Review Exercise 2

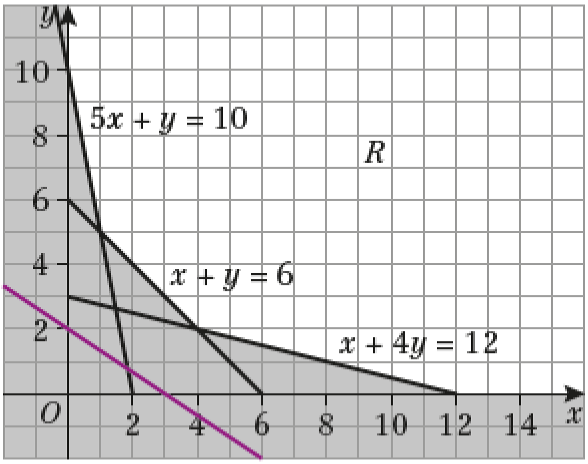
1 a Chemical A $5x + y \geqslant 10$

Chemical B $2x + 2y \geqslant 12$ $\quad [x + y \geqslant 6]$

Chemical C $\frac{1}{2}x + 2y \geqslant 6$ $\quad [x + 4y \geqslant 12]$

$x,\ y \geqslant 0$

b



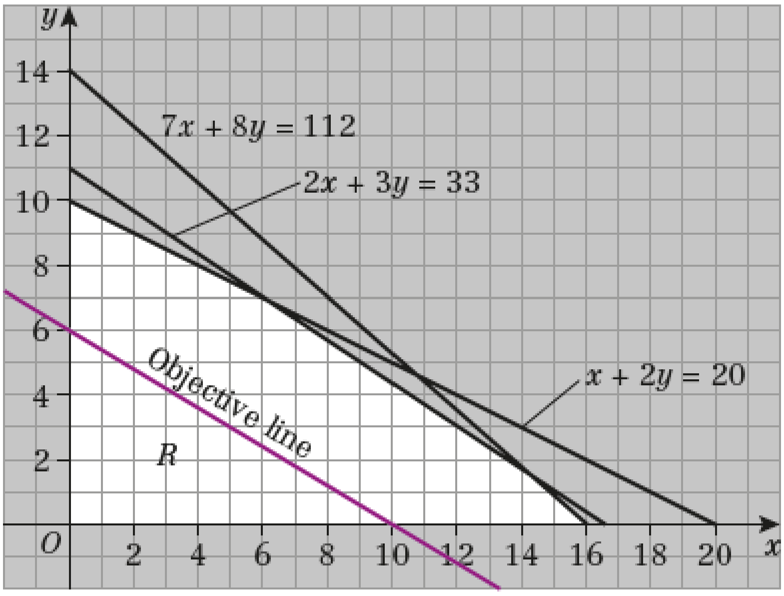
c $T = 2x + 3y$

d $(x, y) = (4,\ 2)T = 14$

2 **a** Maximise $P = 300x + 500y$

b Finishing $3.5x + 4y \leqslant 56 \Rightarrow 7x + 8y \leqslant 112$ (o.e.)
Packing $2x + 4y \leqslant 40 \Rightarrow x + 2y \leqslant 20$ (o.e.)

c



d For example, *point testing*

- Test all corner points in feasible region.
- Find profit at each and select point yielding maximum.

profit line

- Draw profit lines.
- Select point on profit line furthest from the origin.

e Using a correct, complete method.
Making 6 Oxford and 7 York gives a profit = £5300

$(6, 7) \rightarrow 5300$ $(14.4, 1.4) \xrightarrow{\text{integer}} (14, 1) \rightarrow 4700$ $(16, 0) \rightarrow 4800$

$(0, 10) \rightarrow 5000$

f The line $3.5x + 4y = 49$ passes through (6, 7) so reduce *finishing* by 7 hours.

3 **a** Objective: maximise $P = 30x + 40y$ (or $P = 0.3x + 0.4y$)
subject to:

$x + y \geqslant 200$

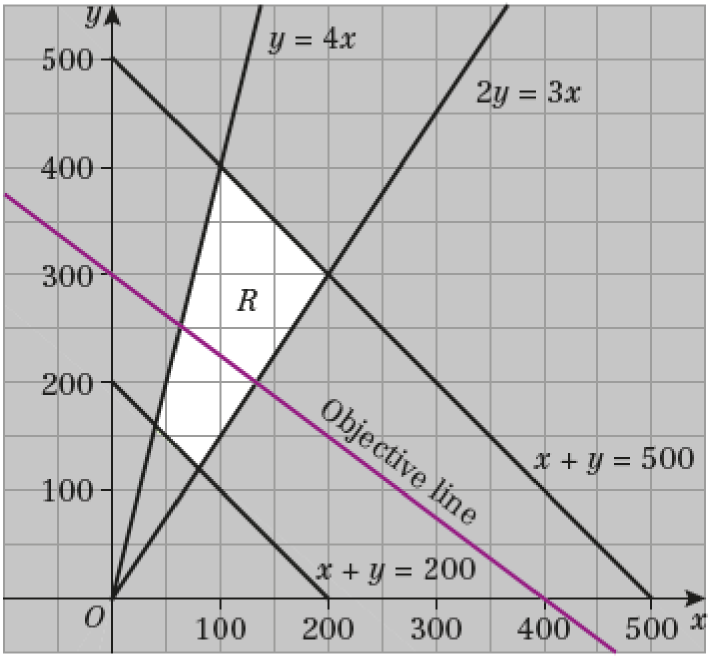
$x + y \leqslant 500$

$x \geqslant \frac{20}{100}(x+y) \Rightarrow 4x \geqslant y$

$x \leqslant \frac{40}{100}(x+y) \Rightarrow 3x \leqslant 2y$

$x,\ y \geqslant 0$

b



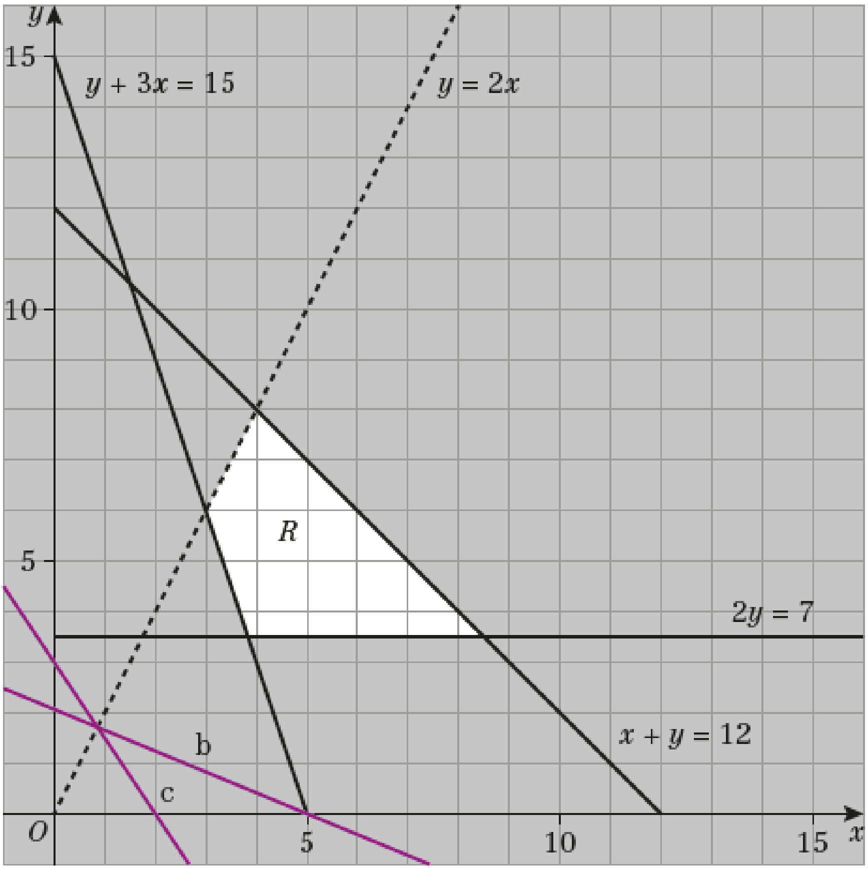
c Visible use of objective line method – objective line drawn or vertex testing – all 4 vertices tested

