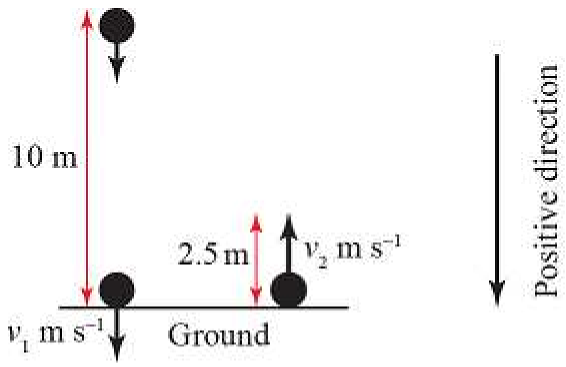
Review Exercise 1

1



As ball descends

$u = 0, a = 9.8, s = 10, v = v_1$

$v^2 = u^2 + 2as$

$v_1^2 = 0^2 + 2 \times 10 \times 9.8 = 196$

$v_1 = \sqrt{196} = 14$

The ball is released from rest 10 m above the ground. The first step is to calculate the speed with which the ball strikes the ground.

After rebound

$v = 0, a = 9.8, s = -2.5, u = v_2$

$v^2 = u^2 + 2as$

$0^2 = v_2^2 + 2 \times 9.8 \times (-2.5) \Rightarrow v_2^2 = 49$

You must then use the fact that the ball reaches a maximum height of 2.5 m to find the velocity with which it rebounds from the ground.

$v_2 = -\sqrt{49} = -7$

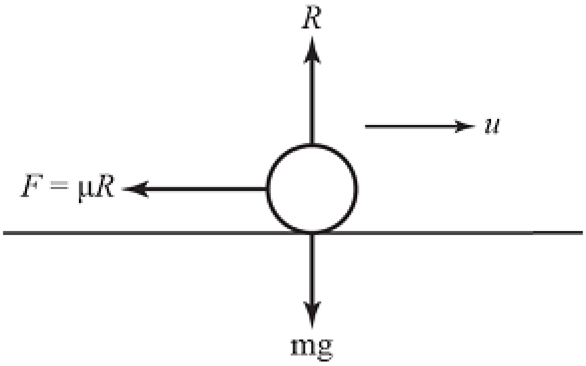
As it rebounds from the ground, the ball is moving upwards. That is in the negative direction. You must take the negative square root of 49, which is –7.

$I = mv_2 - mv_1$

$= 0.3 \times (-7) - 0.3 \times 14 = -6.3$

The magnitude of the impulse is 6.3 N.

2 a $m = 0.250$ kg, $mu = 2$ Ns, $\mu = 0.2$, $v = 0$ ms^{-1}, $s = ?$



Resolving vertically:
$R = mg$

Friction is limiting, so $F = \mu R = \mu mg$

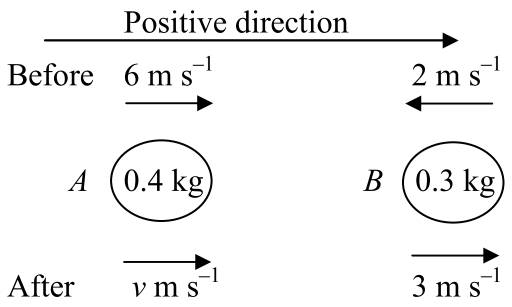
Impulse on car = change of momentum of car:

$$Ft = mv - mu$$
$$\mu mgt = 0 - (-2)$$
$$t = \frac{2}{\mu mg}$$
$$s = \tfrac{1}{2}(u + v)t$$
$$s = \frac{1}{2}\left(\frac{2}{m} + 0\right)\frac{2}{\mu mg}$$
$$s = \frac{2}{\mu m^2 g} = \frac{2}{0.2 \times 0.25^2 \times 9.8} = 16.3265...$$

The racing car travels a distance of 16 m (2 s.f.) past point A before coming to a stop.

b The car stops in a shorter distance because there will be additional frictional forces acting on it (e.g. air resistance) which will increase the deceleration.

3



a Conservation of linear momentum

$$0.4\times 6+0.3\times(-2)=0.4\times v+0.3\times 3$$
$$2.4-0.6=0.4v+0.9$$
$$0.4v=2.4-0.6-0.9=0.9$$
$$v=\frac{0.9}{0.4}=2.25$$

The speed of A after the collision is 2.25 m s^{-1}

The direction of motion of A is unchanged.

b For $B, \mathbf{I}=m\mathbf{v}-m\mathbf{u}$

$$I=0.3\times 3-0.3\times(-2)$$
$$=0.9+0.6=1.5$$

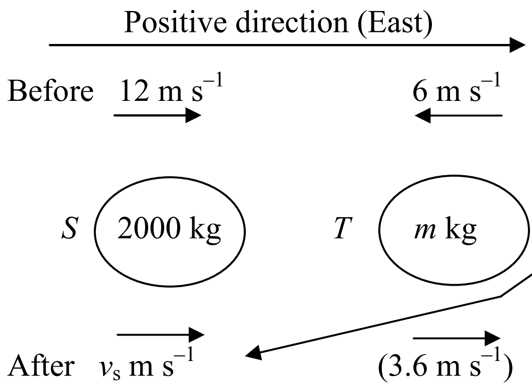
The magnitude of the impulse exerted on B is 1.5 N s

> The total linear momentum before impact must equal the total linear momentum after impact.

> The velocity of B before impact is in the negative direction so it must be entered as –2 in any equations involving linear momentum.

> The velocity of A is positive (2.25 m s^{-1}) after impact and it was positive (6 m s^{-1}) before impact. So the direction of motion of A is unchanged.

4



> You do not know which direction S will be moving in after the impact. Mark the unknown velocity as v m s^{-1} in the positive direction. After you have worked out v, the sign of v will tell you the direction in which S is moving.

a For $S, \mathbf{I}=m\mathbf{v}-m\mathbf{u}$

$$-28\,800=2000\times v_s-2000\times 12$$
$$2000v_s=-28\,800+24\,000=-4800$$
$$v_s=-\frac{4800}{2000}=-2.4$$

The speed of S immediately after the collision is 2.4 m s^{-1}

b Immediately after the collision S is moving due west.

> The sign of v is negative, so S is moving in the negative direction. In this solution, the positive direction has been taken as east, so S is now moving west.

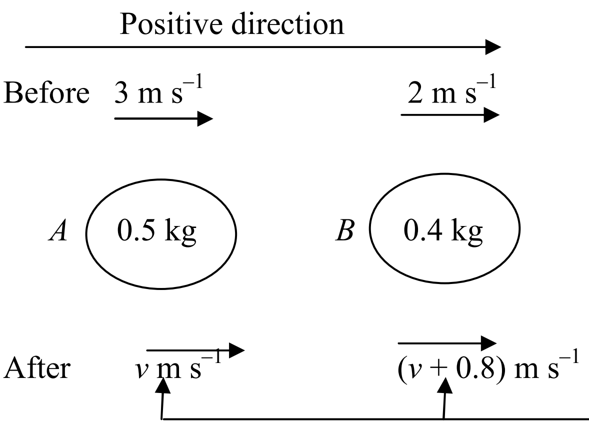
4 c Conservation of linear momentum

$$2000\times 12+m\times(-6)=2000\times(-2.4)+m\times 3.6$$

$$9.6m=24\,000+4800=28\,800\Rightarrow m=\frac{28\,800}{9.6}=3000$$

The mass of T is 3000 kg

5



You need to translate the statement that 'the speed of B is 0.8 m s^{-1} greater than the speed of A' into algebra. If the speed of A after the collision is v m s^{-1} then the speed of B is 0.8 m s^{-1} greater; that is $(v + 0.8)$m s^{-1}

a Conservation of linear momentum

$$0.5\times 3+0.4\times 2=0.5\times v+0.4(v+0.8)$$
$$1.5+0.8=0.5v+0.4v+0.32$$
$$0.9v=1.5+0.8-0.32=1.98$$
$$v=\frac{1.98}{0.9}=2.2$$

All velocities in this part are in the positive direction.

The speed of A after the collision is 2.2 m s^{-1}

The speed of B after the collision is (2.2 + 0.8) m s^{-1} = 3 m s^{-1}

To find the speed of B add 0.8 m s^{-1} to the speed of A.

b The momentum of A before the collision is given by

$$mu=0.5\times 3\,\text{N s}=1.5\,\text{N s}$$

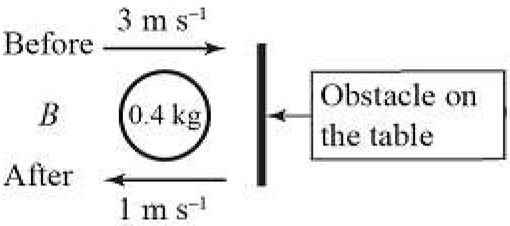
The momentum of A after the collision is given by

$$mv=0.5\times 2.2\,\text{N s}=1.1\,\text{N s}$$

The momentum of a particle is its mass times its velocity. Momentum is a vector quantity.

A loses a momentum of $(1.5-1.1)\,\text{N s}=0.4\,\text{N s}$, as required.

5 c



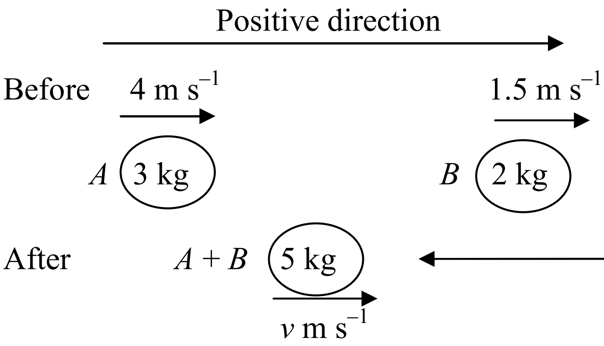
For B, before and after the second impact

$$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$$
$$= 0.4 \times (-1) - 0.4 \times 3$$
$$= -1.6$$

Left to right has been taken as the positive direction throughout the question. The impulse on B is negative as, as the situation is drawn here, the impulse on B is in the direction from right to left.

The magnitude of the impulse received by B in this second impact is 1.6 Ns

6



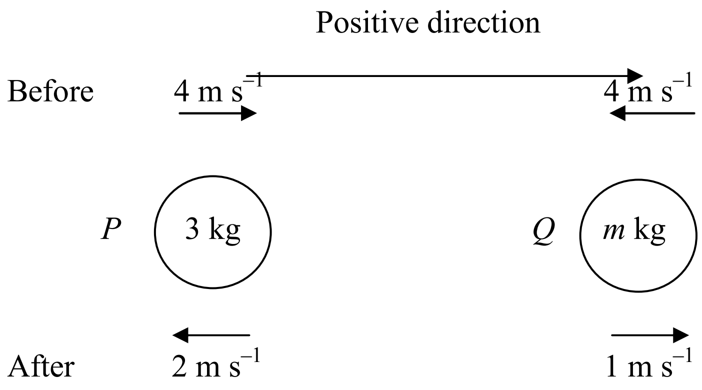
After the collision A (of mass 3 kg) and B (of mass 2 kg) combine to form a single particle. That particle will have the mass which is the sum of the two individual masses, 5 kg.

