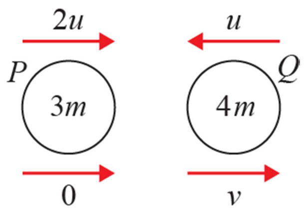
Momentum and impulse Mixed exercise 1

1 a



Using conservation of momentum: $(\rightarrow)$

$6mu - 4mu = 4mv$

$\frac{1}{2}u = v$

After the collision the direction of Q is reversed and its speed is $\frac{1}{2}u$

b Impulse = change in momentum

$I = (3m \times 2u) - 0$

$= 6mu$

The magnitude of the impulse exerted by Q on P is $6mu$

2 a $v^2 = u^2 + 2as$

$v^2 = 2 \times 10 \times 9.8$

$v = 14\,\text{m s}^{-1}$

The speed of the driver immediately before it hits the pile is 14 m s^{-1}

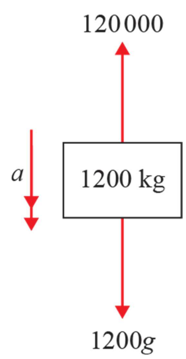
b Using conservation of momentum: $(\downarrow)$

$1000 \times 14 = 1200v$

$v = \frac{35}{3}$

The common speed of the pile and pile driver is $\frac{35}{3}$ m s^{-1}

c First, use $F = ma$ to find the deceleration.



$1200g - 120\,000 = 1200a$

$a = -90.2$

$v^2 = u^2 + 2as \; (\downarrow)$

$0^2 = \left(\frac{35}{3}\right)^2 - 2 \times 90.2 \times s$

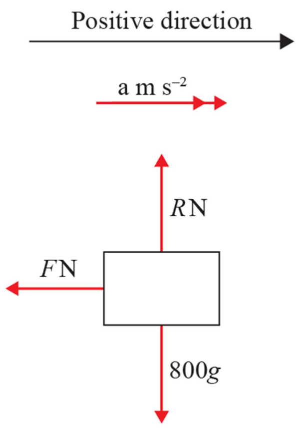
$s = 0.75\,\text{m}$ (2 s.f.)

Use the common speed found in part **b** for u.

2 s.f. as $g = 9.8$ has been used.

2 d The model assumes that the pile driver would not ‘bounce’ upon contact with the pile, i.e. the particles coalesce. Given that the pile driver is much heavier than the pile, this would be a fair assumption.

3



a $u = 18$, $v = 12$, $t = 2.4$, $a = ?$

$$v = u + at$$
$$12 = 18 + 2.4a$$
$$a = \frac{12 - 18}{2.4} = -2.5$$

You are going to have to use $F = ma$ to find F. So the first step of your solution must be to find a.

$$F = ma$$
$$-F = 800 \times (-2.5) = -2000 \Rightarrow F = 2000$$

The retarding force is slowing the car down and is in the negative direction. So, in the positive direction, the force is $-F$.

The value of F is 2000.

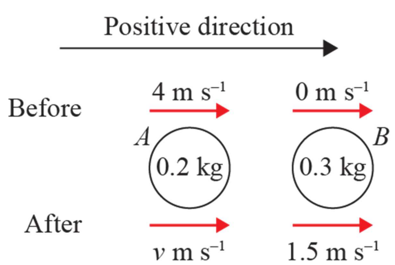
b $u = 18, v = 12, t = 2.4, s = ?$

$$s = \left(\frac{u + v}{2}\right)t$$
$$= \left(\frac{18 + 12}{2}\right) \times 2.4 = 15 \times 2.4 = 36$$

The distance moved by the car is 36 m

You could use the value of a you found in part **a** and another formula. Unless it causes you extra work, it is safer to use the data in the question.

4



a Conservation of momentum

$$0.2 \times 4 = (0.2 \times v) + (0.3 \times 1.5)$$
$$0.8 = 0.2v + 0.45$$
$$v = \frac{0.8 - 0.45}{0.2} = 1.75$$

The speed of A after the impact is 1.75 m s^{-1}

A full formula for the conservation of momentum is $m_A u_A + m_B u_B = m_A v_A + m_B v_B$. In this case the velocity of B is zero.

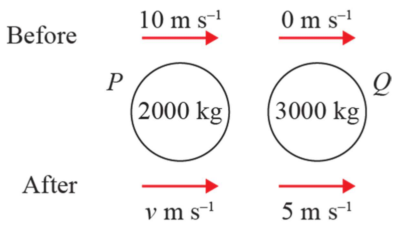
4 b Consider the impulse of B on A

$$I = mv - mu$$
$$= (0.2 \times 1.75) - (0.2 \times 4)$$
$$= 0.35 - 0.8 = -0.45$$

The magnitude of the impulse of B on A during the impact is 0.45 N s

It is a common mistake to mix up the particles. The impulses on the two particles are equal and opposite. Finding the magnitude of the impulse, you can consider either particle – either would give the same magnitude. However, you must work on only one single particle. Here you can work on A or B, but not both.