Vertex testing

(40, 160) → 7600
(80, 120) → 7200
(100, 400) → 19 000
(200, 300) → 18 000

Intersection of $y = 4x$ and $x + y = 500$
(100, 400) profit £190 (or 19 000 p)

4 a



b Visible use of objective line method – objective line drawn or vertex testing.

$\left[\left(3\frac{5}{6}, 3\frac{1}{2}\right) \to 25\frac{1}{6}\left(8\frac{1}{2}, 3\frac{1}{2}\right) \to 34\frac{1}{2}(4, 8) \to 48(3, 6) \to 36\right]$

Optimal point $\left(3\frac{5}{6}, 3\frac{1}{2}\right)$ with value $25\frac{1}{6}$

c Visible use of objective line method – objective line drawn, or vertex testing – all 4 vertices tested.

$\left(3\frac{5}{6}, 3\frac{1}{2}\right)$ not an integer try $(4, 4) \to 20 \quad (4, 8) \to 28$

$\left(8\frac{1}{2}, 3\frac{1}{2}\right)$ not an integer try $(8, 4) \to 32 \quad (3, 6) \to 21$

Optimal point (8, 4) with value £32, so Becky should use 4 kg of bird feeder and 3.5 kg of bird table food.

5 a Objective: maximise $P = 0.4x + 0.2y \quad (P = 40x + 20y)$
subject to:

$$x \leqslant 6.5$$
$$y \leqslant 8$$
$$x + y \leqslant 12$$
$$y \leqslant 4x$$
$$x,\ y \geqslant 0$$

b Visible use of objective line method – objective line drawn (e.g. from (2, 0) to (0, 4)) or all 5 points tested.
vertex testing
$[(0,\ 0) \rightarrow 0; (2,\ 8) \rightarrow 2.4; (4,\ 8) \rightarrow 3.2; (6.5,\ 5.5) \rightarrow 3.7; (6.5,\ 0) \rightarrow 2.6]$
Optimal point is $(6.5,\ 5.5) \Rightarrow 6\,500$ type X and 5500 type Y

c $P = 0.4(6500) + 0.2(5500) = £3700$

6 a Maximise $P = 50x + 80y + 60z$

Subject to $x + y + 2z \leqslant 30$

$x + 2y + z \leqslant 40$

$3x + 2y + z \leqslant 50$

where $x,\ y,\ z \geqslant 0$

b Initialising tableau

b.v.	x	y	z	r	s	t	value
r	1	1	2	1	0	0	30
s	1	(2)	1	0	1	0	40
t	3	2	1	0	0	1	50
P	−50	−80	−60	0	0	0	0

Choose correct pivot, divide R2 by 2

State correct row operation R1 – R2, R3 – 2R2, R4 + 80R2, R2 ÷ 2

c The solution found after one iteration has a slack of 10 units of black per day

d i

b.v.	x	y	z	r	s	t	value
r	$\frac{1}{2}$	0	$\left(\frac{3}{2}\right)$	1	$\frac{-1}{2}$	0	10
y	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	20
t	2	0	0	0	−1	1	10
P	−10	0	−20	0	40	0	1600

(given)

b.v.	x	y	z	r	s	t	value	
z	$\frac{1}{3}$	0	1	$\frac{2}{3}$	$\frac{-1}{3}$	0	$6\frac{2}{3}$	R1 ÷ $\frac{3}{2}$
y	$\frac{1}{3}$	1	0	$\frac{-1}{3}$	$\frac{2}{3}$	0	$16\frac{2}{3}$	R2 $-\frac{1}{2}$ R1
t	2	0	0	0	−1	1	10	R3 – no change
P	$-3\frac{1}{3}$	0	0	$13\frac{1}{3}$	$33\frac{1}{3}$	0	$1733\frac{1}{3}$	R4 + 20R1

ii Not optimal, as there is a negative value in profit row

iii $x = 0 \quad y = 16\frac{2}{3} \quad z = 6\frac{2}{3}$

$P = £1733.33 \quad r = 0,\ s = 0,\ t = 10$

7 a Objective: Maximise $P = 4x + 5y + 3z$

Subject to $3x + 2y + 4z \leqslant 35$

$x + 3y + 2z \leqslant 20$

$2x + 4y + 3z \leqslant 24$

b

b.v.	x	y	z	r	s	t	value	
r	2	0	$\frac{5}{4}$	1	0	$-\frac{1}{2}$	23	R1 – 2R3
s	$-\frac{1}{2}$	0	$-\frac{1}{4}$	0	1	$-\frac{3}{4}$	2	R2 – 3R3
y	$\frac{1}{2}$	1	$\frac{3}{4}$	0	0	$\frac{1}{4}$	6	R3 ÷ 4
P	$-\frac{3}{2}$	0	$\frac{3}{4}$	0	0	$\frac{5}{4}$	30	R4 + 5R3

b.v.	x	y	z	r	s	t	value	
x	1	0	$\frac{5}{4}$	$\frac{1}{2}$	0	$-\frac{1}{4}$	$\frac{23}{2}$	R1 ÷ 2
s	0	0	$\frac{3}{8}$	$\frac{1}{4}$	1	$-\frac{7}{8}$	$\frac{31}{4}$	R2 + $\frac{1}{2}$R1
y	0	1	$\frac{1}{8}$	$-\frac{1}{4}$	0	$\frac{3}{8}$	$\frac{1}{4}$	R3 – $\frac{1}{2}$R1
P	0	0	$\frac{21}{8}$	$\frac{3}{4}$	0	$\frac{7}{8}$	$\frac{189}{4}$	R4 + $\frac{3}{2}$ R1

$P = 47\frac{1}{4} \quad x = 11\frac{1}{2},\ y = \frac{1}{4},\ z = 0$

c There is some slack $\left(7\frac{3}{4}\right)$ on s, so *do not* increase blending: therefore increase Processing and Packing which are both at their limit at present.