Conservation of linear momentum

$$4 \times 3 + 2 \times 1.5 = 5 \times v$$
$$12 + 3 = 5v \Rightarrow v = \frac{15}{5} = 3$$

The speed of C immediately after the collision is 3 m s^{-1}

7



a Conservation of linear momentum

$$3\times4+m\times(-4)=3\times(-2)+m\times1$$

$$12-4m=-6+m\Rightarrow5m=18$$

$$m=\frac{18}{5}=3.6$$

In the equation for the conservation of momentum, you must give the velocities in the negative direction a negative sign.

b For $Q, \mathbf{I}=m\mathbf{v}-m\mathbf{u}$

$$I=3.6\times1-3.6\times(-4)$$

$$=3.6+14.4=18$$

The magnitude of the impulse exerted on Q in the collision is 18 N s

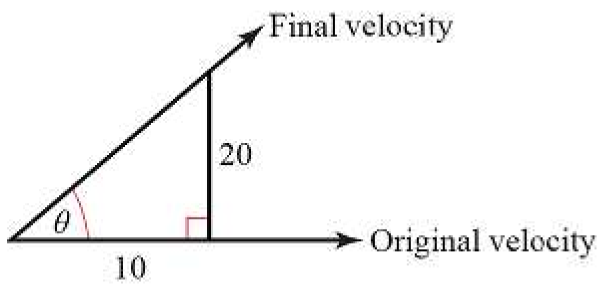
As the magnitude of the impulse exerted on P is the same as the magnitude of the impulse exerted on Q, you could equally correctly work out the change in linear momentum of P. The working then would be $I=3\times(-2)-3\times4=-18,$ which gives the same magnitude, 18 Ns

8 **a** $-4\mathbf{i}+4\mathbf{j}=0.2\mathbf{v}-0.2\times30\mathbf{i}$

$$\mathbf{v}=10\mathbf{i}+20\mathbf{j}\,\text{m s}^{-1}$$

Impulse = change of momentum

b



$$\tan\theta=\frac{20}{10}$$

$$\theta=63.4^\circ$$

c K.E. lost

$$=\frac{1}{2}\times0.2\times30^2-\frac{1}{2}\times0.2(10^2+20^2)$$

$$=40\text{ J}$$

9 a $\mathbf{v} = (t^2 + 2)\mathbf{i} - 6t\mathbf{j}$

$\mathbf{a} = 2t\mathbf{i} - 6\mathbf{j}$ — Differentiate to find **a.**

$\mathbf{F} = 0.75(2t\mathbf{i} - 6\mathbf{j})$ — Use $\mathbf{F} = m\mathbf{a}$ to find **F**.

$t = 4$ — Make $t = 4$

$\mathbf{F} = 0.75(8\mathbf{i} - 6\mathbf{j})$

$= 6\mathbf{i} - 4.5\mathbf{j}$

$|\mathbf{F}| = \sqrt{(6^2 + 4.5^2)} = 7.5\,\text{N}$ — The magnitude of **F** is the modulus of the vector.

b $\mathbf{I} = p\mathbf{i} - p\mathbf{j}$ — Impulse is parallel to **i** – **j**

$|\mathbf{I}| = \sqrt{(p^2 + p^2)} = p\sqrt{2}$

But $|\mathbf{I}| = 9\sqrt{2}\ \therefore p = 9$ — Impulse = change of momentum

$9\mathbf{i} - 9\mathbf{j} = 0.75(\mathbf{v} - (27\mathbf{i} - 30\mathbf{j}))$

$36\mathbf{i} - 36\mathbf{j} = 3\mathbf{v} - 81\mathbf{i} + 90\mathbf{j}$ — For the initial velocity, $t = 5$

$3\mathbf{v} = 117\mathbf{i} - 126\mathbf{j}$

$\mathbf{v} = 39\mathbf{i} - 42\mathbf{j}\,\text{m s}^{-1}$

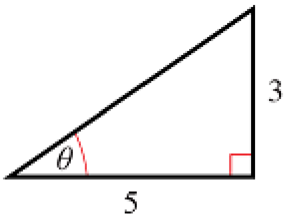
10 a $\mathbf{I} = 0.2((15\mathbf{i} + 15\mathbf{j}) - (-10\mathbf{i}))$ — Use Impulse = change of momentum

$= 5\mathbf{i} + 3\mathbf{j}$

$|\mathbf{I}| = \sqrt{(5^2 + 3^2)} = \sqrt{34}$ — The magnitude of the impulse is the modulus of the vector.

$= 5.83\,\text{N s}$

b



$\tan\theta = \dfrac{3}{5}$

$\theta = 31^\circ$ (nearest degree)

c K.E. gained

$= \dfrac{1}{2} \times 0.2 \times (15^2 + 15^2) - \dfrac{1}{2} \times 0.2 \times 10^2$

$= 35\,\text{J}$

11 a $\mathbf{v} = \int (2\mathbf{i} + 6t\mathbf{j})\,dt$

$= 2t\mathbf{i} + 3t^2\mathbf{j} + \mathbf{c}$

$t = 0\ \dot{\mathbf{v}} = 2\mathbf{i} - 4\mathbf{j}$

$\therefore \mathbf{c} = 2\mathbf{i} - 4\mathbf{j}$

$\mathbf{v} = (2t + 2)\mathbf{i} + (3t^2 - 4)\mathbf{j}$

Don't forget the (vector) constant of integration.

b $t = 2\ \mathbf{v} = 6\mathbf{i} + 8\mathbf{j}$

$3\mathbf{i} - 1.5\mathbf{j} = 0.5(\mathbf{V} - (6\mathbf{i} + 8\mathbf{j}))$

$6\mathbf{i} - 3\mathbf{j} = \mathbf{V} - 6\mathbf{i} - 8\mathbf{j}$

$\mathbf{V} = 12\mathbf{i} + 5\mathbf{j}$

$|\mathbf{V}| = \sqrt{(12^2 + 5^2)} = 13$

Speed $= 13\,\text{m s}^{-1}$

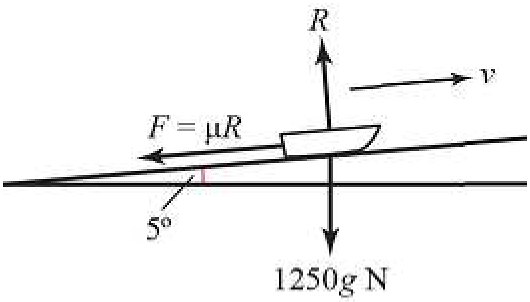
Find the velocity immediately before the impact.

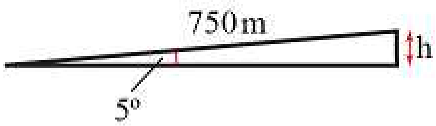
Impulse = change of momentum

Speed is the modulus of the velocity.

12 $m = 1250$ kg, $\mu = 0.05$, $s = 750$ m

a





Resolving perpendicular to the slope:

$R = mg\cos 5$

Friction is limiting, so $F = \mu R$

$F = \mu mg\cos 5$

$F = 0.05 \times 1250 \times 9.8\cos 5 = 610.16...$

The frictional force between the sled and the slope is 610 N (3 s.f.)

b The frictional force acts along the slope, so work done against friction, W_F:

$W_F = Fs$

$W_F = 610 \times 750 = 457500$

The work done against friction is 458 kJ (3 s.f.)

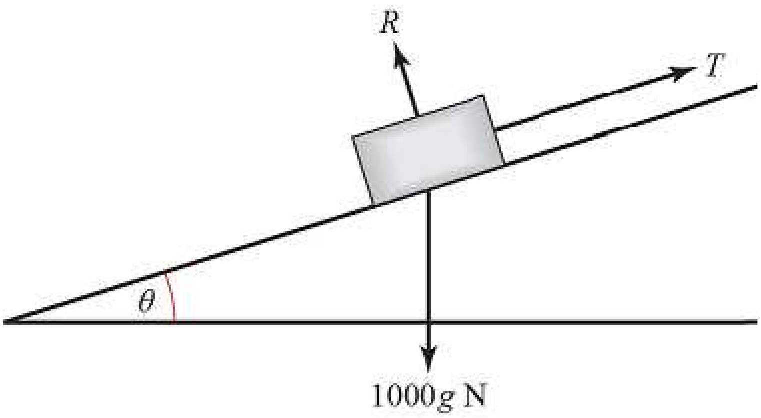
12 c Work done against gravity, $W_G = mgh$

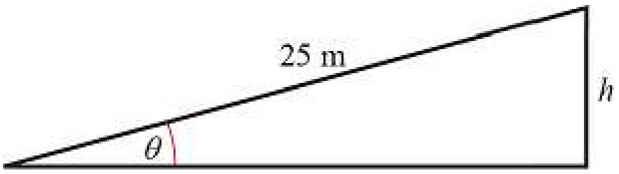
$h = 750\sin 5$

$W_G = 1250 \times 9.8 \times 750\sin 5 = 800743$

The work done against gravity is 801 kJ (3 s.f.)

13 $W_G = 19\,600$ J, $s = 25$ m, $m = 1000$ kg





$W_G = mgh$

$h = 25\sin\theta$

$19600 = 1000 \times 9.8 \times 25\sin\theta$

$25\sin\theta = \frac{19600}{9800} = 2$

$\sin\theta = \frac{2}{25}$ as required.

14 $m = 4$ kg, $h = 40$ m

a From the conservation of energy:
K.E. gained = P.E. lost

Final K.E. $= mgh$

Final K.E. $= 4 \times 9.8 \times 40 = 1568$

When the rock hits the sea, its kinetic energy is 1568 J

b Work will be done against the opposing air resistance and the final kinetic energy will therefore be reduced.

15 $m = 200$ kg, $u = 2$ ms^{-1}, $v = 1.5$ ms^{-1}, $s = 200$ m

a Loss of kinetic energy = initial K.E. – final K.E.

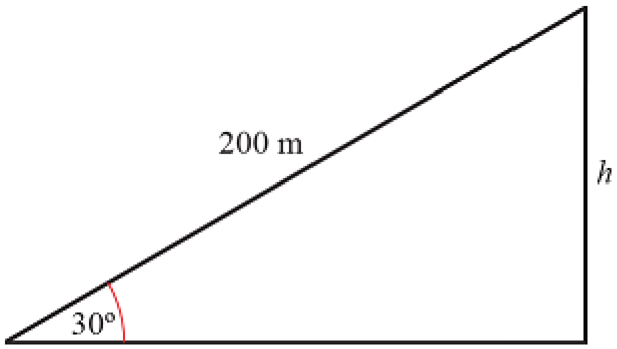
$\text{K.E. lost} = \tfrac{1}{2}mu^2 - \tfrac{1}{2}mv^2$

$\text{K.E. lost} = \tfrac{1}{2}m\left(u^2 - v^2\right)$

$\text{K.E. lost} = \tfrac{1}{2}\times 200\left(2^2 - 1.5^2\right) = 175$

The cable car loses 175 J of kinetic energy.

b Potential energy gained, P.E. = mgh

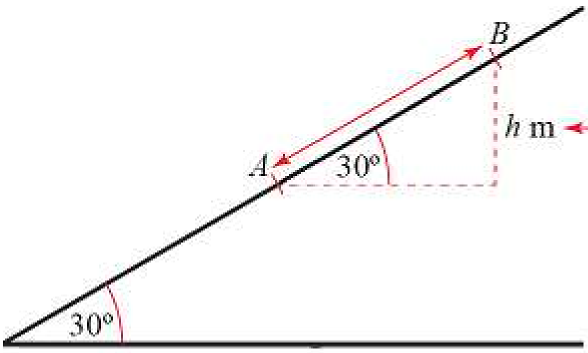


$h = 200\sin 30$

$\text{P.E.} = 200\times 9.8\times 200\sin 30 = 196000$

The potential energy gained is 196 kJ (3 s.f.)

16



The change in the potential energy of P depends on the vertical distance it has moved. You find this using trigonometry.

a Let the vertical distance moved by P be h m.

$$\frac{h}{3} = \sin 30^\circ \Rightarrow h = 3\sin 30^\circ = 1.5$$

The potential energy gained by P is given by
$\text{PE} = mgh = 2 \times 9.8 \times 1.5 = 29.4$

Let the speed of P at B be v m s^{-1}

The kinetic energy lost by P is given by

$$\text{K.E.} = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$$
$$= \frac{1}{2}2\times 10^2 - \frac{1}{2}2v^2 = 100 - v^2$$

Using the principle of conservation of energy

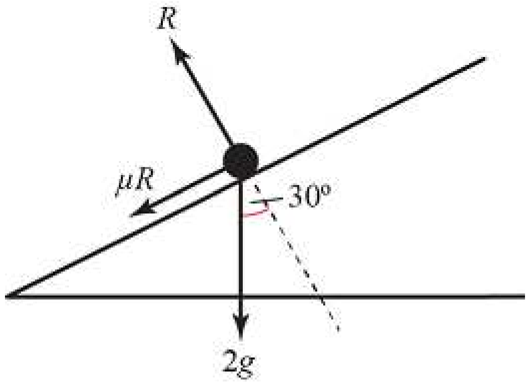
$$100 - v^2 = 29.4$$
$$v^2 = 100 - 29.4 = 70.6$$
$$v = \sqrt{70.6} = 8.402\ldots$$

If no forces other than gravity are acting on the particle, as mechanical energy is conserved, the loss of kinetic energy must equal the gain in potential energy.

The speed of P at B is 8.4 m s^{-1} (2 s.f.)