5



a Conservation of linear momentum

$$2000 \times 10 = (2000 \times v) + (3000 \times 5)$$
$$20\,000 = 2000v + 15\,000$$
$$v = \frac{20\,000 - 15\,000}{2000} = 2.5$$

The speed of P immediately after the collision is 2.5 m s^{-1}

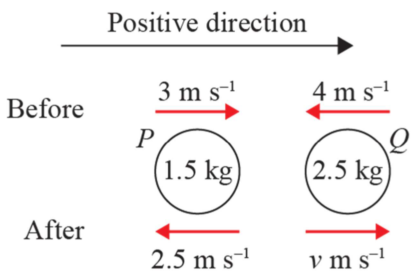
b For Q,

$$I = mv - mu$$
$$I = (3000 \times 5) - (3000 \times 0) = 15\,000$$

The magnitude of the impulse of P on Q is 15 000 N s

To find the magnitude of the impulse you could consider **either** the change in momentum of P **or** the change of momentum of Q. You must not mix them up.

6



You do not know which direction Q will be moving in after the impact. Mark the unknown velocity as v m s^{-1} in the positive direction. After you have worked out v, the sign of v will tell you the direction Q is moving in.

a Conservation of momentum

$$(1.5 \times 3) + (2.5 \times (-4)) = (1.5 \times (-2.5)) + (2.5 \times v)$$
$$4.5 - 10 = -3.75 + 2.5v$$
$$2.5v = 4.5 - 10 + 3.75 = -1.75$$
$$v = -\frac{1.75}{2.5} = -0.7$$

The speed of Q immediately after the impact is 0.7 m s^{-1}

The sign of v is negative, so Q is moving in the negative direction. It was moving in the negative direction before the impact and so its direction has not changed.

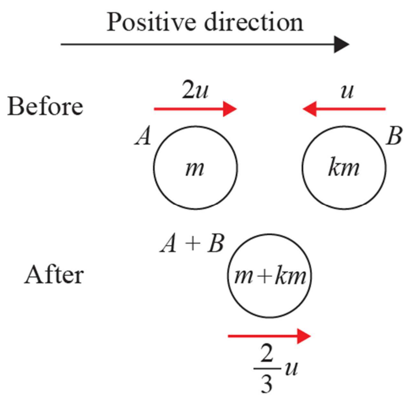
6 b The direction of Q is unchanged.

c For P, $I = mv - mu$

$$I = (1.5\times(-2.5)) - (1.5\times 3) = -8.25$$

The magnitude of the impulse exerted by Q on P is 8.25 N s

7



After the collision A (of mass m) and B (of mass km) combine to form a single particle. That particle will have the mass which is the sum of the two individual masses, $m + km$.

Conservation of momentum: $(\rightarrow)$

The total linear momentum before impact must equal the total linear momentum after impact. Particle B is moving in the negative direction before the collision and so it has a negative linear momentum.

$$(m\times 2u) + (km\times(-u)) = (m + km)\times\frac{2}{3}u$$

$$2mu - kmu = \frac{2}{3}mu + \frac{2}{3}kmu$$

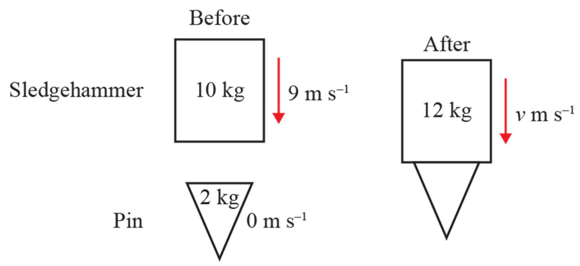
$$2mu - \frac{2}{3}mu = kmu + \frac{2}{3}kmu$$

$$\frac{4}{3}mu = \frac{5}{3}kmu$$

$$k = \frac{4}{3}\times\frac{3}{5} = \frac{4}{5}$$

m and u are common factors on both sides of the equation and can be cancelled.

8 a



After impact, the sledgehammer and the metal pin move together. You model the sledgehammer and pin as a single particle of mass 12 kg.

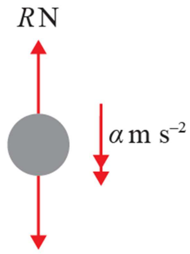
Conservation of momentum: $(\downarrow)$

$$(10\times 9)+(2\times 0)=12\times v$$

$$v=\frac{90}{12}=7.5$$

The speed of the pin immediately after impact is 7.5 m s^{-1}

b



The model given in the question assumes that the pin and sledgehammer stay in contact and move together after impact, before coming to rest. Although the question only refers to the pin, you must consider the pin and the sledgehammer as moving together, with the same velocity and the same acceleration, throughout the motion after the impact.