8 a $x+2y+4z\leqslant 24$

b i $x+2y+4z+s=24$

ii $s(\geqslant 0)$ is the slack time on the machine in hours

c 1 euro

d

b.v.	x	y	z	r	s	value	
r	$\frac{3}{2}$	2	0	1	$-\frac{3}{2}$	14	R1 – 6R2
z	$\frac{1}{4}$	$\frac{1}{2}$	1	0	$\frac{1}{4}$	6	R2 ÷ 4
P	0	−1	0	0	1	24	R3 + 4R2

b.v.	x	y	z	t	s	value	
y	$\frac{3}{4}$	1	0	$\frac{1}{2}$	$\frac{-3}{4}$	7	R1 ÷ 2
z	$\frac{-1}{8}$	0	1	$\frac{-1}{4}$	$\frac{5}{8}$	$\frac{5}{2}$	R2 $-\frac{1}{2}$R1
P	$\frac{3}{4}$	0	0	$\frac{1}{2}$	$\frac{1}{4}$	31	R3 + R1

Profit = 31 euros $\quad y=7\ z=2.5\ x=r=s=0$

e Cannot make $\frac{1}{2}$ a lamp

f e.g. (0, 10, 0) or (0, 6, 3) or (1, 7, 2) checks in **both** inequalities

9 a

	Board (m)	Time (R)
Small (x)	$2\frac{1}{2}$	10
Medium (y)	10	20
Large (z)	15	50
Available	300	1000

Board $2\frac{1}{2}x + 10y + 15z \leqslant 300$

$x + 4y + 6z \leqslant 120$

Time $10x + 20y + 50z \leqslant 1000$

$x + 2y + 5z \leqslant 100$

b $P = 10x + 20y + 28z$

c

b.v.	x	y	z	r	s	values
r	1	4	6	1	0	120
s	1	2	5	0	1	100
P	−10	−20	−28	0	0	0

d $\theta_1 = 30,\ \theta_2 = 50$; pivot 4

b.v.	x	y	z	r	s	value	Row operation
y	$\frac{1}{4}$	1	$1\frac{1}{2}$	$\frac{1}{4}$	0	30	R1 ÷ 4
s	$\frac{1}{2}$	0	2	$\frac{-1}{2}$	1	40	R2 – 2R1
P	−5	0	2	5	0	600	R3 + 20R1

$\theta_1 = 120,\ \theta_2 = 80$; pivot $\frac{1}{2}$

b.v.	x	y	z	r	s	value	Row operation
y	0	1	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	10	R1 $-\frac{1}{2}$ R2
x	1	0	4	−1	2	80	2R2
P	0	0	22	0	10	1000	R3 + 5R2

e This tableau is optimal as there are no negative numbers in the profit line.

f Small 80, medium 10; large 0

Profit £1000

10 a The constraints include a mixture of $\leqslant$ and $\geqslant$ variables.

b $x + y + 2z + s_1 = 10$
$x + 3y + z + s_2 = 15$
$2x + y + z - s_3 + a_1 = 12$

c The purpose is to maximise
$I = -a_1$ which $= 2x + y + z - s_3 - 12$
So $I - 2x - y - z + s_3 = -12$
This gives the final row of the tableau.

d The smallest value in the bottom row is −2 and the smallest θ value is $a_1(6)$ so this is the pivot and we obtain:

b.v.	x	y	z	s_1	s_2	s_3	a_1	value
s_1	0	0.5	1.5	1	0	0.5	−0.5	4
s_2	0	2.5	0.5	0	1	0.5	−0.5	9
x	1	0.5	0.5	0	0	−0.5	0.5	6
P	0	−1.5	−2.5	0	0	−0.5	0.5	6
I	0	0	0	0	0	0	1	0

e There are no negative values in the bottom row, so the optimal value of I is 0 when $a_1 = 0$

f There is a negative value in the bottom row.

g

b.v.	x	y	z	s_1	s_2	s_3	Value	Row operation
z	0	0	1	$\frac{5}{7}$	$-\frac{1}{7}$	$\frac{2}{7}$	$\frac{11}{7}$	$R_1 - \frac{1}{7}R_2$
y	0	1	0	$-\frac{1}{7}$	$\frac{3}{7}$	$\frac{1}{7}$	$\frac{23}{7}$	$\frac{3}{7}R_2$
x	1	0	0	$-\frac{2}{7}$	$-\frac{1}{7}$	$-\frac{5}{7}$	$\frac{25}{7}$	$R_3 - \frac{1}{7}R_2$
P	0	0	0	$\frac{11}{7}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{104}{7}$	$R_4 + \frac{2}{7}R_2$

The maximum value of P is $\frac{104}{7} = 14\frac{6}{7}$ which occurs when

$x = \frac{25}{7},\ y = \frac{23}{7},\ z = \frac{11}{7},\ s_1 = s_2 = s_3 = 0$

11 a $x + 2y + 3z + s_1 = 18$
$3x + y + z - s_2 + a_1 = 6$
$2x + 5y + z - s_3 + a_2 = 20$

b New objective is maximise $I = -(a_1 + a_2)$
$-a_1 = 3x + y + z - s_2 - 6$
$-a_2 = 2x + 5y + z - s_3 - 20$
In terms of non-basic variables, the new objective is maximise $I = 5x + 6y + 2z - s_2 - s_3 - 26$

c

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	value
s_1	1	2	3	1	0	0	0	0	18
a_1	3	1	1	0	−1	0	1	0	6
a_2	2	5	1	0	0	−1	0	1	20
P	−2	1	−1	0	0	0	0	0	0
I	−5	−6	−2	0	1	1	0	0	−26

d 1st iteration
Pivot is y-column a_2 row

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value	Row operation
s_1	$\frac{1}{5}$	0	$\frac{13}{5}$	1	0	$\frac{2}{5}$	0	$-\frac{2}{5}$	10	$R_1 - \frac{2}{5}R_3$
a_1	$\frac{13}{5}$	0	$\frac{4}{5}$	0	−1	$\frac{1}{5}$	1	$-\frac{1}{5}$	2	$R_2 - \frac{1}{5}R_3$
y	$\frac{2}{5}$	1	$\frac{1}{5}$	0	0	$-\frac{1}{5}$	0	$\frac{1}{5}$	4	$\frac{1}{5}R_3$
P	$-\frac{12}{5}$	0	$-\frac{6}{5}$	0	0	$\frac{1}{5}$	0	$-\frac{1}{5}$	−4	$R_4 - \frac{1}{5}R_3$
I	$-\frac{13}{5}$	0	$-\frac{4}{5}$	0	1	$-\frac{1}{5}$	0	$\frac{6}{5}$	−2	$R_5 + \frac{6}{5}R_3$

2nd iteration
Pivot is x column a_1 row

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value	Row operation
s_1	0	0	$\frac{33}{13}$	1	$\frac{1}{13}$	$\frac{5}{13}$	$-\frac{1}{13}$	$-\frac{5}{13}$	$\frac{128}{13}$	$R_1 - \frac{1}{13}R_2$
x	1	0	$\frac{4}{13}$	0	$-\frac{5}{13}$	$\frac{1}{13}$	$\frac{5}{13}$	$-\frac{1}{13}$	$\frac{10}{13}$	$\frac{5}{13}R_2$
y	0	1	$\frac{1}{13}$	0	$\frac{2}{13}$	$-\frac{3}{13}$	$-\frac{2}{13}$	$\frac{3}{13}$	$\frac{48}{13}$	$R_3 - \frac{2}{13}R_2$
P	0	0	$-\frac{6}{13}$	0	$-\frac{12}{13}$	$\frac{5}{13}$	$\frac{12}{13}$	$-\frac{5}{13}$	$-\frac{28}{13}$	$R_4 + \frac{12}{13}R_2$
I	0	0	0	0	0	0	1	1	0	$R_5 + R_2$

Basic feasible solution is $x = \frac{10}{13}$, $y = \frac{43}{13}$, $z = 0$, $s_1 = \frac{128}{13}$, $s_2 = s_3 = a_1 = a_2 = 0$