As a numerical value of g has been used, you should round your final answer to 2 significant figures. Three significant figures are also acceptable.

16 b



Let the normal reaction between the particle and the plane have magnitude R N.

R(↖) $R = 2g\cos 30^\circ$

The frictional force is given by

$F = \mu R = \mu 2g\cos 30^\circ$

The kinetic energy lost by P is given by

$$\text{K.E.} = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$$
$$= \frac{1}{2}2\times 10^2 - \frac{1}{2}2\times 7^2 = 51$$

The potential energy gained by P is the same as in **a**.

The vertical height moved is the same as in **a.**

The total loss of mechanical energy, in J, is $51 - 29.4 = 21.6$

The work done by friction is given by

$$\text{work done} = \text{force} \times \text{distance moved}$$
$$W = \mu R\times 3 = \mu 2g\cos 30^\circ \times 3 = \mu \times 50.922\ldots$$

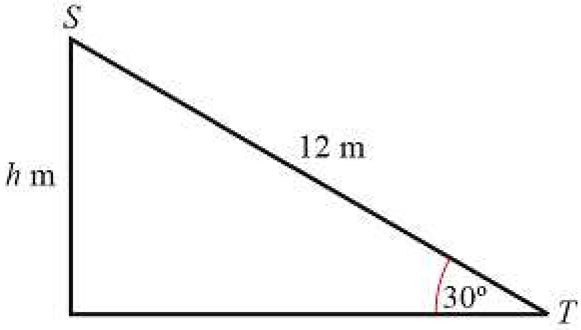
Using the work–energy principle

$\mu \times 50.922\ldots = 21.6 \Rightarrow \mu = 0.424\ldots$

The work done by the friction is equal to the total loss of energy of the particle.

The coefficient of friction is 0.42 (2 s.f.)

17 a



In moving from S to T, P descends a vertical distance of h m, where

$$\frac{h}{12} = \sin 30^\circ \Rightarrow h = 12\sin 30^\circ = 6$$

The change in the potential energy of P depends on the vertical distance it has moved. You find this using trigonometry.

The potential energy, in J, lost by P is given by
$mgh = 0.6 \times 9.8 \times 6 = 35.28$

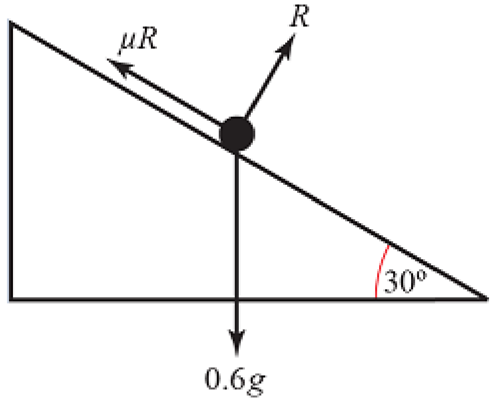
The kinetic energy, in J, lost by P is given by

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = \frac{1}{2}m\left(u^2 - v^2\right)$$
$$= \frac{1}{2} \times 0.6 \times \left(10^2 - 9^2\right) = 5.7$$

The total loss of energy of P is
$(35.28 + 5.7)\,\text{J} = 40.98\,\text{J} = 41\,\text{J}\ (2\text{ s.f.})$

As P moves from S to T both kinetic and potential energy are lost.

17 b



Let the normal reaction between the particle and the plane have magnitude R N.

R(↗) $R = 0.6g\cos 30^\circ$

The frictional force is given by

$F = \mu R = \mu 0.6g\cos 30^\circ = \mu \times 5.09229\ldots$

The work done by friction is given by

$\text{work done} = \text{force} \times \text{distance moved}$

$$W = F \times 12 = \mu \times 61.106\ldots$$

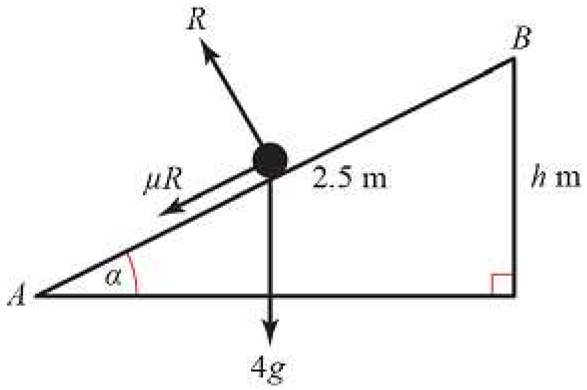
Using the work–energy principle

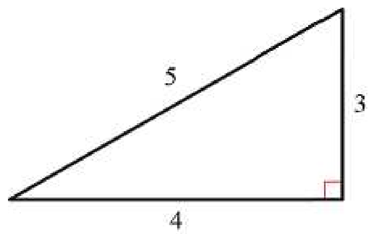
$\mu \times 61.106\ldots = 40.98 \Rightarrow \mu = 0.6706$

The coefficient of friction is 0.67 (2 s.f.)

Friction opposes motion and acts up the plane. The work done by friction against the motion of the particle equals the total loss of energy of the particle. You should use the unrounded answer from **a** for the total energy loss.

18 a $\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{3}{5}, \cos\alpha = \frac{4}{5}$





This diagram illustrates that if

$\tan\alpha = \frac{3}{4}, \sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$

Let the normal reaction between the particle and the plane have magnitude R N.

$\text{R}(\nwarrow) R = 4g\cos\alpha = 4 \times 9.8 \times \frac{4}{5} = 31.36$

The work done by friction is given by
work done = force × distance moved

$$W = \mu R \times AB$$
$$= \frac{2}{7} \times 31.36 \times 2.5 = 22.4$$

The magnitude of the frictional force is given by $F = \mu R$ for the particle's motion both up and down the plane. The direction of the frictional force changes but its magnitude does not.

The work done by friction in moving from A to B is 22.4 J

b Let the vertical distance moved by P in moving from A to B be h m.

$\frac{h}{2.5} = \sin\alpha \Rightarrow h = 2.5 \times \frac{3}{5} = 1.5$

The potential energy, in J, gained by P in moving from A to B is given by
$mgh = 4 \times 9.8 \times 1.5 = 58.8$

Let the speed of P at A be u m s^{-1}

The kinetic energy, in J, lost by P in moving from A to B is given by

$\frac{1}{2}mu^2 = 2u^2$

At B the particle is instantaneously at rest and has no kinetic energy. So all of the initial kinetic energy has been lost.

The mechanical energy, in J, lost by P in moving from A to B is given by
$2u^2 - 58.8$

In moving from A to B, kinetic energy has been lost and potential energy gained. The difference between the values is the net loss.

Using the work–energy principle

$$22.4 = 2u^2 - 58.8$$
$$u^2 = \frac{58.8 + 22.4}{2} = 40.6$$
$$u = \sqrt{40.6} = 6.371\ldots$$

The speed of P at A is 6.4 m s^{-1} (2 s.f.)

18 c Let the speed of P when it returns to A be v m s^{-1}

The work done by friction as P moves from B to A is the same that it does as P moves from A to B. Hence the total work done by friction is
$2 \times 22.4 = 44.8\,\text{J}$

The work done depends on the normal reaction and the distance moved. Both the reaction and the distance are the same as when the particle moves from A to B and from B to A. So you can find the total work done by friction by doubling your answer to **a**.

By the work–energy principle

$$44.8 = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$$
$$= 2 \times 40.6 - 2v^2$$
$$v^2 = \frac{81.2 - 44.8}{2} = 18.2$$
$$v = \sqrt{18.2} = 4.266\ldots$$

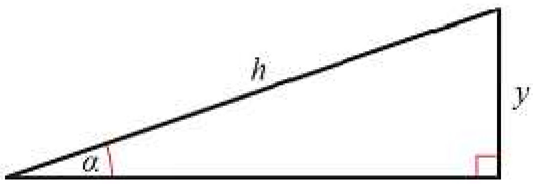
As the particle is now at the same level as it started, there is no change in its potential energy. So the change in mechanical energy is just the loss in kinetic energy.

The speed of P when it returns to A is 4.3 m s^{-1} (2 s.f.)

19 a $\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha\frac{3}{5}, \cos\alpha = \frac{4}{5}$

You can sketch a 3, 4, 5 triangle to check these relations.

As B descends a distance h, A moves a distance h up the plane. Let the vertical displacement of A be y.



$$\frac{y}{h} = \sin\alpha\frac{3}{5} \Rightarrow y = \frac{3}{5}h$$

The potential energy lost by B is $2mgh$

B has mass $2m$ and descends a distance h.

The potential energy gained by A is

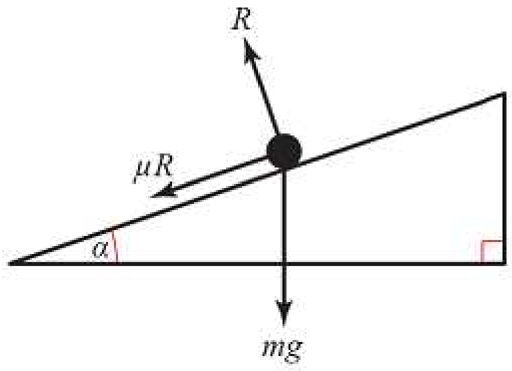
$$mgy = mg \times \frac{3}{5}h = \frac{3}{5}mgh$$

A has mass m and ascends a vertical distance $\frac{3}{5}h$

The net loss in potential energy of the system is

$$2mgh - \frac{3}{5}mgh = \frac{7}{5}mgh$$

19 b For A



Let the normal reaction between the particle and the plane have magnitude R N

$$R(\nwarrow)\; R = mg\cos\alpha = mg \times \frac{4}{5} = \frac{4}{5}mg$$

The work done by friction is given by
work done = force × distance moved

$$W = \mu R \times h$$
$$= \frac{5}{8} \times \frac{4}{5}mg \times h = \frac{1}{2}mgh$$

The gain in kinetic energy is

$$\frac{1}{2}mv^2 + \frac{1}{2}(2m)v^2 = \frac{3}{2}mv^2$$

Both particles start from rest, so the system has no initial kinetic energy.

The net loss of mechanical energy is

$$\frac{7}{5}mgh - \frac{3}{2}mv^2$$

The total loss in mechanic energy is the difference between the loss in potential energy you worked out in **a,** less the kinetic energy gained.

Using the work–energy principle

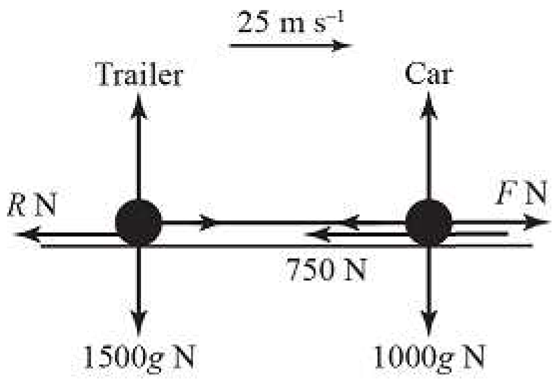
$$\frac{1}{2}mgh = \frac{7}{5}mgh - \frac{3}{2}mv^2$$

$$\frac{3}{2}mv^2 = \left(\frac{7}{5} - \frac{1}{2}\right)mgh = \frac{9}{10}mgh$$

$$v^2 = \frac{2}{3} \times \frac{9}{10}gh = \frac{3}{5}gh$$

The work done by friction against the motion of the particle A equals the total loss of energy of the system. Other than gravity, there is no force acting on B. The only force causing the loss of mechanical energy is the friction acting on A.

20 a



Let F N be the magnitude of the driving force produced by the engine of the car.

$$50\text{kW} = 50\,000\text{ W}$$
$$P = Fv$$
$$50\,000 = F \times 25 \Rightarrow F = 2000$$

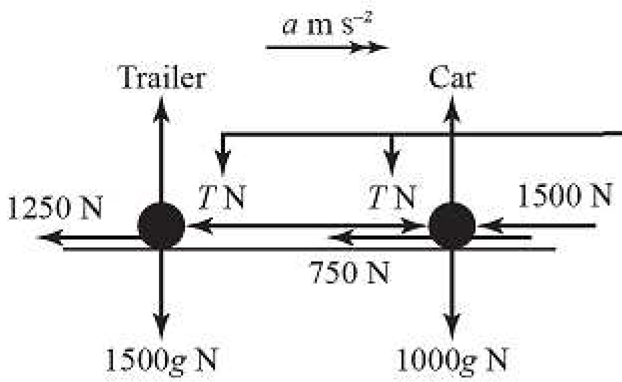
For the car and trailer combined

$R(\rightarrow) F - 750 - R = 0$

$R = F - 750 = 2000 - 750 = 1250,$ as required

When you consider the car and trailer combined, the tensions at the ends of the tow–bar cancel one another out and can be ignored.