$u=7.5,\ v=0,\ s=0.03,\ a=?$

$$v^2=u^2+2as$$

$$0^2=7.5^2+(2\times a\times 0.03)$$

$$a=-\frac{7.5^2}{0.06}=-937.5$$

Using $F=ma$:

$$12g-R=12\times(-937.5)$$

$$R=(12\times 9.8)+(12\times 937.5)$$

$$=11\,367.6$$

The value of R is 11 000 (2 s.f.)

c The resistance (R) could be modelled as varying with speed.

9 Impulse = change in momentum

$$\begin{aligned}&= 0.5(23\mathbf{i} + 20\mathbf{j}) - 0.5(-25\mathbf{i})\\&= (24\mathbf{i} + 10\mathbf{j}) \text{ N s}\end{aligned}$$

$$\begin{aligned}\therefore \text{ Magnitude of the impulse} &= \sqrt{24^2 + 10^2} \text{ N s}\\&= 26 \text{ N s}\end{aligned}$$

Angle between the impulse and the direction **i** is α where

$$\tan\alpha = \frac{10}{24}$$

$$\therefore \quad \alpha = 23° \text{(nearest degree)}$$

10 Let velocity before being hit be **u** m s^{-1}

impulse = change in momentum

$$\begin{aligned}2.4\mathbf{i} + 3.6\mathbf{j} &= 0.2(12\mathbf{i} + 5\mathbf{j}) - 0.2\boldsymbol{u}\\0.2\mathbf{u} &= 2.4\mathbf{i} + \mathbf{j} - 2.4\mathbf{i} - 3.6\mathbf{j}\\&= -2.6\mathbf{j}\\\therefore \quad \mathbf{u} &= -13\mathbf{j}\end{aligned}$$

The velocity of the ball immediately before it is hit is –13**j** m s^{-1}

11 Let the velocity of Q after the collision be **v** m s^{-1}
Use conservation of momentum:

$$\begin{aligned}4(2\mathbf{i} + 16\mathbf{j}) + 3(-\mathbf{i} - 8\mathbf{j}) &= 4(-4\mathbf{i} - 32\mathbf{j}) + 3\mathbf{v}\\5\mathbf{i} + 40\mathbf{j} &= -16\mathbf{i} - 128\mathbf{j} + 3\mathbf{v}\\3\mathbf{v} &= 21\mathbf{i} + 168\mathbf{j}\\\mathbf{v} &= 7\mathbf{i} + 56\mathbf{j}\end{aligned}$$

The velocity of Q immediately after the collision is (7**i** + 56**j**) m s^{-1}

12 a

$$\mathbf{r} = (t^3 + t^2 + 4t)\mathbf{i} + (11t)\mathbf{j}$$

$$\therefore \mathbf{v} = \dot{\mathbf{r}} = (3t^2 + 2t + 4)\mathbf{i} + 11\mathbf{j}$$

Differentiate displacement vector to give the velocity vector.

When $t = 4, \mathbf{v} = (60)\mathbf{i} + 11\mathbf{j}$

$$\begin{aligned}|\mathbf{v}| &= \sqrt{60^2 + 11^2}\\&= 61\end{aligned}$$

$\therefore$ The speed of P when $t = 4$ is 61 m s^{-1}

12 b Let the velocity immediately after the impulse be **V** m s^{-1}

Then as impulse = change in momentum

$$
\begin{aligned}
2.4\mathbf{i}+3.6\mathbf{j} &= 0.3\mathbf{V}-0.3(60\mathbf{i}+11\mathbf{j}) \\
0.3\mathbf{V} &= 2.4\mathbf{i}+3.6\mathbf{j}+18\mathbf{i}+3.3\mathbf{j} \\
&= 20.4\mathbf{i}+6.9\mathbf{j} \\
\mathbf{V} &= 68\mathbf{i}+23\mathbf{j}
\end{aligned}
$$

Use impulse = change in momentum.

$\therefore$ velocity of P immediately after the impulse is (68**i** + 23**j**) m s^{-1}

Challenge

Using equations for impulse

1 Q changes direction after impact:

$km(v+u) = m(u-v)$ so $k = \dfrac{u-v}{u+v}$

2 P changes direction after impact

$km(u-v) = m(v+u)$ so $k = \dfrac{u+v}{u-v}$

a k must be positive so $u > v$

b If $k = \dfrac{u-v}{u+v}$ then Q changes direction after impact.

If $k = \dfrac{u+v}{u-v}$ then P changes direction after impact.