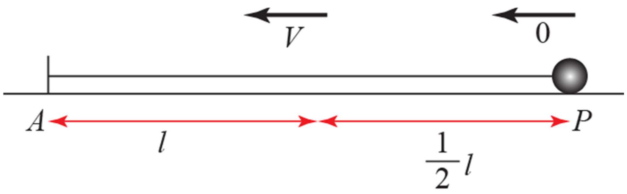
Elastic strings and springs 3D

1



Conservation of energy

K.E. gain = E.P.E. loss

$$\frac{1}{2}mV^2 = \frac{mg\left(\frac{1}{2}l\right)^2}{2l}$$

$$V^2 = \frac{1}{4}gl$$

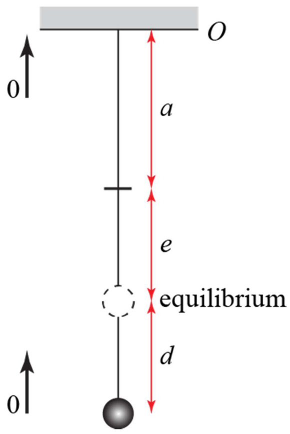
$$V = \frac{1}{2}\sqrt{gl}$$

2 At equilibrium, $T = mg$

$$\frac{4mgx}{a} = mg \Rightarrow x = \frac{1}{4}a$$

When the particle reaches O it has risen by $\left(a + \frac{1}{4}a + d\right)$

Conservation of energy



P.E. gain = E.P.E. loss

$$mg\left(a + \frac{1}{4}a + d\right) = \frac{4mg\left(\frac{1}{4}a + d\right)^2}{2a}$$

$$\frac{5a^2}{4} + ad = 2\left(\frac{a^2}{16} + \frac{ad}{2} + d^2\right)$$

$$\frac{5a^2}{4} = \frac{a^2}{8} + 2d^2$$

$$\frac{9a^2}{16} = d^2$$

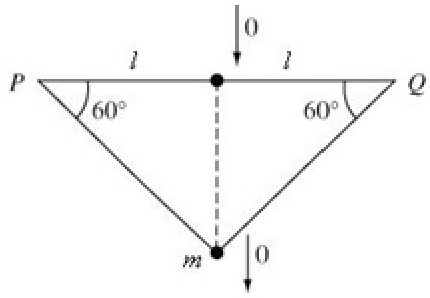
$$\frac{3a}{4} = d$$

(ignore solution $d = -\frac{3a}{4}$)

The distance d is $\frac{3a}{4}$.

3 a Conservation of energy

P.E. loss = E.P.E. gain



$$mgl\tan 60° = \frac{2\times\lambda\left(\frac{l}{\cos 60°}-l\right)^2}{2l}$$

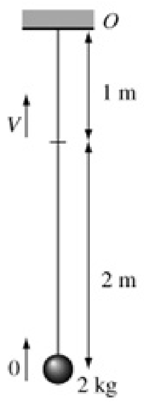
$$mgl\sqrt{3} = \lambda l$$

$$mg\sqrt{3} = \lambda$$

The modulus of elasticity of the spring is $mg\sqrt{3}$.

b Take into account the mass of the spring.

4 a



Conservation of energy
K.E. gain + P.E. gain = E.P.E. loss

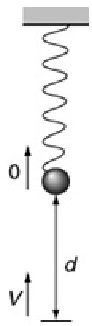
$$\frac{1}{2}\times 2\times V^2 + 2g\times 2 = \frac{21.6\times 2^2}{2\times 1}$$

$$V^2 = 43.2 - 39.2$$

$$= 4$$

$$V = 2\text{ m s}^{-1}$$

b



Conservation of energy
K.E. loss = P.E. gain

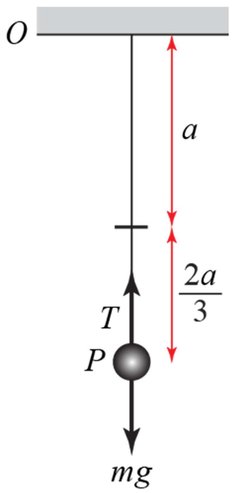
$$\frac{1}{2}\times mV^2 = mgd$$

$$2 = gd$$

$$\frac{2}{g} = d = 0.20\ldots$$

Distance from O is $(1 - d) = 0.80$ m (2 s.f.).

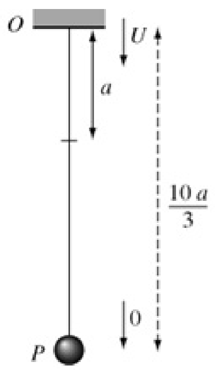
5



$(\uparrow)\quad T = mg$

$$\frac{\lambda}{a} \times \frac{2a}{3} = mg$$

$$\lambda = \frac{3mg}{2}$$



K.E. loss + P.E. loss = E.P.E. gain

$$\frac{1}{2}mU^2 + mg\frac{10a}{3} = \frac{3mg}{2} \times \frac{\left(\frac{7a}{3}\right)^2}{2a}$$