20 b



As the car brakes, the forces in the tow-bar are thrusts and act in the directions shown in this diagram. The forces in the tow-bar in **a** are tensions and act in the opposite directions to thrusts.

Let the acceleration of the car while braking be a m s^{-2}
For the car and trailer combined:

$$\text{R}(\rightarrow)\ \mathbf{F} = m\mathbf{a}$$
$$-1500 - 750 - 1250 = 2500a$$
$$2500a = -3500 \Rightarrow a = -\frac{3500}{2500} = -1.4$$

The deceleration of the car is therefore 1.4 m s^{-2}

c Let the magnitude of the thrust in the tow-bar while braking be T N.

For the trailer alone
$$\text{R}(\rightarrow)\ \mathbf{F} = m\mathbf{a}$$
$$-1250 - T = 1500a = 1500 \times (-1.4)$$
$$T = 1500 \times 1.4 - 1250 = 850$$
The magnitude of the thrust in the tow-bar while braking is 850 N

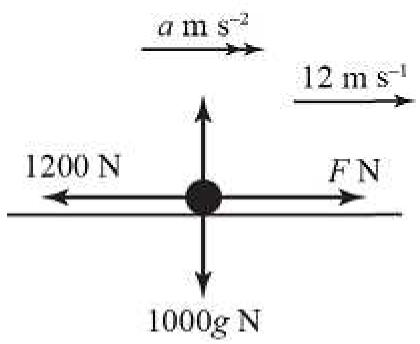
20 d To find the distance travelled in coming to rest
$$v^2 = u^2 + 2as$$
$$0^2 = 25^2 + 2 \times (-1.4)s$$
$$s = \frac{25^2}{2.8}$$
The work done, in J, by the braking force of 1500 N is given by
$$W = 1500 \times s = 1500 \times \frac{25^2}{2.8} = 334821$$

Work done = force × distance moved

The work done by the braking force in bringing the car and the trailer to rest is 335 kJ (3 s.f.)

e The resistance could be modelled as varying with speed.

21



a Let F N be the magnitude of the driving force produced by the engine of the car.

$$24\,\text{kW} = 24\,000\,\text{W}$$
$$P = Fv$$
$$24\,000 = F \times 12 \Rightarrow F = 2000$$

$$\text{R}(\rightarrow)\ \mathbf{F} = m\mathbf{a}$$
$$F - 1200 = 1000a$$
$$2000 - 1200 = 1000a \Rightarrow a = \frac{800}{1000} = 0.8\ \text{m s}^{-2}$$

21 b The kinetic energy, in J, lost as the car is brought to rest is

$$\frac{1}{2}mu^2 = \frac{1}{2}1000\times14^2 = 98\,000$$

The final kinetic energy is zero.

$$\text{Work done by resistance} = \text{Energy lost}$$
$$\text{Resistance}\times\text{distance} = \text{Energy lost}$$
$$1200d = 98\,000$$
$$d = \frac{98\,000}{1200} = 81\frac{2}{3}$$

You use the work-energy principle. The work done by the resistance (1200 N) in bringing the car to rest is equal to the kinetic energy lost.

c Resistance usually varies with speed.

As the speed slows down, the resistance to motion usually decreases. In this case, this might mean that the car would travel further.

22 a From A to B, the cyclist descends
$(20-12)\text{m} = 8\,\text{m}$

Whatever the path taken, the potential energy lost in travelling from A to B depends solely on the difference in levels between A and B.

The potential energy, in J, lost in travelling from A to B is given by

$$mgh = 80\times9.8\times8 = 6272$$

The kinetic energy, in J, lost in travelling from A to B is given by

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = \frac{1}{2}m\left(u^2 - v^2\right)$$
$$= 40\left(8^2 - 5^2\right) = 1560$$

The total mechanical energy lost is
$(6272 + 1560)\,\text{J} = 7832\,\text{J}$

The work done by resistance due to non-gravitational forces is given by

$$W = \text{force}\times\text{distance moved}$$
$$= 20\times500 = 10\,000$$

$(10\,000 - 7832)\,\text{J} = 2168\,\text{J}$

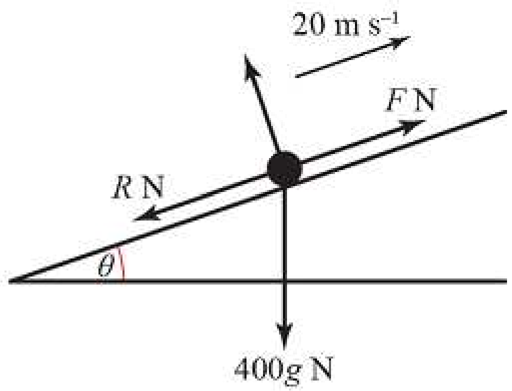
The non-gravitational resistances to motion have worked 10 000 J against the motion. However, the mechanical energy lost is only 7832 J. The difference between these values is the work that has been done by the cyclist.

The work done by the cyclist in moving from A to B is 2200 J (2 s.f.)

22 b At B, let the force generated by the cyclist be FN.

$$R(\rightarrow)\ \mathbf{F} = m\mathbf{a}$$
$$F - 20 = 80 \times 0.5$$
So $F = 60N$
$$P = Fv$$
$$= 60 \times 5 = 300$$
The power generated by the cyclist is 300 W

23



Let FN be the magnitude of the driving force produced by the engine.

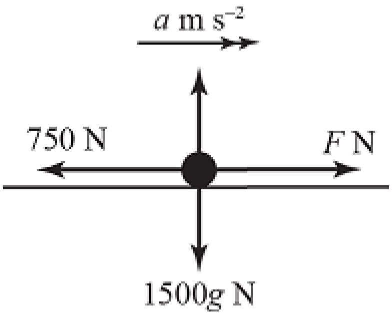
$$10\,\text{kW} = 10\,000\,\text{W}$$
$$P = Fv$$
$$10\,000 = F \times 20 \Rightarrow F = 500$$

Before you use the formula $P = Fv$, you have to convert kilowatts to watts.

$$R(\nearrow)\ \mathbf{F} = m\mathbf{a}$$
$$F - R - 400g\sin\theta = 0$$
$$R = F - 400g\sin\theta$$
$$= 500 - 400 \times 9.8 \times \frac{1}{14} = 220$$

As the car is travelling at a constant speed, its acceleration is zero.

24 a



Let the acceleration of the lorry be a m s^{-2} and the driving force of the engine have magnitude FN.

$$36\text{kW} = 36\,000\text{ W}$$

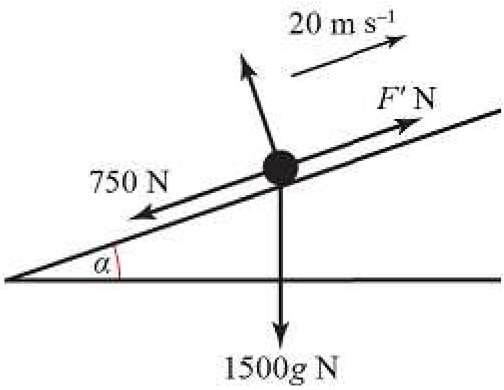
kW must be converted to W.

$$P = Fv$$
$$36\,000 = F \times 20 \Rightarrow F = 1800$$
$$\text{R}(\rightarrow)\ \mathbf{F} = m\mathbf{a}$$
$$F - 750 = 1500a$$
$$1800 - 750 = 1500a$$
$$a = \frac{1800 - 750}{1500} = 0.7$$

The acceleration of the lorry when the speed is 20 m s^{-1} is 0.7 m s^{-2}

This result is only true at one instant in time. The speed would now increase and the driving force and acceleration decrease.

24 b



Let the driving force of the engine have magnitude F' N.

The driving forces in **a** and **b** are different and it is a good idea to avoid confusion by using different symbols for the forces.

$R(\nearrow)\ \mathbf{F} = m\mathbf{a}$

$F' - 750 - 1500g\sin\alpha = 0$

In this part of the question the lorry is moving at a constant speed and the acceleration is zero.

$F' = 750 + 1500 \times 9.8 \times \frac{1}{10} = 2220$

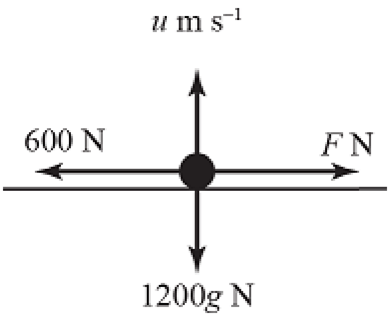
$P = Fv$

$\quad = 2220 \times 20 = 44\,400$

The rate at which the lorry is now working is 44.4 kW.

This question asks for no particular form of the answer, so you could give your answer in either W or kW. Two or three significant figures are acceptable.

25 a



Let the speed of the car be u m s^{-1} and the driving force of the engine have magnitude F N.

$21\text{kW} = 21000\text{ W}$

$R(\rightarrow)\ F - 600 = 0 \Rightarrow F = 600$

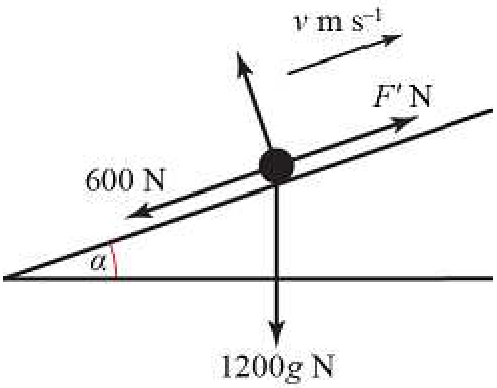
$P = Fv$

$21000 = 600v \Rightarrow v = \dfrac{21000}{600} = 35$

The speed of the car is 35 m s^{-1}

In both parts of this question, as the car is moving with constant speed, the acceleration is zero. So the vector sum of the forces acting on the car is zero.

b



Let the speed of the car be u ms^{-1} and the driving force of the engine have magnitude F' N.

The driving forces in **a** and **b** are different and it is a good idea to avoid confusion by using different symbols for the forces.

$R(\nearrow) F' - 1200g\sin\alpha - 600 = 0$

$F' = 1200 \times 9.8 \times \dfrac{1}{14} + 600 = 1440$

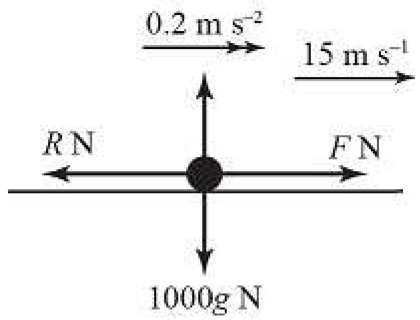
$P = Fv$

$2100 = 1440v \Rightarrow v = 14.58\dot{3}$

The constant speed of the car as it moves up the hill is 15 m s^{-1} (2 s.f.)

Although there is an exact answer, $14\frac{7}{12}$, a numerical value for g has been used in the question and the answer should be rounded to 2 significant figures. Three significant figures (14.6) is also acceptable.

26

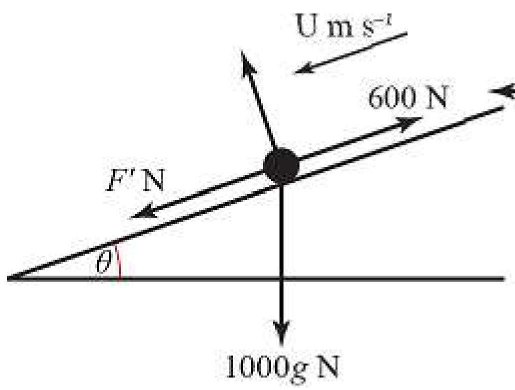


a Let F N be the magnitude of the driving force produced by the engine of the car.

$$12\text{ kW} = 12\,000\text{ W}$$
$$P = Fv$$
$$12\,000 = F \times 15$$
$$F = \frac{12\,000}{15} = 800$$
$$\text{R}(\rightarrow)\ \mathbf{F} = m\mathbf{a}$$
$$F - R = 1000 \times 0.2$$
$$R = F - 1000 \times 0.2$$
$$= 800 - 200 = 600, \text{ as required}$$

Using Newton's second law, the vector sum of the forces on the car equals the mass times acceleration.

b



Resistance acts against motion. As the car is travelling down the hill, the resistance of 600 N acts up the hill.