12 a $3x + 2y + z + s_1 = 24$
$5x + 3y + 2z + s_2 = 60$
$x - s_3 + a_1 = 2$

b $P = x + 3y + 4z - Ma_1$
$= x + 3y + 4z - M(2 - x + s_3)$
$P = x(1 + M) + 3y + 4z - 2M - Ms_3$
$P - (1 + M)x - 3y - 4z + Ms_3 = -2M$

c

b.v.	x	y	z	s_1	s_2	s_3	a_1	value
s_1	3	2	1	1	0	0	0	24
s_2	5	3	2	0	1	0	0	60
a_1	1	0	0	0	0	−1	1	2
P	$-(1 + M)$	−3	−4	0	0	M	0	$-2M$

d The most negative value in the P row is in the x-column so, in the first iteration, x enters the basic variables.

e 1st iteration
x column a_1 row is the pivot

b.v.	x	y	z	s_1	s_2	s_3	a_1	Value	Row operation
s_1	0	2	1	1	0	3	−3	18	$R_1 - 3R_3$
s_2	0	3	2	0	1	5	−5	50	$R_2 - 5R_3$
x	1	0	0	0	0	−1	1	2	
P	0	−3	−4	0	0	−1	$(1 + M)$	2	$R_4 + (1 + M)R_3$

2nd iteration z column s_1 row is the pivot

b.v.	x	y	z	s_1	s_2	s_3	a_1	Value	Row operation
z	0	2	1	1	0	3	−3	18	
s_2	0	−2	0	−2	1	−1	−1	14	$R_2 - 2R_1$
x	1	0	0	0	0	−1	1	2	
P	0	5	0	4	0	11	$M - 11$	74	$R_4 + 4R_1$

All entries in the P row are non-negative so the tableau represents the optimal solution.
$x = 2, y = 0, z = 18, s_1 = 0, s_2 = 14, s_3 = 0, a_1 = 0$

13 a $4x + 3y + 2z + s_1 = 36$
$x + 4z + s_2 = 52$
$x + y - s_3 + a_1 = 10$

b Maximise $P = -2x + 3y - z - Ma_1$
$= -2x + 3y - z - M(10 - x - y + s_3)$
$= x(M - 2) + y(M + 3) - z - 10M - Ms_3$
Rearranging gives
$P - (M - 2)x - (M + 3)y + z + Ms_3 = -10M$

c

b.v.	x	y	z	s_1	s_2	s_3	a_1	value
s_1	4	3	2	1	0	0	0	36
s_2	1	0	4	0	1	0	0	52
a_1	1	1	0	0	0	−1	1	10
P	$-(M-2)$	$-(M+3)$	1	0	0	M	0	$-10M$

d 1st iteration y column a_1 row is the pivot

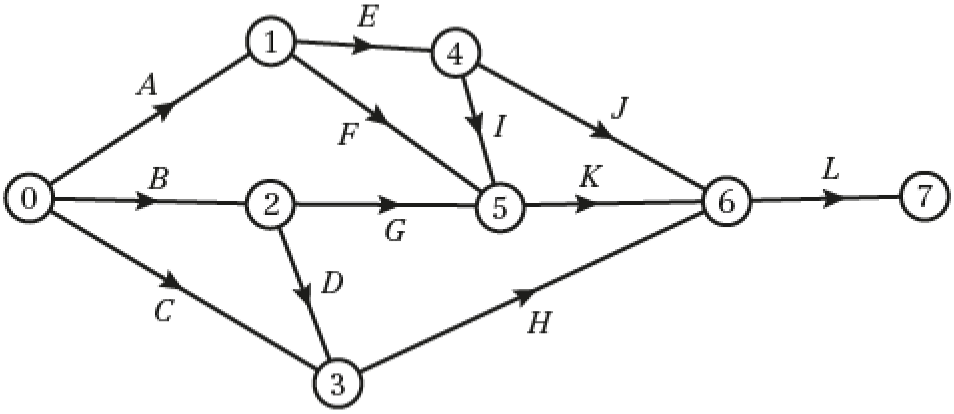
b.v.	x	y	z	s_1	s_2	s_3	a_1	Value	Row operation
s_1	1	0	2	1	0	3	−3	6	$R_1 - 3R_3$
s_2	1	0	4	0	1	0	0	52	
y	1	1	0	0	0	−1	1	10	
P	5	0	1	0	0	−3	$M+3$	30	$R_4 + (M+3)R_3$

2nd iteration s_3 column s_1 row is the pivot

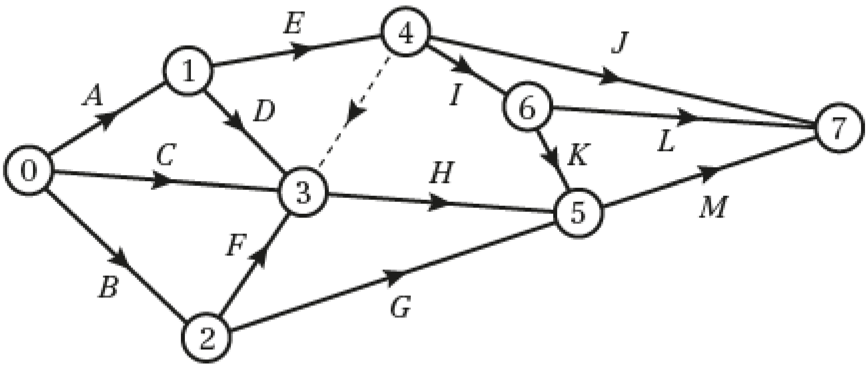
b.v.	x	y	z	s_1	s_2	s_3	a_1	Value	Row operation
s_3	$\frac{1}{3}$	0	$\frac{2}{3}$	$\frac{1}{3}$	0	1	−1	2	$\frac{1}{3}R_1$
s_2	1	0	4	0	1	0	0	52	
y	$\frac{4}{3}$	1	$\frac{2}{3}$	$\frac{1}{3}$	0	0	0	12	$\frac{1}{3}R_1 + R_3$
P	6	0	3	1	0	0	M	36	$R_4 + R_1$

Maximum value of $P = 36$
Minimum value of $C = -36$
This occurs when $x = 0$, $y = 12$, $z = 0$, $s_1 = 0$, $s_2 = 52$, $s_3 = 2$

14

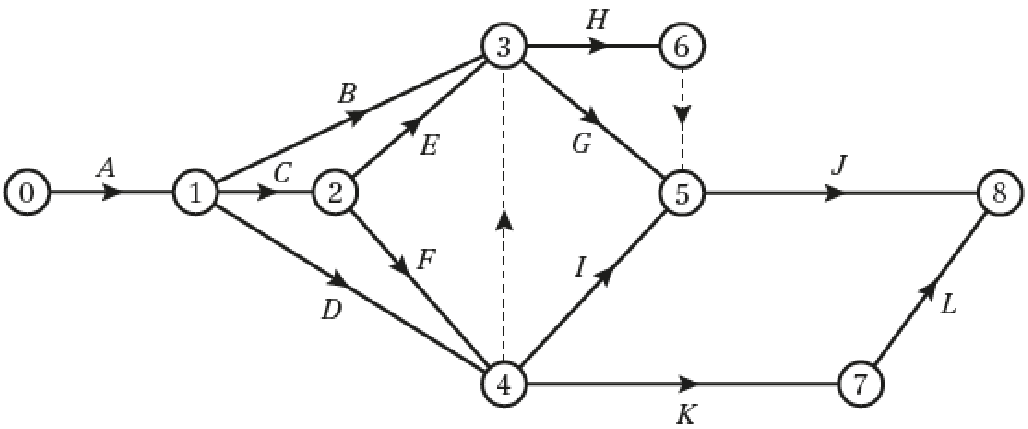


15 a



b Here we have that *I* and *J* depend only on *E*, whereas *H* depends on *C*, *D*, *E* and *F*. Hence we need separate nodes with a dummy.

