The simplex algorithm 7B

1

b.v.	x	y	z	r	s	value	θ values
r	1	(2)	0	1	0	6	3 *
s	5	3	3	0	1	24	8
P	−5	-6	−4	0	0	0	

b.v.	x	y	z	r	s	value	Row operations
y	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	3	$R1 \div 2$
s	$\frac{7}{2}$	0	(3)	$-\frac{3}{2}$	1	15	$R2 - 3R1$
P	−2	0	−4	3	0	18	$R3 + 6R1$

b.v.	x	y	z	r	s	value	Row operations
y	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	3	R1 (no change)
z	$\frac{7}{6}$	0	1	$-\frac{1}{2}$	$\frac{1}{3}$	5	$R2 \div 3$
P	$\frac{8}{3}$	0	0	1	$\frac{4}{3}$	38	$R3 + 4R2$

$P = 38 \quad x = 0 \quad y = 3 \quad z = 5 \quad r = 0 \quad s = 0$

2

b.v.	x	y	z	r	s	value	θ values
r	1	2	2	1	0	100	50
s	1	0	(4)	0	1	40	10*
P	−3	−4	−10	0	0	0	

b.v.	x	y	z	r	s	value	Row operations
y	$\frac{1}{2}$	2	0	1	$-\frac{1}{2}$	80	R1 − 2R2
z	$\frac{1}{4}$	0	1	0	$\frac{1}{4}$	10	R2 ÷ 4
P	$-\frac{1}{2}$	−4	0	0	$\frac{5}{2}$	100	R3 + 10R2

b.v.	x	y	z	r	s	value	Row operations
y	$\frac{1}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	40	R1 ÷ 2
z	$\frac{1}{4}$	0	1	0	$\frac{1}{4}$	10	R2 (no change)
P	$\frac{1}{2}$	0	0	2	$\frac{3}{2}$	260	R3 + 4R1

$P = 260 \quad x = 0 \quad y = 40 \quad z = 10 \quad r = 0 \quad s = 0$

$P = 260 \quad x = 0 \quad y = 40 \quad z = 10 \quad r = 0 \quad s = 0$

3

b.v.	x	y	z	r	s	t	value	θ values
r	3	4	5	1	0	0	10	2.5
s	1	(3)	10	0	1	0	5	$1\frac{2}{3}$*
t	1	−2	0	0	0	1	1	negative pivot
P	−3	−5	−2	0	0	0	0	

b.v.	x	y	z	r	s	t	value	Row operations
r	$\frac{5}{3}$	0	$-\frac{25}{3}$	1	$-\frac{4}{3}$	0	$\frac{10}{3}$	R1 − 4R2
y	$\frac{1}{3}$	1	$\frac{10}{3}$	0	$\frac{1}{3}$	0	$\frac{5}{3}$	R2 ÷ 3
t	$\frac{5}{3}$	0	$\frac{20}{3}$	0	$\frac{2}{3}$	1	$\frac{13}{3}$	R3 + 2R2
P	$-\frac{4}{3}$	0	$\frac{44}{3}$	0	$\frac{5}{3}$	0	$\frac{25}{3}$	R4 + 5R2

b.v.	x	y	z	r	s	t	value	Row operations
x	1	0	−5	$\frac{3}{5}$	$-\frac{4}{5}$	0	2	R1 ÷ $\frac{5}{3}$
y	0	1	5	$-\frac{1}{5}$	$\frac{3}{5}$	0	1	R2 − $\frac{1}{3}$R1
t	0	0	15	−1	2	1	1	R3 − $\frac{5}{3}$R1
P	0	0	8	$\frac{4}{5}$	$\frac{3}{5}$	0	11	R4 − $\frac{4}{3}$R1

$P = 11 \quad x = 2 \quad y = 1 \quad z = 0 \quad r = 0 \quad s = 0 \quad t = 1$

4

Basic variable	x_1	x_2	x_3	x_4	r	s	t	u	Value	θ values
r	1	4	3	1	1	0	0	0	95	95
s	2	1	2	3	0	1	0	0	67	$33\frac{1}{2}$
t	1	3	2	2	0	0	1	0	75	75
u	3	2	1	2	0	0	0	1	72	24
P	−4	3	−2	−3	0	0	0	0	0	

Basic variable	x_1	x_2	x_3	x_4	r	s	t	u	Value	Row operations
r	0	$3\frac{1}{3}$	$2\frac{2}{3}$	$\frac{1}{3}$	1	0	0	$-\frac{1}{3}$	71	R1 − R4
s	0	$-\frac{1}{3}$	$\frac{4}{3}$	$1\frac{2}{3}$	0	1	0	$-\frac{2}{3}$	19	R2 − 2R4
t	0	$2\frac{1}{3}$	$1\frac{2}{3}$	$1\frac{1}{3}$	0	0	1	$-\frac{1}{3}$	51	R3 − R4
x_1	1	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	0	0	0	$\frac{1}{3}$	24	R4 ÷ 3
P	0	$5\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	0	0	$\frac{4}{3}$	96	R5 + 4R4

Basic variable	x_1	x_2	x_3	x_4	r	s	t	u	Value	θ values
r	0	$3\frac{1}{3}$	$2\frac{2}{3}$	$\frac{1}{3}$	1	0	0	$-\frac{1}{3}$	71	$26\frac{5}{8}$
s	0	$-\frac{1}{3}$	$\frac{4}{3}$	$1\frac{2}{3}$	0	1	0	$-\frac{2}{3}$	19	$14\frac{1}{4}$
t	0	$2\frac{1}{3}$	$1\frac{2}{3}$	$1\frac{1}{3}$	0	0	1	$-\frac{1}{3}$	51	$30\frac{3}{5}$
x_1	1	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	0	0	0	$\frac{1}{3}$	24	72
P	0	$5\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	0	0	$\frac{4}{3}$	96	