Let the driving force of the engine have magnitude F' N.

$$\text{R}(\swarrow)\ F' + 1000g \sin\theta - 600 = 0$$
$$F' = 600 - 1000 \times 9.8 \times \frac{1}{40} = 355$$

As the car is travelling at a constant speed, there is no acceleration.

$$7\text{ kW} = 7000\text{ W}$$
$$P = Fv$$
$$7000 = 355U$$
$$U = \frac{7000}{355} = 19.718\ldots = 20\ (2\text{ s.f.})$$

27 $m = 600$ kg, $R = \left(500 + 2v^2\right)$ N, $v = 15$ ms^{-1}, $P = ?$

a The engine must create a force F where $F = R$

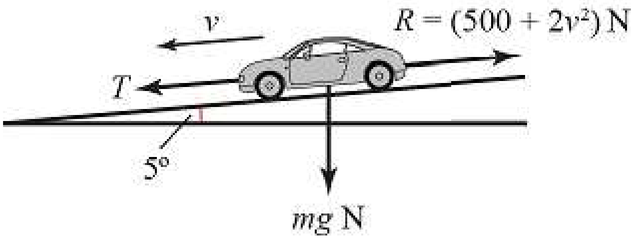
$P = Fv$

$P = \left(500 + 2v^2\right)v$

$P = \left(500 + \left(2 \times 15^2\right)\right) \times 15 = 14250$

For the motorcycle to maintain a constant speed of 15 ms^{-1} on a horizontal road, the engine must deliver 14.3 kW (3 s.f.)

b



Resolving parallel to the slope:

$T = \left(500 + 2v^2\right) - mg\sin 5$

So the power required is:

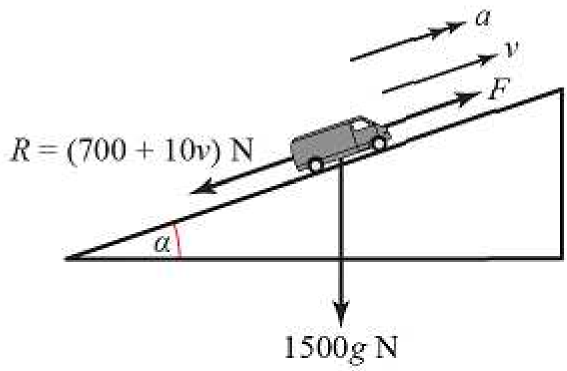
$P = \left(500 + 2v^2 - mg\sin 5\right)v$

$P = \left(500 + \left(2 \times 15^2\right) - (600 \times 9.8\sin 5)\right) \times 15 = 6562.8...$

For the motorcycle to maintain a constant speed of 15 ms^{-1} when travelling down a road inclined at 5° to the horizontal, the engine must deliver 6.6 kW (2 s.f.)

28 $m = 1500$ kg, $R = (700 + 10v)$ N, $v = 30$ ms^{-1}

a



$P = 60\,000$ W

The force provided by the engine, F, is given by:

$$P = Fv$$
$$60000 = 30F$$
$$F = 2000$$

Using Newton's second law of motion up the hill:

$$2000 - (700 + 10v) - mg\sin\alpha = ma$$
$$2000 - (700 + 300) - \left(1500 \times 9.8 \times \tfrac{1}{12}\right) = 1500a$$
$$a = -\frac{225}{1500} = -0.15$$

The initial deceleration of the van is 0.15 ms^{-2}

b $P = 80\,000$ W

The force provided by the engine is now given by:

$$P = F'v$$
$$80000 = F'v$$
$$F' = \frac{80000}{v}$$

When the van reaches its maximum speed, the acceleration will be zero.
Therefore, by Newton's second law,
the resultant force on the van (in the direction of the acceleration)
will be zero.

Only the positive root is relevant.

When the engine operates at 80 kW, the van maintains a constant uphill speed of 35.1 m s^{-1} (3 s.f.)

29 a The vertical distance fallen by P in moving from A to C is (45 – 30) m =15 m

Using the principle of conservation of energy, kinetic energy gained = potential energy lost

$$\frac{1}{2}\cancel{m}v^2 - \frac{1}{2}\cancel{m}u^2 = \cancel{m}gh$$

$$\frac{1}{2}\times 24.5^2 - \frac{1}{2}u^2 = 9.8\times 15$$

$$u^2 = 24.5^2 - 2\times 9.8\times 15 = 306.25$$

$$u = \sqrt{306.25} = 17.5, \text{as required}$$

The mass of the particle cancels throughout this equation. The calculations in this question are independent of the mass of P.

This equation has a similar form to $v^2 = u^2 + 2as$. However, it would be an error to use this formula, which is a formula for motion in a straight line, as P is not moving in a straight line.

b $R(\rightarrow)\, u_x = u\cos\theta = 17.5\times\frac{4}{5} = 14$

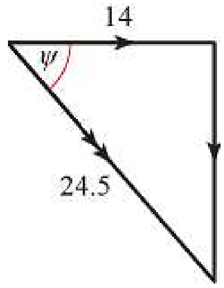
The horizontal component of the velocity is constant throughout the motion.

Let the required angle be ψ

$$\cos\psi = \frac{14}{24.5} = \frac{4}{7}$$

$$\psi = 55.15\ldots^\circ = 55^\circ \text{ (nearest degree)}$$

At C, the velocity of P and its components are illustrated in this diagram.



ψ can now be found using trigonometry. There is no need to find the vertical component of the velocity at C.

c $R(\uparrow)\, u_y = u\sin\theta = 17.5\times\frac{3}{5} = 10.5$

To find the time taken for P to move from A to D

$$R(\uparrow)\, s = ut + \frac{1}{2}at^2$$

$$-45 = 10.5t - 4.9t^2$$

$$4.9t^2 - 10.5t - 45 = 0$$

$$49t^2 - 105t - 450 = 0$$

$$(7t - 30)(7t + 15) = 0$$

$$t = \frac{30}{7}, \text{as } t > 0$$

These factors are difficult to spot and you can use the formula for a quadratic. You should, however, obtain an exact answer.

$R(\rightarrow)$ distance = speed × time

$$= 14\times\frac{30}{7} = 60$$

$$BD = 60\text{ m}$$

30 a The kinetic energy, in J, gained in moving from A to B is

$$\frac{1}{2}mv^2 = \frac{1}{2}80 \times 20^2 = 16\,000$$

The potential energy, in J, lost in moving from A to B is

$$mgh = 80 \times 9.8 \times (32.5 - 8.1) = 19129.6$$

The net loss of mechanical energy is
$(19129.6 - 16\,000)\text{J} = 3129.6\text{J}$

The net loss in mechanical energy is the work done by the resistance to motion.

The work done by the resisting force of R newtons, in J, is given by

$$\begin{aligned}\text{Work} &= \text{force} \times \text{distance}\\ &= R \times 60\end{aligned}$$

By the work–energy principle

$$60R = 3129.6$$

$$R = \frac{3129.6}{60} = 52.16 = 52\text{N}\,(2\,\text{s.f.})$$

b $\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{3}{5}, \cos\alpha = \frac{4}{5}$

You can sketch a 3, 4, 5, triangle to check these relations.

$$\text{R}(\rightarrow)\, u_x = 20\cos\alpha = 20 \times \frac{4}{5} = 16$$

$$R(\uparrow)\, u_y = 20\sin\alpha = 20 \times \frac{3}{5} = 12$$

To find the time taken to move from B to C

$$\text{R}(\uparrow)\, s = ut + \frac{1}{2}at^2$$

$$-8.1 = 12t - 4.9t^2$$

$$4.9t^2 - 12t - 8.1 = 0$$

$$49t^2 - 120t - 81 = 0$$

$$(t-3)(49t+27) = 0$$

$$t = 3, \text{as } t > 0$$

Rearranging the quadratic and multiplying by 10.

The time taken to move from B to C is 3 s.

c $\rightarrow \text{distance} = \text{speed} \times \text{time}$

$$= 16 \times 3 = 48$$

The horizontal distance from B to C is 48 m.

30 d Let the speed of the skier immediately before reaching C be w m s^{-1}

Using the conservation of energy

$$\frac{1}{2}\cancel{m}w^2 - \frac{1}{2}\cancel{m}v^2 = \cancel{m}gh$$

$$w^2 = v^2 + 2gh$$

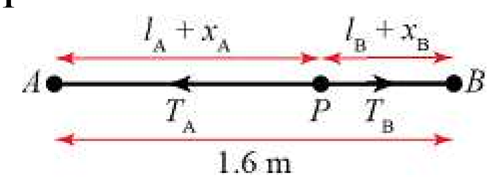
$$= 20^2 + 2\times 9.8\times 8.1 = 558.76$$

$$w = \sqrt{558.76} = 23.638\ldots$$

Cancelling the m and rearranging the formula. This result is similar to $v^2 = u^2 + 2as$. However, it would be an error to use this formula, which is a formula for motion in a straight line, as the skier is not moving in a straight line. You must establish the result using the principle of conservation of energy.

The speed of the skier immediately before reaching C is 24 m s^{-1} (2 s.f.)

31



AB = 1.6 m

For spring AP: $l = l_A = 0.8$ m, $\lambda = \lambda_A = 24$ N, $x = x_A$, $T = T_A$
For spring PB: $l = l_B = 0.4$ m, $\lambda = \lambda_A = 20$ N, $x = x_B$, $T = T_B$

$l_A + l_B + x_A + x_B = 1.6$

$x_A + x_B = 1.6 - 0.8 - 0.4 = 0.4$
$x_B = 0.4 - x_A$

Since P is in equilibrium, $T_A = T_B$

a Using Hooke's law for each spring:

$$T = \frac{\lambda x}{l}$$

$$\frac{\lambda_A x_A}{l_A} = \frac{\lambda_B x_B}{l_B}$$

$$\frac{24x_A}{0.8} = \frac{20x_B}{0.4}$$

$30x_A = 50x_B$ substituting for x_B

$$3x_A = 5(0.4 - x_A)$$

$$8x_A = 2$$

$$x_A = 0.25$$

$$AP = l_A + x_A$$

$$AP = 0.8 + 0.25 = 1.05$$

The distance AP is 1.05 m.

31 b Substituting the value for x_A into the expression for T_A

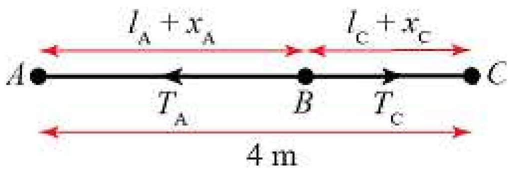
$$T_A = \frac{\lambda_A x_A}{l_A}$$

$$T_A = \frac{24 \times 0.25}{0.8} = 7.5$$

Since P is in equilibrium, $T_A = T_B$

The tension in each spring is 7.5 N

32



AC = 4 m

For spring AB: $l = l_A = 1.5$ m, $\lambda = \lambda_A = 20$ N, $x = x_A$, $T = T_A$

For spring BC: $l = l_C = 0.75$ m, $\lambda = \lambda_A = 15$ N, $x = x_C$, $T = T_C$

$l_A + l_C + x_A + x_C = 4$

$x_A + x_C = 4 - 1.5 - 0.75 = 1.75$

$x_C = 1.75 - x_A$

Since system is in equilibrium, $T_A = T_C$

Using Hooke's law for each spring:

$$T = \frac{\lambda x}{l}$$

$$\frac{\lambda_A x_A}{l_A} = \frac{\lambda_C x_C}{l_C}$$

$$\frac{20x_A}{1.5} = \frac{15x_C}{0.75}$$

$$\frac{40x_A}{3} = 20x_C$$

$2x_A = 3x_C$ substituting for x_C

$$2x_A = 3(1.75 - x_A)$$

$$5x_A = 5.25$$

$$x_A = 1.05$$

$$AB = l_A + x_A$$

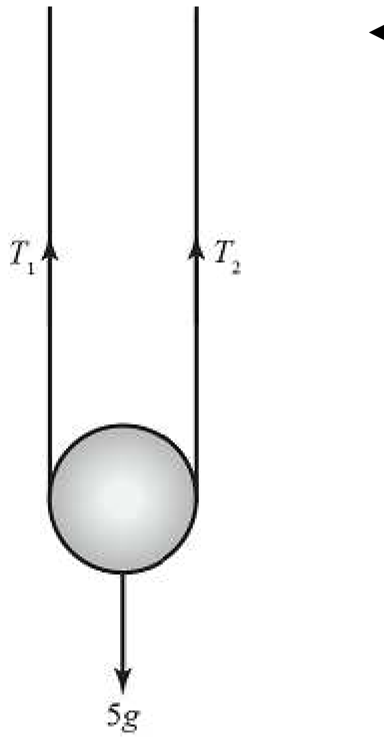
$$AB = 1.5 + 1.05 = 2.55$$

$$BC = 4 - AB$$

$$BC = 4 - 2.55 = 1.45$$

The distances *AB* and BC are 2.55 m and 1.45 m respectively.