16 a



b *D* will only be critical if it lies on the longest path

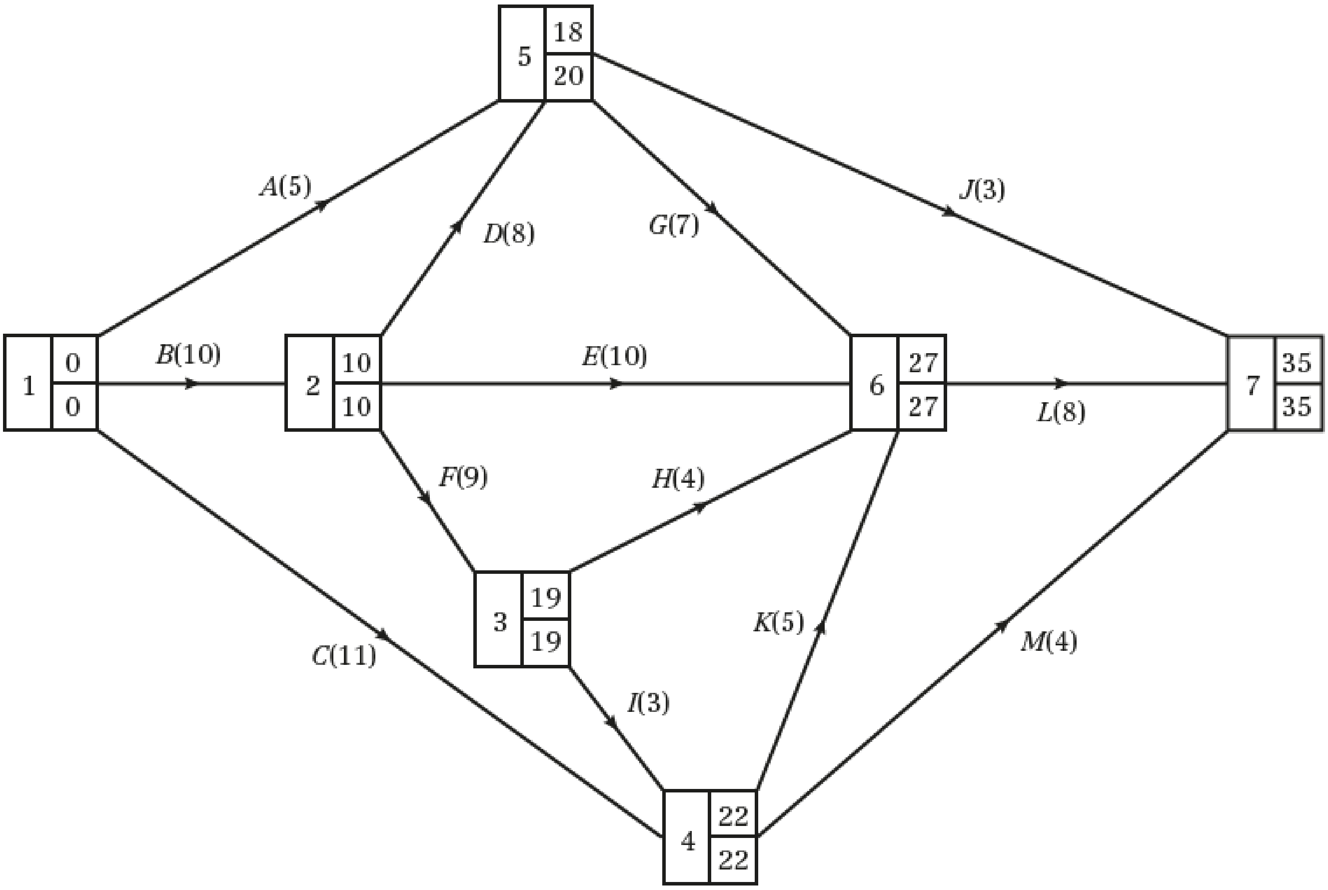
Path A to G	Length
$A-B-E-G$	14
$A-C-F-G$	15
$A-C-D-E-G$	$13+x$

So we need $13+x$ to be the longest, or equal longest

$13+x \geqslant 15$

$x \geqslant 2$

17 a



b Total float on $A = 20 - 0 - 5 = 15$
Total float on $B = 10 - 0 - 10 = 0$
Total float on $C = 22 - 0 - 11 = 11$
Total float on $D = 20 - 10 - 8 = 2$
Total float on $E = 27 - 10 - 10 = 7$
Total float on $F = 19 - 10 - 9 = 0$
Total float on $G = 27 - 18 - 7 = 2$
Total float on $H = 27 - 19 - 4 = 4$
Total float on $I = 22 - 19 - 3 = 0$
Total float on $J = 35 - 18 - 3 = 14$
Total float on $K = 27 - 22 - 5 = 0$
Total float on $L = 35 - 27 - 8 = 0$
Total float on $M = 35 - 22 - 4 = 9$

c Critical activities: B, F, I, K and L
length of critical path is 35 days

d New critical path is $B - F - H - L$
length of new critical path is 36 days

18 a $x = 0$

$y = 7$ [latest out of (3 + 2) and (5 + 2)]

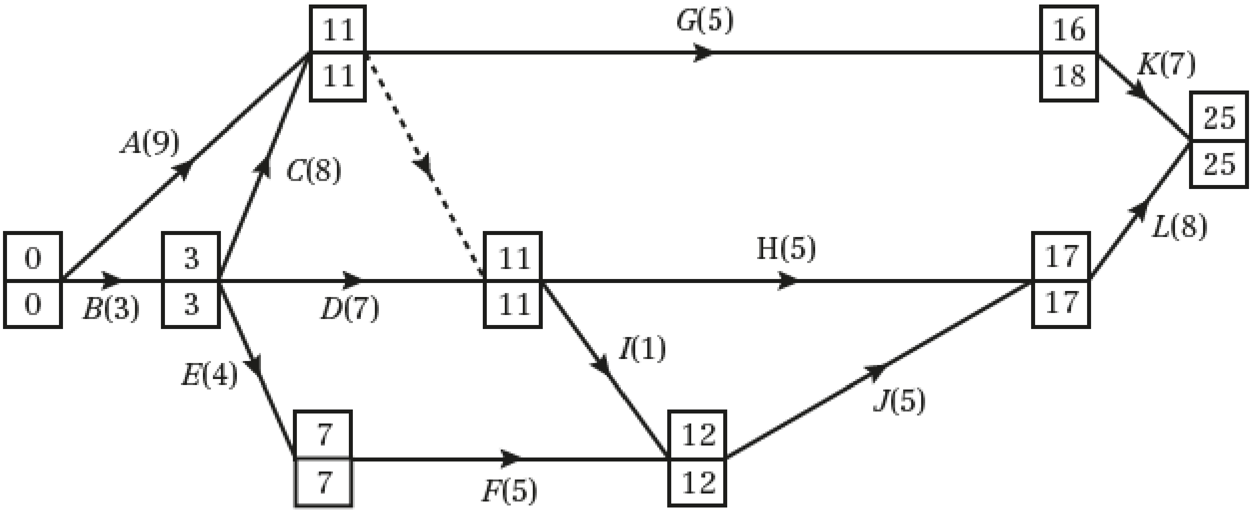
$z = 9$ [Earliest out of (13 – 4) and (19 – 7) and (16 – 2)]

b Length is 22
Critical activities: B, D, E and L

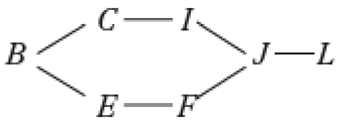
c **i** Total float on $N = 22 - 14 - 3 = 5$
ii Total float on $H = 16 - 5 - 3 = 8$

