u}{2}(5e - 1)$

As $v_2 > 0, \; \dfrac{u}{2}(5e-1) > 0$

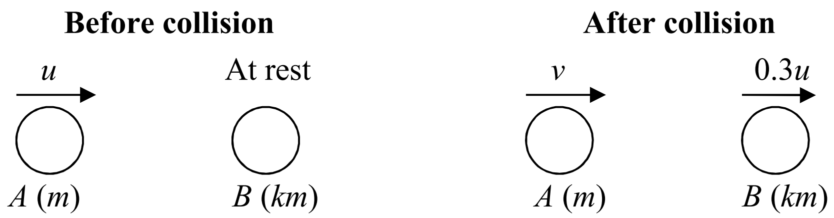
And as $u > 1 \Rightarrow (5e - 1) > 0 \Rightarrow 5e > 1 \Rightarrow e > \dfrac{1}{5}$

Note that subtracting equation **(1)** from equation **(2)** gives:

$2v_1 = 5eu - (-u) \; \Rightarrow \; v_1 = \dfrac{u}{2}(5e + 1)$

So $v_1 > 0$ for any value of *e* as required.

8 a



Using conservation of linear momentum for the system $(\rightarrow)$:

$mu = mv + km0.3u$

$\Rightarrow \; v = u(1 - 0.3k)$ (cancelling out the common factor *m*)

8 b Using Newton's law of restitution:

$$\frac{0.3u - v}{u - 0} = e$$

So using the result from part **a**

$$0.3u - u(1 - 0.3k) = eu$$

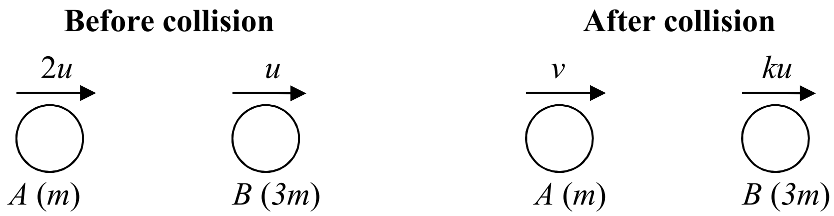
$$\Rightarrow e = 0.3k - 0.7$$

As $0 \leqslant e \leqslant 1$, therefore $0 \leqslant 0.3k - 0.7 \leqslant 1$

$$\Rightarrow 0.7 \leqslant 0.3k \leqslant 1.7$$

$$\Rightarrow \frac{7}{3} \leqslant k \leqslant \frac{17}{3}$$

9 a



Using conservation of linear momentum for the system $(\rightarrow)$:

$$2mu + 3mu = vm + 3kmu$$

$\Rightarrow \ v = u(5 - 3k)$ (cancelling out the common factor m)

b Using Newton's law of restitution:

$$\frac{ku - v}{2u - u} = e$$

So using the result from part **a**

$$ku - u(5 - 3k) = eu$$

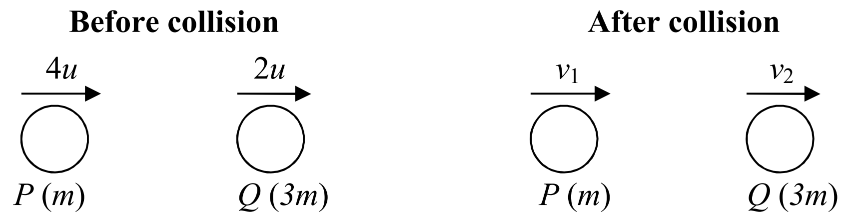
$$\Rightarrow e = 4k - 5$$

As $0 \leqslant e \leqslant 1$, therefore $0 \leqslant 4k - 5 \leqslant 1$

$$\Rightarrow 5 \leqslant 4k \leqslant 6$$

$$\Rightarrow \frac{5}{4} \leqslant k \leqslant \frac{3}{2}$$

10 a



Using conservation of linear momentum for the system $(\rightarrow)$:

$m\times 4u+3m\times 2u=mv_1+3mv_2$

$\Rightarrow\ 3v_2+v_1=10u$ (cancelling out the common factor m) **(1)**

Using Newton's law of restitution:

$e=\dfrac{v_2-v_1}{4u-2u}=\dfrac{v_2-v_1}{2u}$

$\Rightarrow v_2-v_1=2ue$ **(2)**

Adding equations **(1)** and **(2)** gives:

$4v_2=10u+2ue$

$\Rightarrow v_2=\dfrac{u}{4}(10+2e)=\dfrac{u}{2}(5+e)$

b Substituting into equation **(1)** gives:

$\dfrac{3u}{2}(5+e)+v_1=10u$

$2v_1=20u-15u-3ue$

$\Rightarrow v_1=\dfrac{u}{2}(5-3e)$

c The direction of motion of P is unchanged provided that $\dfrac{u}{2}(5-3e)>0$, i.e. $e<\dfrac{5}{3}$

This must be the case as $0\leqslant e\leqslant 1$

d Change of momentum of $Q=3m(v_2-2u)$

$$=3m\left(\frac{5u}{2}+\frac{eu}{2}-2u\right)$$

$$=\frac{3mu}{2}(1+e)$$

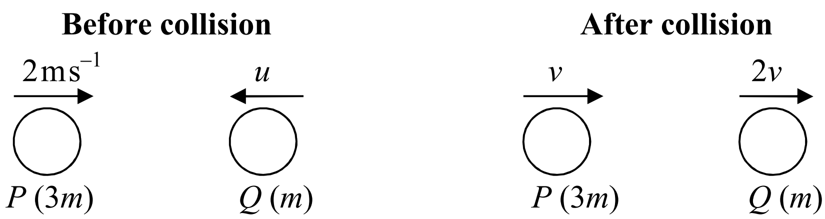
As impulse of P = change in momentum of Q, this gives:

$2mu=\dfrac{3mu}{2}(1+e)$

$1+e=\dfrac{4}{3}$

$\Rightarrow e=\dfrac{1}{3}$

Challenge



Using conservation of linear momentum for the system $(\rightarrow)$:

$3m \times 2 + m \times (-u) = 3mv + 2mv$

$\Rightarrow\ 5v = 6 - u$ (cancelling out the common factor m) **(1)**

Using Newton's law of restitution:

$\frac{1}{4} = \frac{2v - v}{2 + u}$

$\Rightarrow\ 4v = 2 + u$ **(2)**

Eliminating v from equations **(1)** and **(2)** gives:

$\frac{6 - u}{5} = \frac{2 + u}{4}$

So $24 - 4u = 10 + 5u$

$14 = 9u$

$\Rightarrow u = \frac{14}{9}$