33



You must consider the two strings separately. Here the string of natural length 100 cm is drawn on the left. The extension of this string is (120 – 100) cm = 20 cm and the tension in this string is taken as T_1 newtons. The string of natural length 90 cm is drawn on the right. The extension of this string is (120 – 90) cm = 30 cm and the tension in this string is taken as T_2 newtons.

For the string of natural length 100 cm

Hooke's law $T = \frac{\lambda x}{l}$

$$T_1 = \frac{175 \times 20}{100} = 35$$

For the string of natural length 90 cm

Hooke's law $T = \frac{\lambda x}{l}$

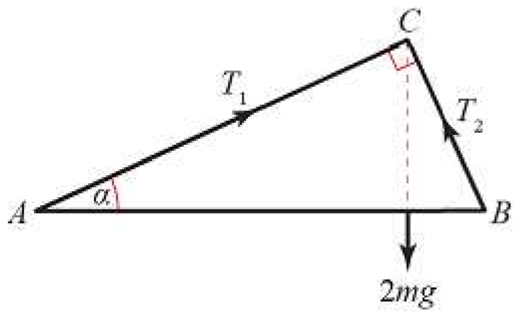
$$T_2 = \frac{\lambda \times 30}{90} = \frac{\lambda}{3}$$

$$R(\uparrow) T_1 + T_2 = 5g$$

$$35 + \frac{\lambda}{3} = 5 \times 9.8$$

$$\lambda = 3(5 \times 9.8 - 35) = 42$$

34



a The line of action of the weight must pass through C which is not above the centre of the rod.

For three forces to be in equilibrium the lines of action of all three forces must pass through the same point. As the lines of action of both tensions pass through C, the line of action of the weight has to pass through C as well, so the rod cannot be uniform.

34 b $\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha \frac{3}{5}, \cos\alpha = \frac{4}{5}$

Let the tension in AC be T_2 newtons and the tension in BC be T_2 newtons.

$R(\rightarrow) \quad T_1\cos\alpha = T_2\sin\alpha$

$\frac{4}{5}T_1 = \frac{3}{5}T_2 \Rightarrow T_1 = \frac{3}{4}T_2$

$R(\uparrow)\ T_1\sin\alpha + T_2\cos\alpha = 2mg$

You substitute $T_1 = \frac{3}{4}T_2$ and the values of sin α and cos α into this equation and solve for T_2

$\frac{3}{4}T_2 \times \frac{3}{5} + T_2 \times \frac{4}{5} = 2\,mg$

$\left(\frac{9}{20} + \frac{4}{5}\right)T_2 = \frac{5}{4}T_2 = 2\,mg$

$T_2 = \frac{8}{5}\,mg$

The tension in BC is $\frac{8}{5}mg$, as required.

$T_1 = \frac{3}{4}T_2 = \frac{3}{4} \times \frac{8}{5}mg = \frac{6}{5}mg$

The tension in AC is $\frac{6}{5}mg$.

c $BC = AB\sin\alpha = 2a \times \frac{3}{5} = \frac{6}{5}a$

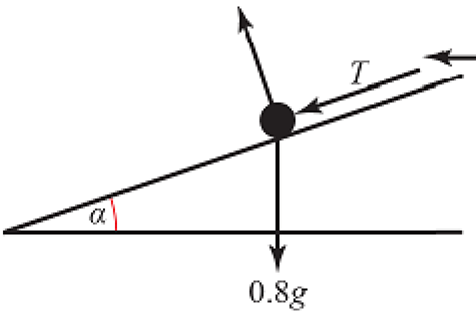
For BC

Hooke's law $T_2 = \frac{\lambda x}{l}$

You find the length of BC by trigonometry. Then the extension of the elastic string BC is $\frac{6}{5}a - a = \frac{1}{5}a$

$\frac{8}{5}mg = \frac{kmg \times \frac{1}{5}a}{a} \Rightarrow k = 8$

35



Initially the spring is in compression and the force of the spring on the particle is acting down the plane.

Let the thrust in the spring be T newtons.

Hooke's law $T = \frac{\lambda x}{l}$

$= \frac{20 \times 0.4}{2} = 4$

The compression is (2 – 1.6) m = 0.4 m

R(↙) $\mathbf{F} = m\mathbf{a}$

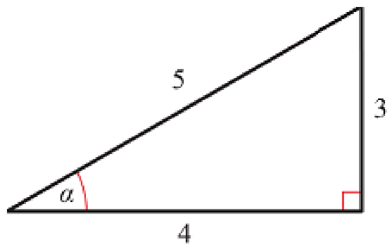
$mg \sin\alpha + T = ma$

$0.8 \times 9.8 \times \frac{3}{5} + 4 = 0.8a$

$0.8a = 8.704$

$a = 10.88$

When you know tan α you can draw a triangle to find cos α and sin α.



$\tan\alpha = \frac{3}{4}$

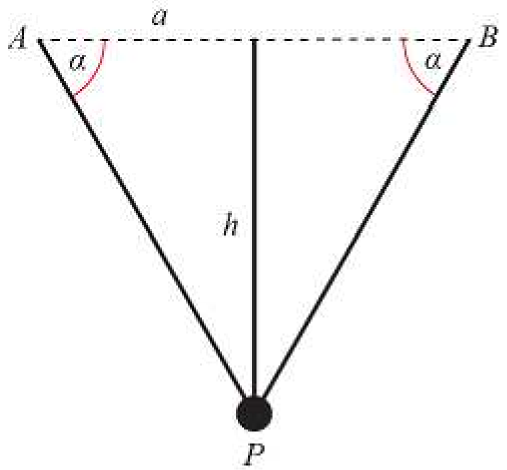
$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

The initial acceleration of the particle is 11 m s^{-1} (2 s.f.)

As you have used an approximate value of g, you should round your answer to a sensible accuracy. Either 2 or 3 significant figures is acceptable.

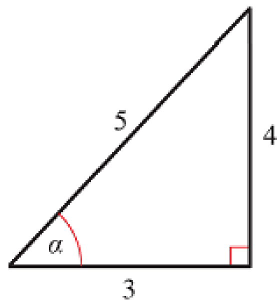
36



When P comes instantaneously to rest, it is not in equilibrium and so the question cannot easily be solved by resolving. It is a common error to attempt the solution of this, and similar questions, by resolving.

$$\tan\alpha = \frac{4}{3} \Rightarrow \cos\alpha = \frac{3}{5}$$

When you know tan α you can draw a triangle to find cos α.



$$\tan\alpha = \frac{4}{3}$$

$$\cos\alpha = \frac{3}{5}$$

Let the distance fallen by P be h.

$$h = a\tan\alpha = \frac{4a}{3}$$

$$AP^2 = h^2 + a^2 = \left(\frac{4a}{3}\right)^2 + a^2 = \frac{25a^2}{9}$$

$$AP = \frac{5a}{3}$$

When P first comes to rest the energy stored in one string is given by

$$E = \frac{\lambda x^2}{2l}$$

$$= \frac{\lambda\left(\frac{2a}{3}\right)^2}{2a} = \frac{2\lambda a}{9}$$

The extension in one string is

$$\text{AP} - \text{natural length} = \frac{5a}{3} - a$$

$$= \frac{2a}{3}$$

When P first comes to rest the potential energy lost is given by

$$mgh = mg \times \frac{4}{3}a$$

Conservation of energy

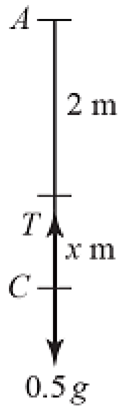
Elastic energy gained = potential energy lost

Initially P is at rest and, when it has fallen $\frac{5a}{3}$, it is at rest again. So there is no change in kinetic energy. Elastic energy is gained by both strings and potential energy is lost by the particle.

$$\frac{4\lambda a}{9} = \frac{4mga}{3}$$

$$\lambda = \frac{4mga}{3} \times \frac{9}{4a} = 3\,mg$$

37



In solving nearly all questions involving elastic strings and springs you need to find the value of, or an expression for, the extension. If no symbol is given in the question, you should introduce a symbol, here x m, yourself.

a Let the extension of string when B is at C be x m.

By the conservation of energy

elastic energy gained = potential energy lost

$$\frac{\lambda x^2}{2l} = mgh$$
$$\frac{19.6x^2}{4} = 0.5 \times 9.8(2+x)$$

In filling from A to C, the ball moves a distance of $(2 + x)$m and so the potential energy lost is, in Joules, $mg\,(2 + x)$.

$$4.9x^2 = 4.9(2+x)$$
$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$
$$x = 2$$
$$AC = 4\text{ m}$$

For there to be elastic energy in a string, the extension must be positive, so you can discard the solution $x = -1$

b At C

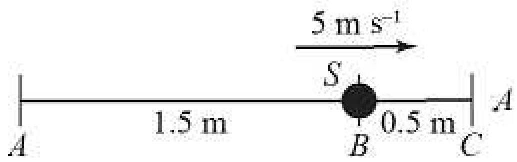
Hooke's law

$$T = \frac{\lambda x}{l} = \frac{19.6 \times 2}{2} = 19.6$$
$$R(\downarrow)\mathbf{F} = m\mathbf{a}$$
$$mg - T = m\mathbf{a}$$
$$0.5 \times 9.8 - 19.6 = 0.5\mathbf{a}$$
$$\mathbf{a} = \frac{0.5 \times 9.8 - 19.6}{0.5} = -29.4$$

The negative acceleration shows you that the acceleration is in the direction of x decreasing, that is towards A.

The instantaneous acceleration of B at C is 29.4 m s^{-2} directed towards A.

38



a Let $AC = 2$ m. When S is at C, the elastic energy stored in the string is given by

$$E = \frac{\lambda x^2}{2l}$$
$$= \frac{20\times(0.5)^2}{2\times 1.5} = \frac{5}{3} \text{ J}$$

Let the speed of S at C be v m s^{-1}

Conservation of energy

Kinetic energy lost = elastic potential energy gained

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = \frac{5}{3}$$
$$\frac{1}{2}\times 0.2\times 5^2 - \frac{1}{2}\times 0.2v^2 = \frac{5}{3}$$
$$0.1v^2 = 0.1\times 25 - \frac{5}{3} = \frac{5}{6}$$
$$v^2 = \frac{25}{3} \Rightarrow V = \frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3} \approx 2.886\ldots$$

The exact answer $\frac{5\sqrt{3}}{3}$ m s^{-1} is also accepted.

The speed of S when $AS = 2$ m is 2.89 m s^{-1} (3 s.f.)

b Let the extension of the string immediately before the string breaks be x m.

To find the tension in the string when the speed of S is 1.5 m s^{-1}, you first need to find the extension of the string at this speed. The extension is found using conservation of energy.