19 a For example, it shows dependence but it is not an activity. *G* depends on *A* and *C* *only* but *H* and *I* depend on *A*, *C* and *D*.

b



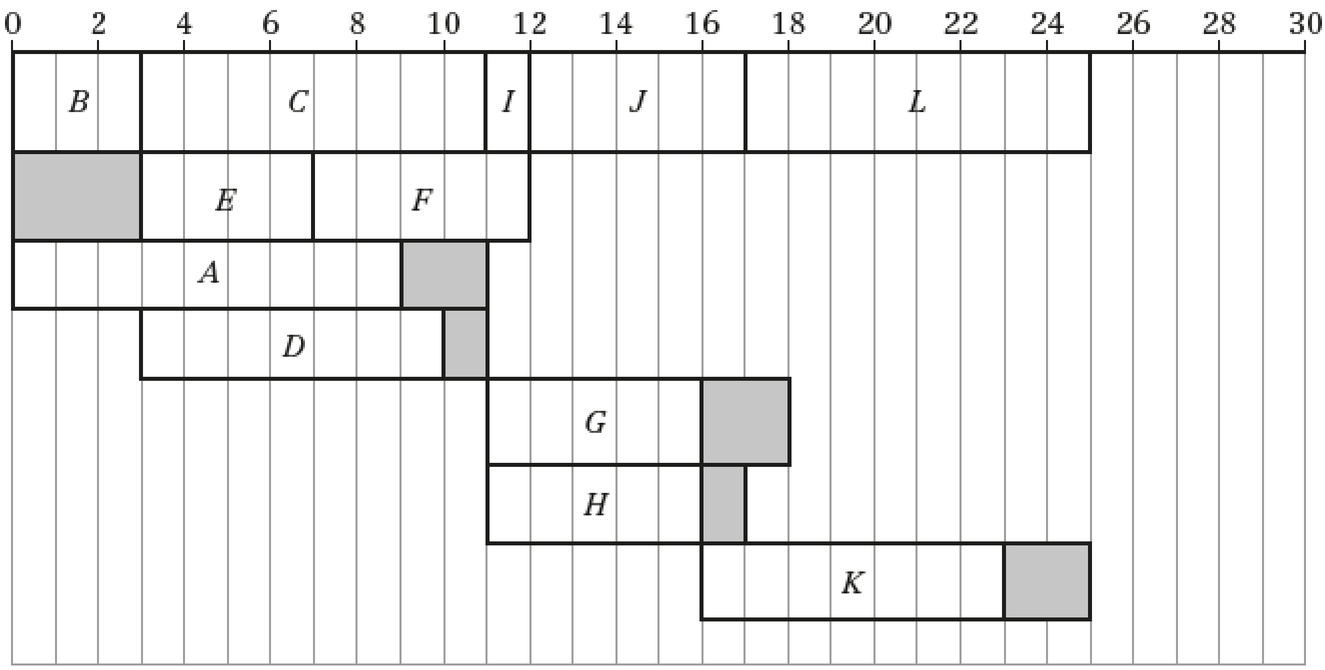
c



So *B*, *C*, *E*, *F*, *I*, *J* and *L*

d Total float on $A = 11 - 0 - 9 = 2$
Total float on $D = 11 - 3 - 7 = 1$
Total float on $G = 18 - 11 - 5 = 2$
Total float on $H = 17 - 11 - 5 = 1$
Total float on $K = 25 - 16 - 7 = 2$

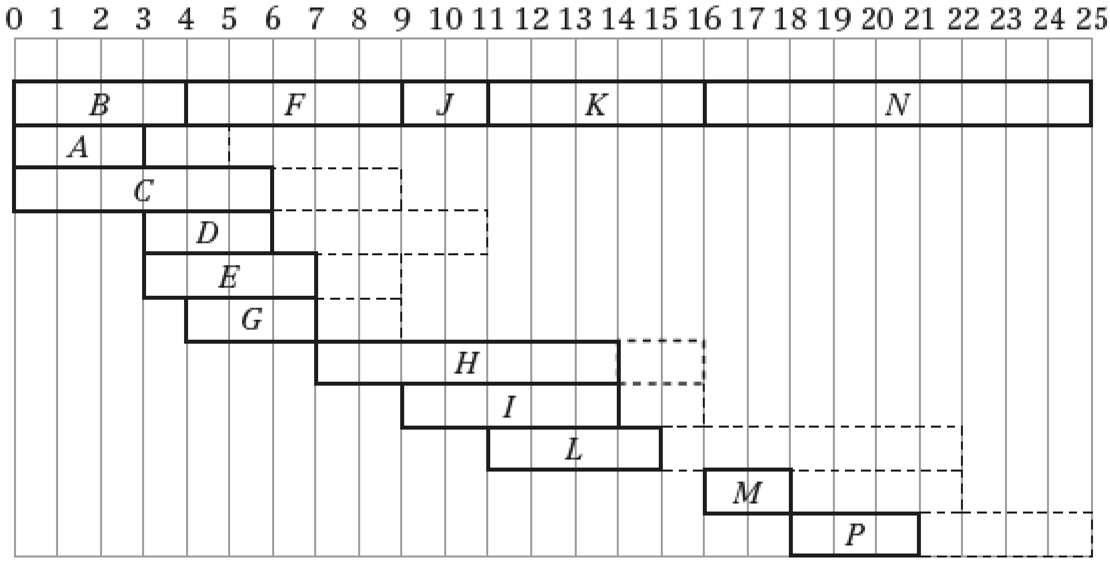
e



20 a Critical activities are *B*, *F*, *J*, *K* and *N* length of critical path is 25 hours
I is not critical.

