The simplex algorithm 7E

1 a $a_1 \neq 0 \Rightarrow$ not in feasible region

b 2 in z column is the pivot as $-2(1+M)$ is the most negative entry in the P row and corresponding $\theta = \frac{6}{2} = 3$ value is the least positive.

c In the next iteration we have $a_1 \neq 0$, remains as a basic variable and so cannot represent a feasible solution as it does not lie in the feasible region.

d

	x	y	z	s_1	s_2	a_1	Value	Row operation
z	$\frac{1}{2}$	$\frac{1}{2}$		$\frac{1}{2}$	0	0	3	$R_1 \div 2$
s_2	$\frac{3}{2}$	$-\frac{7}{2}$	0	$-\frac{1}{2}$	−1	1	2	$R_2 - R_1$
P	0	$3(1+M)$	0	$1+M$	$-M$	0	$6-9M$	$R_3 + R_1(2+2M)$

2 a $x+3y+z+s_1 = 100$

$3x - y + s_2 = 52$

$x - s_3 + a_1 = 20$

b

	x	y	z	s_1	s_2	s_3	a_1	Value
s_1	1	3	1	1	0	0	0	100
s_2	3	−1	0	0	1	0	0	52
a_1	1	0	0	0	0	−1	1	20
P	$-(4+M)$	−2	1	0	0	M	0	$-20M$

c M represents an arbitrarily large number

d $-(4+M)$ most negative $\Rightarrow$ x is the pivot column

θ values are respectively $100, 17\frac{1}{3}, 20 \Rightarrow 3$ in the s_2 row is the pivot

3 a This problem contains a mixture of $\geq$ and $\leq$ inequalities, (hence $(0,0,0)$ is not in the feasible region) so we cannot use the standard simplex algorithm.

b $3x - y + s_1 = 110$

$x + 2y - s_2 + a_1 = 45$

3 c Turn into maximisation problem and add the factor involving M.

$$D = -4x - 3y - Ma_1 = -4x - 3y - M(45 - x - 2y + s_2)$$

$$\Rightarrow D + x(4 - M) + y(3 - 2M) + Ms_2 = -45M$$

	x	y	s_1	s_2	a_1	Value
s_1	3	−1	1	0	0	110
a_1	1	2	0	−1	1	45
D	$(4 - M)$	$(3 - 2M)$	0	M	0	$-45M$

4 a $x + y + z + s_1 = 20$

$3x + y + 2z - s_2 + a_1 = 24$

b $P = 3x + 5y - z - Ma_1 = 3x + 5y - M(24 - 3x - y - 2z + s_2)$

$\Rightarrow P - (3 + 3M)x - (5 + M)y - (2M + 1)z + Ms_2 = -24M$

c

Basic variable	x	y	z	s_1	s_2	a_1	Value	θ values
s_1	1	1	1	1	0	0	20	
a_1	3	1	2	0	−1	1	24	
P	$-(3+3M)$	$-(5+M)$	$-(2M-1)$	0	M	0	$-24M$	

d

Basic variable	x	y	z	s_1	s_2	a_1	Value	θ values
s_1	1	1	1	1	0	0	20	20
a_1	3	1	2	0	−1	1	24	8
P	$-(3+3M)$	$-(5+M)$	$-(2M-1)$	0	M	0	$-24M$	

Basic variable	x	y	z	s_1	s_2	a_1	Value	Row operations
s_1	0	$\frac{2}{3}$	$\frac{1}{3}$	1	$\frac{1}{3}$	$-\frac{1}{3}$	12	R1 − R2
x	1	$\frac{1}{3}$	$\frac{2}{3}$	0	$-\frac{1}{3}$	$\frac{1}{3}$	8	R2 ÷ 3
P	0	−4	3	0	−1	$1+M$	24	R3 + $(3+3M)$ R2

Basic variable	x	y	z	s_1	s_2	a_1	Value	θ values
s_1	0	$\frac{2}{3}$	$\frac{1}{3}$	1	$\frac{1}{3}$	$-\frac{1}{3}$	12	18
x	1	$\frac{1}{3}$	$\frac{2}{3}$	0	$-\frac{1}{3}$	$\frac{1}{3}$	8	24
P	0	–4	3	0	–1	$1+M$	24	

Basic variable	x	y	z	s_1	s_2	a_1	Value	Row operations
y	0	1	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	18	$\frac{3}{2}$ R1
x	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	2	R2 $-\frac{1}{3}$ R1
P	0	0	5	6	1	$M-1$	96	R3 + 4R1

Optimal $P = 96$ when $x = 2, y = 18, z = s_1 = s_2 = a_1 = 0$