When the extension in the string is x m, the elastic energy stored in the string is given by

$$E = \frac{\lambda x^2}{2l} = \frac{20x^2}{3}$$

Conservation of energy

Kinetic energy lost = elastic energy gained

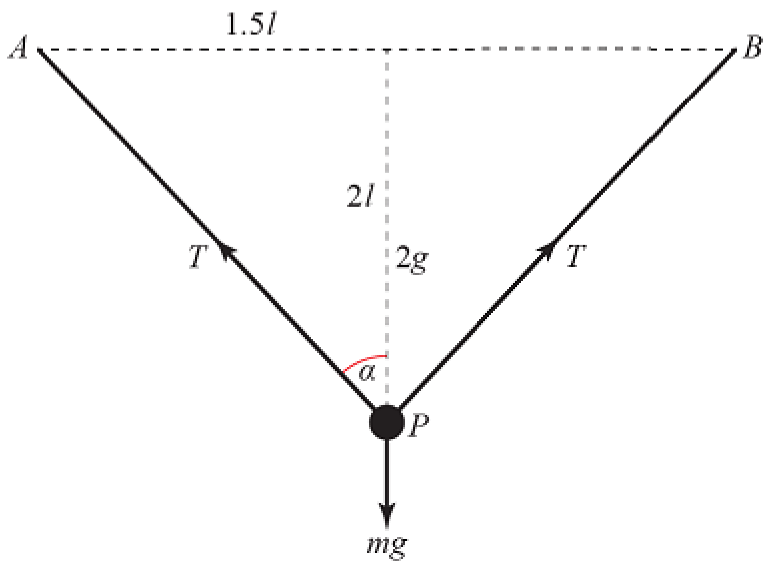
$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = \frac{20x^2}{3}$$
$$\frac{1}{2}\times 0.2\times 5^2 - \frac{1}{2}\times 0.2\times 1.5^2 = \frac{20x^2}{3}$$
$$\frac{20x^2}{3} = 2.275 \Rightarrow x^2 = 0.34125$$
$$x = \sqrt{(0.34125)}$$

Hooke's law

$$T = \frac{\lambda x}{l} = \frac{20\sqrt{(0.34125)}}{1.5} = 7.788\ldots$$

The tension in the string immediately before the string breaks is 7.79 N (3 s.f.)

39 a



$AP^2 = (1.5l)^2 + (2l)^2 = 6.25l^2 \Rightarrow AP = 2.5l$

It is acceptable just to write down $AP = 2.5l$, using the 3, 4, 5 triangle.

Let α be the angle between AP and the vertical.

$\cos\alpha = \frac{2l}{2.5l} = \frac{4}{5}$

The extension of half of the string, AP, is $2.5l - 1.5l = l$

Hooke's law

$$T = \frac{\lambda \times \text{extension}}{\text{natural length}}$$

$$= \frac{\lambda l}{1.5l} = \frac{2\lambda}{3} \qquad (\mathbf{1})$$

You find two equations in T and λ by resolving vertically and using Hooke's law. Eliminating T between the two equations gives λ

$$R(\uparrow)\ 2T\cos\alpha = mg$$

$$2T \times \frac{4}{5} = mg$$

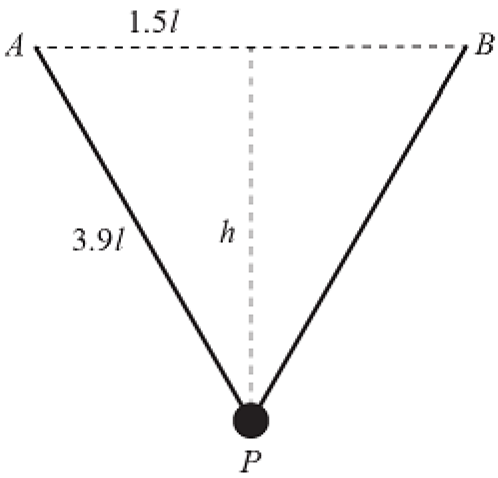
$$T = \frac{5mg}{8} \qquad (\mathbf{2})$$

Eliminating T between (**1**) and (**2**)

$$\frac{2\lambda}{3} = \frac{5mg}{8}$$

$$\lambda = \frac{5mg}{8} \times \frac{3}{2} = \frac{15mg}{16}, \text{ as required.}$$

39 b



Let the perpendicular distance from the original position of P to AB be h.

$h^2 = (3.9l)^2 - (1.5l)^2 = 12.96l^2 \Rightarrow h = 3.6l$

Let the speed of P as it reaches AB be v m s^{-1}

To prove that the speed of P at AB is zero, the speed of P at AB is taken as v m s^{-1}. You then use conservation of energy to obtain an equation for v. You complete the proof by solving the equation for v and showing the solution is zero.

Conservation of energy

kinetic energy gained + potential energy gained = elastic energy lost

$$\frac{1}{2}mv^2 + mgh = 2\times\frac{\lambda x^2}{2\times\text{natural length}}$$

Each of the two halves of the string have natural length $1.5l$ and extension $2.4l$

$$\frac{1}{2}mv^2 + mg\times 3.6l = 2\times\frac{\left(\frac{15mg}{16}\right)(3.9l-1.5l)^2}{2\times 1.5l}$$

$$\frac{1}{2}mv^2 + 3.6mgl = \frac{5mg}{8l}\times(2.4l)^2 = 3.6mgl$$

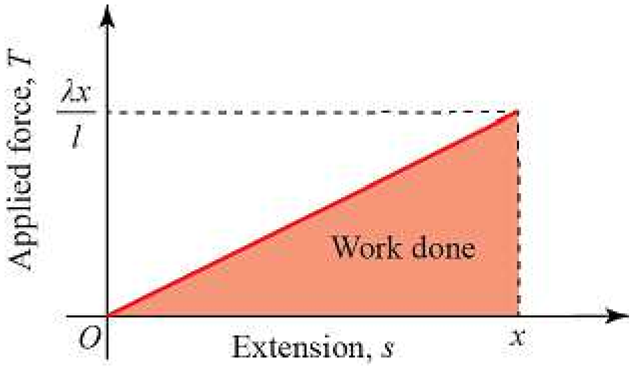
Hence $\frac{1}{2}mv^2 = 0 \Rightarrow v = 0$

P comes to instantaneous rest on the line AB, as required.

40 Due to equivalence of work and energy:
energy stored = work done in stretching the string.
Work done in stretching the string is given by the area under the line (see graph):

$$\text{energy stored} = \frac{1}{2}x\left(\frac{\lambda x}{l}\right)$$

$$\text{energy stored} = \frac{\lambda x^2}{2l}$$



At equilibrium, the tension in the spring, $T = mg$
Using Hooke's law:

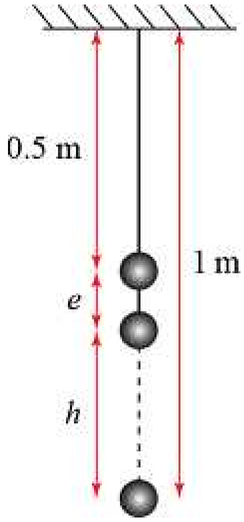
$$T = \frac{\lambda x}{l} = mg$$

$$x = \frac{lmg}{\lambda}$$

$$\text{so energy stored} = \frac{\lambda}{2l}\left(\frac{lmg}{\lambda}\right)^2$$

$$\text{energy stored} = \frac{lm^2g^2}{2\lambda} \quad \text{as required.}$$

41 $l = 0.5$ m, $\lambda = 20$ N, $m = 0.5$ kg



Due to the equivalence of work and energy:
work done in stretching the string
= energy stored when total length is 1.0 m – total energy stored at equilibrium length

When the string is stretched to a total length of 1.0 m, $x = 1.0 - 0.5 = 0.5$ m
and energy stored in the string at this length $= \dfrac{\lambda x^2}{2l}$

When the string is at equilibrium, the tension, $T = mg$
Let the extension at this point be e

Using Hooke's law:

$$T = \frac{\lambda x}{l} = mg$$

$$mg = \frac{\lambda e}{l}$$

$$e = \frac{mgl}{\lambda} = \frac{0.5 \times 0.5 \times 9.8}{20} = 0.1225$$

However, in this position there is also additional gravitational potential energy as particle is further above the ground.
gravitational potential energy $= mgh = mg(x - e)$

So work done in stretching the string from equilibrium to 1.0 m:

work done = final EPE – initial (EPE + PE)

$$\text{work done} = \frac{\lambda x^2}{2l} - \left(\frac{\lambda e^2}{2l} + mg(x - e)\right)$$

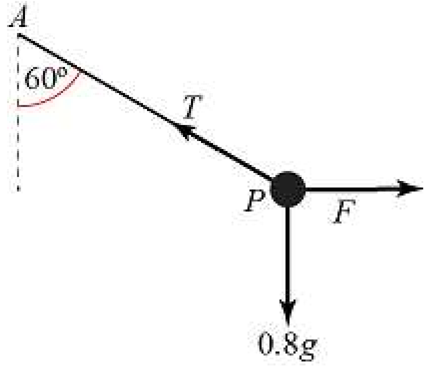
$$\text{work done} = \frac{\lambda}{2l}(x^2 - e^2) - mg(x - e)$$

$$\text{work done} = \frac{20}{2 \times 0.5}(0.5^2 - 0.1225^2) - 0.5 \times 9.8(0.5 - 0.1225)$$

$$\text{work done} = 20(0.25 - 0.01500\ldots) - 4.9(0.5 - 0.1225) = 2.8501\ldots$$

The work done in stretching the string is 2.85 J (3 s.f.)

42 a



$$R(\uparrow)\ T\cos 60^\circ = 0.8g$$

$$\frac{1}{2}T = 0.8g \Rightarrow T = 1.6g$$

Resolving vertically gives you the tension in the string.

$$R(\leftarrow)\ F = T\cos 30^\circ = 1.6g \times \frac{\sqrt{3}}{2}$$

$$= 13.579$$

$$= 14\ (2\text{ s.f.})$$

Substituting for the tension into the equation obtained by resolving horizontally gives the value of *F*.

b Hooke's law $T = \frac{\lambda x}{l}$

$$1.6g = \frac{24x}{1.2}$$

$$x = \frac{1.6g \times 1.2}{24} = 0.784$$

Substituting for the tension into Hooke's Law gives you an equation for the extension.

The extension of the string is 0.78 m (2 s.f.)

c The elastic energy sotred in the string is given by

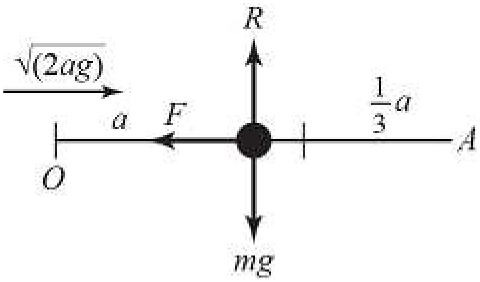
$$E = \frac{\lambda x^2}{2l}$$

You need to remember the formula for the energy stored in an elastic string.

$$= \frac{24 \times (0.784)^2}{2 \times 1.2} = 6.14656$$

The elastic energy stored in the string is 6.1 J (2 s.f.)

43



a At A, the elastic energy stored in the string is given by

$$E = \frac{\lambda x^2}{2l}$$
$$= \frac{3.6\,mg \times (\frac{1}{3}a)^2}{2a}$$
$$= 0.2mga$$

At A, the extension of the string is $\frac{4}{3}a - a = \frac{1}{3}a$

b The total energy lost is

$$\frac{1}{2}mu^2 - 0.2mga = \frac{1}{2} \times 2mga - 0.2mga$$
$$= 0.8mga$$

As P is at rest at A, then net loss of energy is the loss in kinetic energy minus the gain in elastic energy.

At any point in the motion

R(↑) $R = mg$

The friction is given by

$F = \mu R = \mu mg$

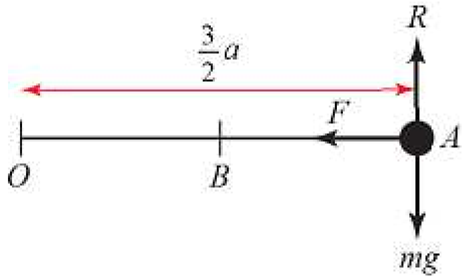
By the work–energy principle

$$0.8mga = \mu mg \times \frac{4}{3}a$$
$$\mu = 0.8 \times \frac{3}{4} = 0.6$$

By the work–energy principle, the net loss in energy is equal to the work done by friction. You find the work done by friction by multiplying the magnitude of the friction, μmg, by the distance the particle moves, $\frac{4}{3}a$

This gives you an equation in μ, which you solve.

44



At any point in the motion

R(↑) $R = mg$

The friction is given by

$$F = \mu R = \frac{2}{3}mg$$

At A, the elastic energy stored in the string is given by

$$E = \frac{\lambda x^2}{2l}$$
$$= \frac{4mg \times (\frac{1}{2}a)^2}{2a}$$
$$= \frac{1}{2}mga$$

At A, the extension of the string is $\frac{3}{2}a - a = \frac{1}{2}a$

By the work–energy principle

$$\frac{1}{2}mga = \frac{2}{3}mg \times AB$$
$$AB = \frac{3}{4}a$$

When P comes to rest, as $OB < a$, the string is slack so all of the elastic energy has been lost. This lost energy must equal the work done by friction, which is the magnitude of the friction, $\frac{2}{3}mg$, multiplied by the distance moved by P, which is AB.