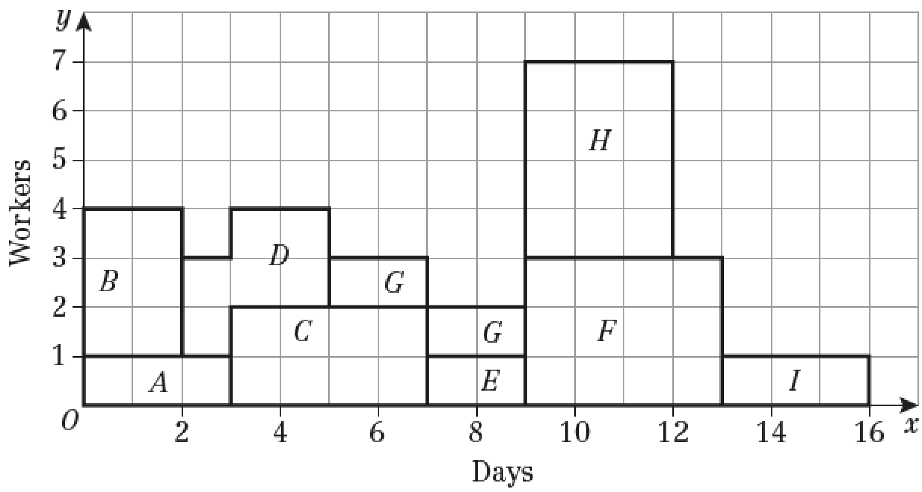
b Total float on *A* = 5 – 0 – 3 = 2
Total float on *C* = 9 – 0 – 6 = 3
Total float on *D* = 11 – 3 – 3 = 5
Total float on *E* = 9 – 3 – 4 = 2
Total float on *G* = 9 – 4 – 3 = 2
Total float on *H* = 16 – 7 – 7 = 2
Total float on *I* = 16 – 9 – 5 = 2
Total float on *L* = 22 –11 – 4 = 7
Total float on *M* = 22 – 16 – 2 = 4
Total float on *P* = 25 – 18 – 3 = 4

c



d Look at 6.5 in the chart in c. *F*, *E* and *G*

21 a

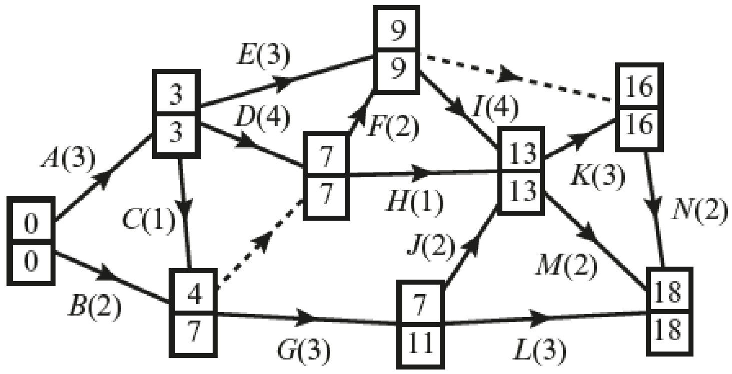


b 16 days, 7 workers

c Delay the start of *H* until time 13

d Activity *H* would have to take place on its own so the project will be delayed by at least 3 days.

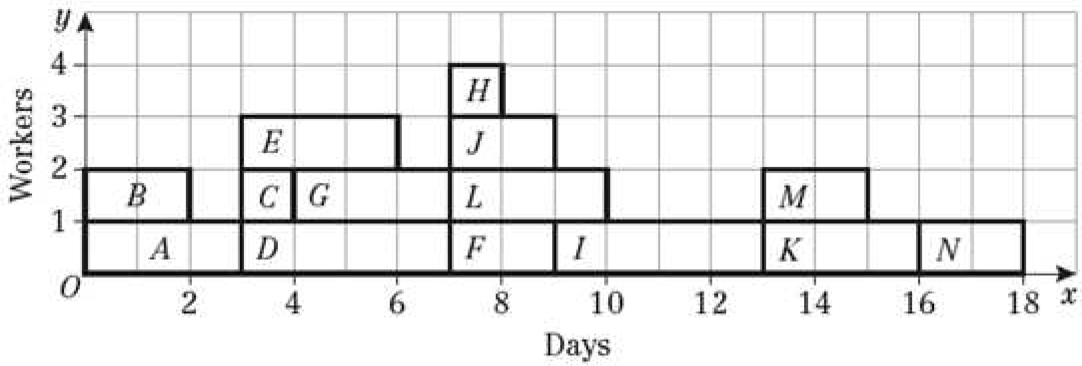
22 a



b 18 days

c *ADFIKN*

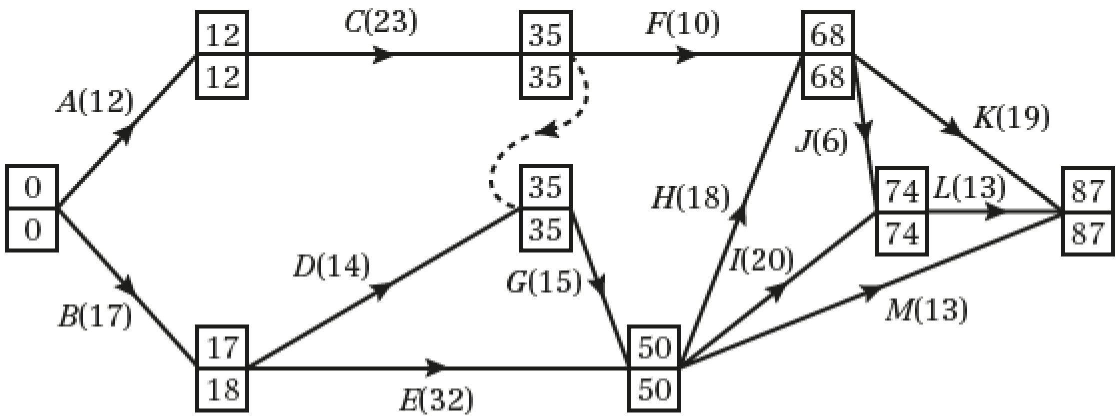
d



e 4 workers

f e.g. delay the start times of:
E to time 4
G to time 7
H to time 12
J to time 10
L to time 13
M to time 16

23 a



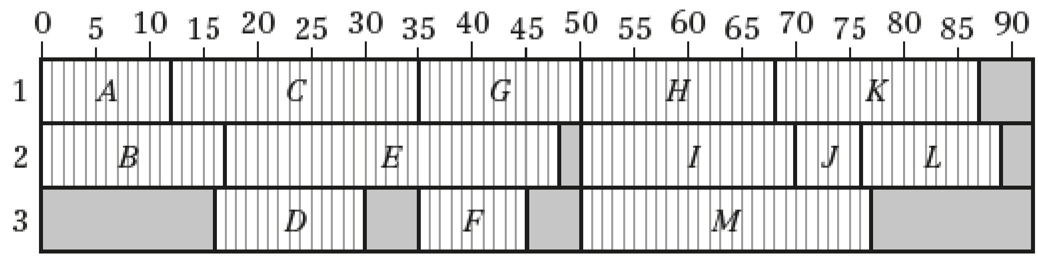
b *A*, *C*, *G*, *H*, *J*, *K* and *L*

All critical activities have a zero total float.

c Total float = 35 – 17 – 14 = 4

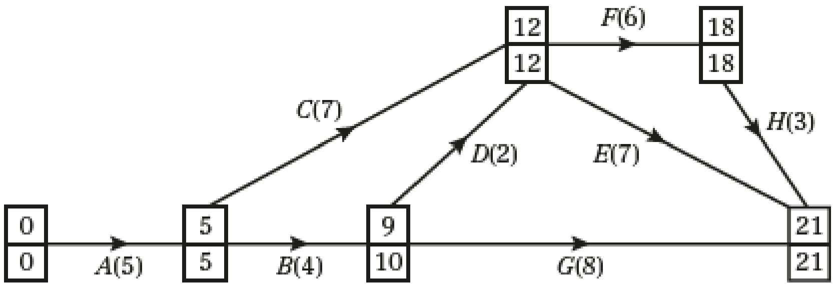
d *Either* 226 ÷ 87 = 2.6 (1 d.p.) so at least 3 workers needed (here 226 is the total number of hours required for all the activities) *or* 69 hours into the project activities J, K, I and M *must* be happening so at least 4 workers will be needed.

e



New shortest time is 89 hours.