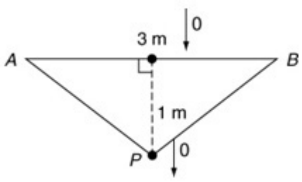
$$\frac{U^2}{2} + \frac{10ag}{3} = \frac{3g}{4a} \times \frac{49a^2}{9}$$

$$\frac{U^2}{2} = \frac{49ag}{12} - \frac{10ag}{3}$$

$$U^2 = \frac{9ag \times 2}{12}$$

$$U = \sqrt{\frac{3ag}{2}}$$

6



$$AP = \sqrt{1.5^2 + 1^2} = \sqrt{\frac{13}{4}}$$

$$= \frac{\sqrt{13}}{2}$$

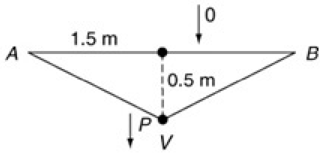
a P.E.loss = E.P.E.gain

$$g \times 1 = \frac{2\lambda\left(\frac{\sqrt{13}}{2} - \frac{3}{2}\right)^2}{2 \times 1.5}$$

$$\lambda = \frac{2 \times 3g}{\left(\sqrt{13} - 3\right)^2} = 160.35\ldots$$

The value of λ is 160 N (2 s.f.).

6 b



$$AP = \sqrt{1.5^2 + 0.5^2}$$
$$= \frac{\sqrt{10}}{2}$$

K.E. gain + E.P.E. gain = P.E. loss

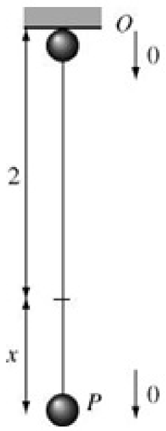
$$\frac{1}{2}V^2 + \frac{2\lambda\left(\frac{\sqrt{10}}{2} - \frac{3}{2}\right)^2}{2\times 1.5} = 0.5g$$

$$V^2 = g - \frac{\left(\sqrt{10}-3\right)^2}{3}\times\lambda$$

$$V = 2.896\ldots$$

When P is 0.5 m below the initial position its speed is 2.9 m s^{-1} (2 s.f.).

7 a



P.E. loss = E.P.E. gain

$$3g(2+x) = \frac{117.6}{4}x^2$$

$$\frac{4\times 3g}{117.6}(2+x) = x^2$$

$$0 = x^2 - x - 2$$

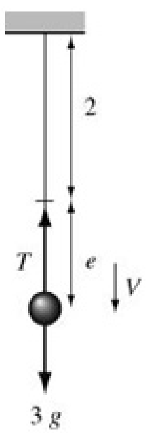
$$0 = (x-2)(x+1)$$

$$x = 2 \quad \text{or} \quad x - 1$$

Ignore negative root.

The distance fallen is 4 m.

b Greatest speed at equilibrium position



$$(\uparrow)\ T = 3g$$

$$\frac{117.6\times e}{2} = 3g$$

$$e = 0.5$$

E.P.E. gain + K.E. gain = P.E. loss

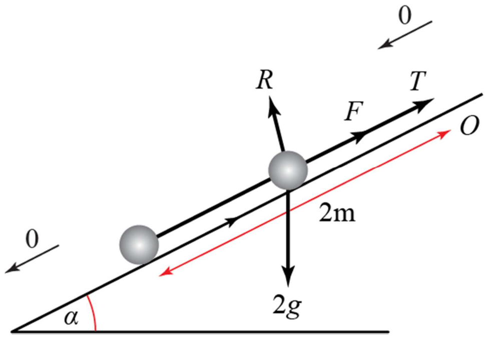
$$\frac{117.6(0.5)^2}{2\times 2} + \frac{1}{2}\times 3V^2 = 3g\times 2.5$$

$$7.35 + 1.5V^2 = 73.5$$

$$V = 6.640\ldots$$

The greatest speed is 6.6 m s^{-1} (2 s.f.).

8



$\tan\alpha = \frac{3}{4}$ so $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$.

$(\nwarrow)\ R = 2g\cos\alpha = \frac{8g}{5}$

$F = \mu R = \mu\frac{8g}{5}$

Work done against friction = P.E. loss − E.P.E. gain

$$\mu\frac{8g}{5}\times 2 = 2g\times 2\sin\alpha - \frac{40\times 1^2}{2\times 1}$$

$$\mu\frac{16g}{5} = \frac{12g}{5} - 20$$

$$\mu = \frac{12g - 100}{16g}$$

$$= 0.112\ldots$$

The coefficient of friction is 0.11 (2 s.f.).

Challenge

The extension of the string with one mass attached is $\frac{l}{10}$m.

By Hooke's law, $Mg = k\frac{l}{10}$

$$\Rightarrow k = 10\frac{Mg}{l}$$

Let x be the extension of the string with two masses attached.
Hooke's Law $\Rightarrow 2Mg = kx$

Substituting $k = 10\frac{Mg}{l}$ from above, we see that

$$2Mg = 10\frac{Mg}{l}x$$

$$x = \frac{l}{5}$$

The work done in producing the additional extension is given by:

$$\Delta\text{EPE} = \frac{1}{2}k\left(\frac{l}{5}\right)^2 - \frac{1}{2}k\left(\frac{l}{10}\right)^2$$

$$= \frac{1}{2}kl^2\left(\frac{1}{25} - \frac{1}{100}\right)$$

$$= \frac{1}{2}\left(10\frac{Mg}{l}\right)l^2\left(\frac{3}{100}\right) \qquad \left[\text{using } k = 10\frac{Mg}{l}\right]$$

$$= \frac{3}{20}Mgl \text{ J}$$