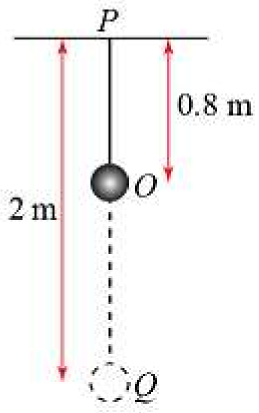
45 $l = 1.0$ m, $\lambda = 75$ N, $m = 5$ kg, $x = 1.5 - 1.0 = 0.5$ m

Energy stored in the spring is transferred to the kinetic energy of the particle.

By the conservation of energy

$$\frac{\lambda x^2}{2l} = \frac{1}{2}mv^2$$
$$\frac{\lambda x^2}{l} = mv^2$$
$$v^2 = \frac{\lambda x^2}{ml}$$
$$v^2 = \frac{75 \times 0.5^2}{5 \times 1} = \frac{15}{4}$$
$$v = \frac{\sqrt{15}}{2} \quad \text{as required.}$$

46 $l = 0.8$ m, $\lambda = 15$ N, $m = 0.5$ kg, $x = 2 - 0.8 = 1.2$ m



Energy stored in string when stretched to total length 2 m

$$E = \frac{\lambda x^2}{2l}$$

$$E = \frac{15 \times 1.2^2}{2 \times 0.8} = 13.5$$

As particle moves upwards, this is converted into gravitational potential energy and kinetic energy.

a When string first becomes slack, the particle is $h = 1.2$ m above initial position.
Initial elastic potential energy = final potential energy + final kinetic energy

$$E = mgh + \frac{1}{2}mv^2$$

$$E = m\left(gh + \frac{1}{2}v^2\right)$$

$$\frac{1}{2}v^2 = \frac{E}{m} - gh$$

$$v = \sqrt{2\left(\frac{E}{m} - gh\right)}$$

$$v = \sqrt{2\left(\frac{13.5}{0.5} - (9.8 \times 1.2)\right)} = 5.5208...$$

When the string first becomes slack, the particle is travelling at 5.5 ms^{-1} (2 s.f.)

b When particle reaches *P*, $h = 2$ m

$$v = \sqrt{2\left(\frac{13.5}{0.5} - (9.8 \times 2)\right)} = 3.8470...$$

When the particle reaches *P*, it is travelling at 3.8 m s^{-1} (2 s.f.)

47 a When P comes to rest for the first time, let the extension of the string be x m

Conservation of energy

elastic energy gained = potential energy lost

$$\frac{\lambda x^2}{2l} = mgh$$
$$\frac{58.8x^2}{8} = 0.5 \times 9.8 \times (4 + x)$$
$$7.5x^2 = 19.6 + 4.9x$$
$$3x^2 - 2x - 8 = 0$$
$$(x - 2)(3x + 4) = 0$$
$$x = 2$$

Divide this equation throughout by 2.45 and rearrange the terms. If you cannot see this simplification, you can use the quadratic formula but you would be expected to obtain an exact answer.

For the string to have elastic energy, it has to be stretched so you can ignore the negative solution $-\frac{4}{3}$

The distance fallen by P is (4 + 2) m = 6 m

b P will first become slack when it has moved 3 m vertically.

Let the velocity at this point be v m s^{-1}

Conservation of energy

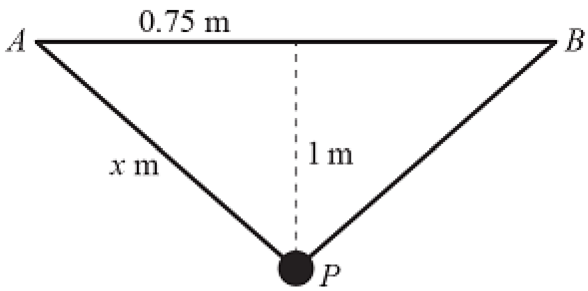
kinetic energy gained + potential energy gained = elastic energy lost

$$\frac{1}{2}mv^2 + mgh = \frac{\lambda x^2}{2l}$$
$$\frac{1}{2}0.5v^2 + 0.5 \times 9.8 \times 3 = \frac{58.8 \times 3^2}{8}$$
$$0.25v^2 = 14.7 = 66.15$$
$$v^2 = \frac{66.15 - 14.7}{0.25} = 205.8$$
$$v = \sqrt{(205.8)} = 14.345\ldots$$

Initially P is at rest and then rise 3 m. So both kinetic and potential energy are gained. Initially the string is stretched but, after rising 3 m, it is slack. So elastic energy is lost. By Conservation of energy, the net gain of kinetic and potential energies must equal the elastic energy lost.

The speed of the particle when the string first becomes slack is 14 m s^{-1} (2 s.f.)

48 a



When P has fallen 1 m, let $AP = x$ m

$x^2 = 0.75^2 + 1^2 = 1.5625 \Rightarrow x = 1.25$

At this point the extension of the string AP is
$(1.25 - 0.75)\text{ m} = 0.5\text{ m}$

To find the elastic energy in the string, you need to calculate the extension in the string. First find AP (you could just use the 3, 4, 5 triangle) and then subtract the natural length, 0.75 m.

Let the velocity of P when it has fallen 1 m be v m s^{-1}

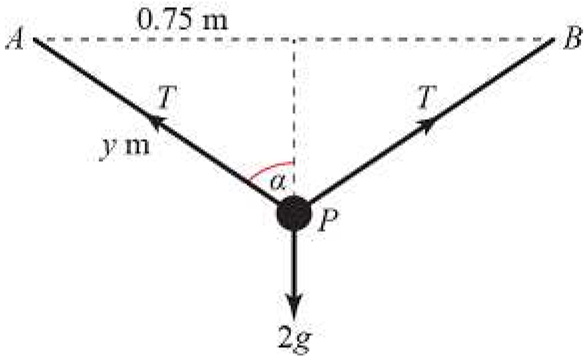
Conservation of energy
kinetic energy gained + elastic energy gained = potential energy lost

$$\frac{1}{2}mv^2 + 2\times\frac{\lambda x^2}{2l} = mgh$$
$$\frac{1}{2}2\times v^2 + 2\times\frac{49\times 0.5^2}{2\times 0.75} = 2\times 9.8\times 1$$
$$v^2 + \frac{49}{3} = 19.6 \Rightarrow v^2 = 3.26\dot{6}$$
$$v = 1.807\ldots$$

Both strings have elastic energy stored in them. By symmetry, the energy in both strings is the same.

The speed of P when it has fallen 1 m is 1.8 m s^{-1} (2 s.f.)

48 b



Let $AP = y\ m$ and the angle AP makes with the vertical be α.
By trigonometry

$$\sin\alpha = \frac{0.75}{y} \Rightarrow y = \frac{0.75}{\sin\alpha}$$

$$R(\uparrow)\ 2T\cos\alpha = 2g$$

$$T = \frac{g}{\cos\alpha} = \frac{9.8}{\cos\alpha}$$

Hooke's law

$$T = \frac{\lambda x}{l}$$

$$= \frac{49}{0.75}(y - 0.75) = \frac{49}{0.75}\left(\frac{0.75}{\sin\alpha} - 0.75\right)$$

$$= 49\left(\frac{1}{\sin\alpha} - 1\right) = 49\left(\frac{1-\sin\alpha}{\sin\alpha}\right)$$

$$\frac{9.8}{\cos\alpha} = 49\left(\frac{1-\sin\alpha}{\sin\alpha}\right)$$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{49}{9.8}(1-\sin\alpha) = 5(1-\sin\alpha)$$

$$\tan\alpha = 5 - 5\sin\alpha$$

$$\tan\alpha + 5\sin\alpha = 5, \text{as required.}$$

You find two separate expression for T, one by resolving vertically and the other from Hooke's law. Equating the two expressions gives you an equation in α.

Eliminating T gives an equation in α. You have to manipulate this equation to obtain the printed answer.

Challenge

1 a The method used by the student is not suitable because the equation for potential energy used assumes that the weight of the object (the gravitational force acting on its mass) remains constant.

In practice, the force the earth exerts on an object far above the surface of the earth is significantly less than that at sea level.

This means that less work is done in raising a mass to a great height than suggested by use of this equation.

Challenge

1 b $m = 420\,000$ kg

initial height = r_i = radius of the earth = 6380 km = 6.38×10^6 m

final height = r_f = orbital radius = 6380 km + 405 km = 6.785×10^6 m

For a variable force, the work done, E, is area under the curve:

$$E = \int_{r_i}^{r_f} F\, dr$$

$$E = \int_{r_i}^{r_f} 3.99\times10^{14}\frac{m}{r^2}\, dr$$

$$E = 3.99\times10^{14}\left[-\frac{m}{r}\right]_{r_i}^{r_f}$$

$$E = 3.99\times10^{14}\left[-\frac{m}{r}\right]_{r_i}^{r_f}$$

$$E = 3.99\times10^{14}\times4.2\times10^{5}\left[-\frac{1}{6.785\times10^{6}}-\left(-\frac{1}{6.38\times10^{6}}\right)\right]$$

$$E = 16.758\times10^{19}\times9.3558...\times10^{-9}$$

$$E = 1.5678...\times10^{12}$$

The work done against gravity in raising the ISS to its orbital height is 1.57×10^{12} J (3 s.f.)

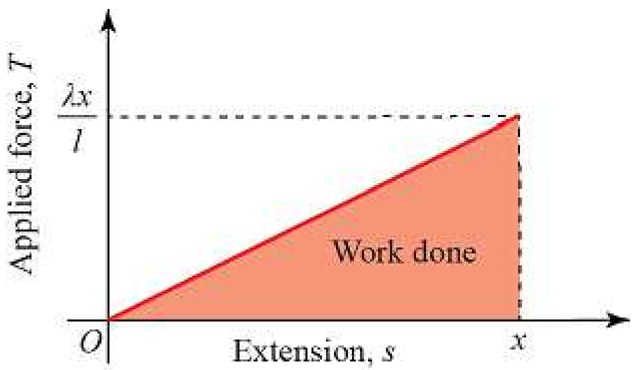
2 a Due to equivalence of work and energy:
energy stored = work done in stretching the string.

Work done in stretching the string is given by the area under the line (see graph):

$$\text{area} = \int_0^x \frac{\lambda s}{l}\, ds$$

$$\text{area} = \left[\frac{\lambda s^2}{2l}\right]_0^x$$

$$\text{area} = \frac{\lambda x^2}{2l}$$



Challenge

2 b Work done = change in elastic potential energy stored by string

$$\text{Work done} = \frac{\lambda}{2l}\left(b^2 - a^2\right)$$

$$\text{Work done} = \frac{\lambda}{2l}(b+a)(b-a)$$

$$\text{Work done} = \frac{1}{2}\left(\frac{\lambda b}{l} + \frac{\lambda a}{l}\right)(b-a)$$

$$\text{Work done} = \tfrac{1}{2}\left(T_b + T_a\right)(b-a)$$

Work done = mean tension × distance moved as required.

3 $l = 0.8$ m, $\lambda = 120$ N, $m = 1.2$ kg, extension of string, $x = ?$

If the particle first comes to rest a distance h below P, $h = l + x$

Initially, energy is all gravitational potential energy.

When particle instantaneously comes to rest, this has all been converted to elastic potential energy.

$$\frac{\lambda x^2}{2l} = mgh$$

$$\frac{\lambda x^2}{2l} = mg(l + x)$$

$$\lambda x^2 = 2lmg(l + x)$$

$$120x^2 = (2 \times 0.8 \times 1.2 \times 9.8)(0.8 + x)$$

$$0 = 120x^2 - 18.816x - 15.0528$$

$$x = \frac{18.816 \pm \sqrt{18.816 + (4 \times 120 \times 15.0528)}}{2 \times 120}$$

$$x = 0.44114\ldots \text{ or } x = -0.28434\ldots$$

Only the positive root is relevant

$h = l + x = 0.8 + 0.44114 \ldots = 1.24114 \ldots$

The particle falls a distance of 1.24 m (3 s.f.) before coming instantaneously to rest.