24 a

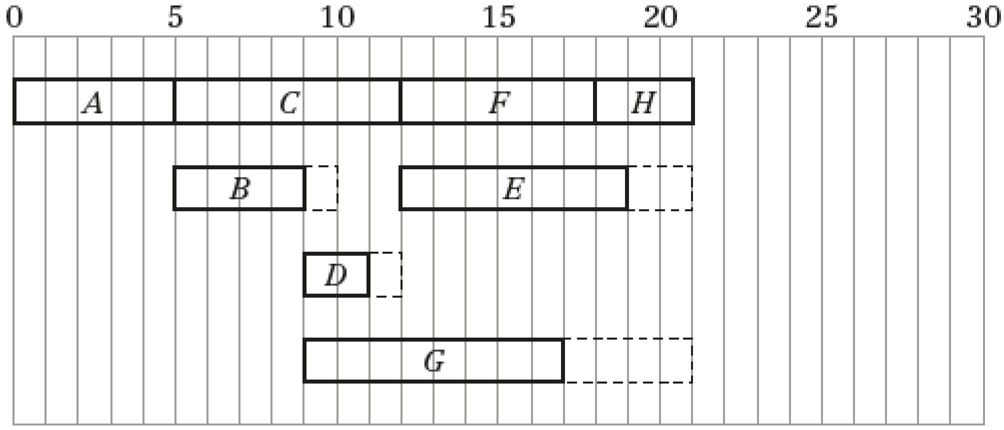


b Critical activities: A, C, F and H; length of critical path = 21

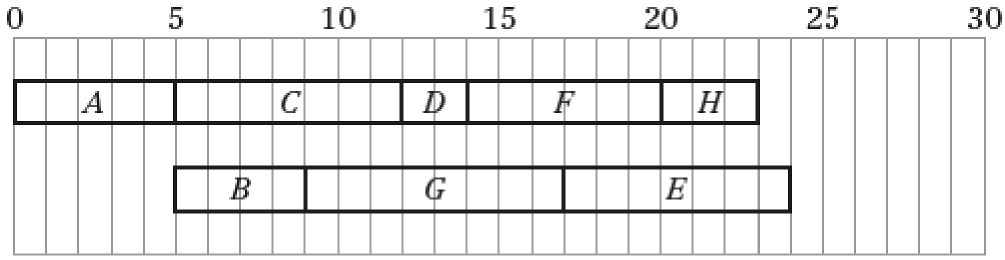
c Total float on $B = 10 - 5 - 4 = 1$ Total float on $E = 21 - 12 - 7 = 2$

Total float on $D = 12 - 9 - 2 = 1$ Total float on $G = 21 - 9 - 8 = 4$

d



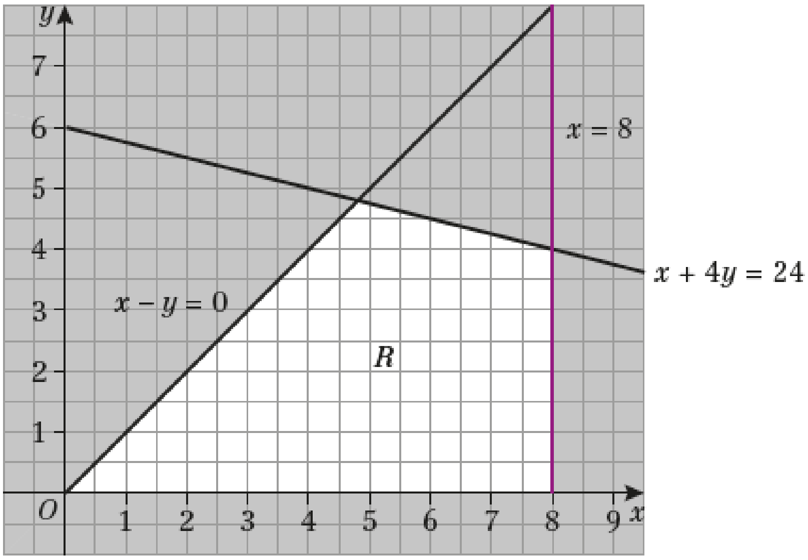
e For example;



Minimum time for 2 workers is 24 days.

Challenge

1 a



Optimal values $x = y = \frac{24}{5}$

b $x = 4, y = 4 \Rightarrow P = 24$
$x = 5, y = 4 \Rightarrow P = 25$

c $x = 7, y = 4 \Rightarrow P = 27$

d If the gradient of the objective line is similar to the gradient of a constraint that runs through the optimal vertex, then the optimal integer solution may not lie close to the optimal vertex.

Challenge

2 We need to maximise $D = -2x + 3y - z$ subject to the constraints

$4x + 3y + 2z \leqslant 36$

$x + 4z \leqslant 52$

$x + y \geqslant 10$

We rewrite these using slack, surplus and artificial variables to obtain:

$4x + 3y + 2z + s_1 = 36$

$x + 4z + s_2 = 52$

$x + y - s_3 + a_1 = 10$

The new objective function is:

$I = -a_1 = x + y - s_3 - 10$

So $I - x - y + s_3 = -10$

So the initial tableau is:

b.v.	x	y	z	s_1	s_2	s_3	a_1	Value
s_1	4	3	2	1	0	0	0	36
s_2	1	0	4	0	1	0	0	52
a_1	1	1	0	0	0	−1	1	10
D	2	−3	1	0	0	0	0	0
I	−1	−1	0	0	0	1	0	−10

Both the x and y columns have the same value in the I row. But s_1 in the x-column has the smallest θ value (9) so we use that as the pivot. We then obtain:

b.v.	x	y	z	s_1	s_2	s_3	a_1	Value	Row operation
x	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0	0	0	9	$\frac{1}{4}R_1$
s_2	0	$-\frac{3}{4}$	$\frac{7}{2}$	$-\frac{1}{4}$	1	0	0	43	$R_2 - \frac{1}{4}R_1$
a_1	0	$-\frac{15}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	0	-1	1	1	$R_3 - \frac{1}{4}R_1$
D	0	$-\frac{9}{2}$	−1	$-\frac{1}{2}$	0	0	0	−18	$R_4 - \frac{1}{2}R_1$
I	0	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0	1	0	−1	$R_5 + \frac{1}{4}R_1$

Challenge

2 continued

The y-column still has a negative value in the I row and only the x-row has a positive θ value so we need to use that as the pivot giving.

b.v.	x	y	z	s_1	s_2	s_3	a_1	Value	Row operation
y	$\frac{4}{3}$	1	$\frac{2}{3}$	$\frac{1}{3}$	0	0	0	12	$\frac{4}{3}R_1$
s_2	0	0	4	0	1	0	0	52	$R_2 + R_1$
a_1	5	0	3	$-\frac{1}{4}$	1	1	0	46	$R_3 + 5R_1$
D	6	0	2	1	0	0	0	36	$R_4 + 6R_1$
I	$\frac{1}{3}$	0	$\frac{2}{3}$	$\frac{1}{3}$	0	1	0	2	$R_5 + \frac{1}{3}R_1$

There are now no negative entries in the bottom two rows so we have reached the optimal solution:
$y = 12$, $x = z = 0$ and $C = -36$