4 continued

Basic variable	x_1	x_2	x_3	x_4	r	s	t	u	Value	Row operations
r	0	4	0	–3	1	–2	0	1	33	$R1-2\frac{2}{3}R2$
x_3	0	$-\frac{1}{4}$	1	$1\frac{1}{4}$	0	$\frac{3}{4}$	0	$-\frac{1}{2}$	$14\frac{1}{4}$	$\frac{3}{4}$ R2
t	0	$2\frac{3}{4}$	0	$-\frac{3}{4}$	0	$-1\frac{1}{4}$	1	$\frac{1}{2}$	$27\frac{1}{4}$	$R3-1\frac{2}{3}R2$
x_1	1	$\frac{3}{4}$	0	$\frac{1}{4}$	0	$-\frac{1}{4}$	0	$\frac{1}{2}$	$19\frac{1}{4}$	$R4-\frac{1}{3}R2$
P	0	$5\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$105\frac{1}{2}$	$R1+\frac{2}{3}R2$

Maximum $P=105\frac{1}{2}$, when $x_1=19\frac{1}{4}, x_2=0, x_3=14\frac{1}{4}, x_4=0, r=33, s=0, t=27\frac{1}{4}, u=0$

5 Let $S=-P$, so maximise $S=-3x-6y+32z$

Basic variable	x	y	z	r	s	t	u	Value	θ values
r	1	6	24	1	0	0	0	672	28
s	3	1	24	0	1	0	0	336	14
t	1	3	16	0	0	1	0	168	$10\frac{1}{2}$
u	2	3	32	0	0	0	1	352	11
P	3	6	−32	0	0	0	0	0	

Basic variable	x	y	z	r	s	t	u	Value	Row operations
r	$-\frac{1}{2}$	$1\frac{1}{2}$	0	1	0	$-1\frac{1}{2}$	0	420	R1 – 24R3
s	$1\frac{1}{2}$	$-3\frac{1}{2}$	0	0	1	$-1\frac{1}{2}$	0	84	R2 – 24R3
z	$\frac{1}{16}$	$\frac{3}{16}$	1	0	0	$\frac{1}{16}$	0	$10\frac{1}{2}$	R3 ÷ 16
u	0	–3	0	0	0	–2	1	16	R4+ 32R3
S	5	12	0	0	0	2	0	336	

Minimum $P=-336$ ($S=336$), when $x=0, y=0, z=10\frac{1}{2}, r=420, s=84, t=0, u=16$

6 Let $S = -P$, so maximise $S = -2x + 2y - 3z$

Basic variable	x	y	z	r	s	t	Value	θ values
r	4	2	1	1	0	0	2	1
s	2	4	2	0	1	0	8	2
t	3	3	4	0	0	1	4	$1\frac{1}{3}$
S	2	–2	3	0	0	0	0	

Basic variable	x	y	z	r	s	t	Value	Row operations
y	2	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	1	R1 ÷ 2
s	–6	0	0	–2	1	0	4	R2 – 4R1
t	–3	0	$2\frac{1}{2}$	$-1\frac{1}{2}$	0	1	1	R3 – 1R1
S	6	0	4	1	0	0	2	R4 + 2R1

Minimum $P = -2$ ($S = 2$), when $x = 0, y = 1, z = 0, r = 0, s = 4, t = 1$

7 *For Q1*

a

$$P = 5x + 6y + 42 \qquad 5(0) + 6(3) + 4(5) = 38$$
$$x + 2y + r = 6 \qquad 0 + 2(3) + 0 = 6$$
$$5x + 3y + 32 + 5 = 24 \qquad 5(0) + 3(3) + 3(5) + 0 = 24$$

b $P + \frac{8}{3}x + r + \frac{4}{3}s = 38$

$\frac{1}{2}x + y + \frac{1}{2}r = 3$

$\frac{7}{6}x + z - \frac{1}{2}r + \frac{1}{3}s = 5$

c $P = 38 - \frac{8}{3}x - r - \frac{4}{3}s$ so increasing x, r or s would decrease P.

7 *For Q2*

a $P = 3x + 4y + 10z \Rightarrow 3(0) + 4(10) + 10(10) = 260$

$x + 2y + 2z + r = 100 \Rightarrow 0 + 2(40) + 2(10) + 0 = 100$

$x + 4z + s = 40 \Rightarrow 0 + 4(10) + 0 = 40$

b $P + \frac{1}{2}x + 2r + \frac{3}{2}s = 260$

$\frac{1}{4}x + y + \frac{1}{2}r - \frac{1}{4}s = 40$

$\frac{1}{4}x + z + \frac{1}{4}s = 10$

c $P = 260 - \frac{1}{2}x - 2r - \frac{3}{2}s$, so increasing x, r, or s would decrease P.

7 *For Q3*

a $P = 3x + 5y + 2z \Rightarrow 3(2) + 5(1) + 2(0) = 11$

$3x + 4y + 5z + r = 10 \Rightarrow 3(2) + 4(1) + 5(0) + 0 = 10$

$x + 3y + 10z + s = 5 \Rightarrow 2 + 3(1) + 10(0) + 0 = 5$

$x - 2y + t = 1 \Rightarrow 2 - 2(1) + 1 = 1$

b $P + 8z + \frac{4}{5}r + \frac{3}{5}s = 11$

$x - 5z + \frac{3}{5}r - \frac{4}{5}s = 2$

$y + 5z - \frac{1}{5}r + \frac{3}{5}s = 1$

$15z - r + 2s + t = 1$

c $P = 11 - 8z - \frac{4}{5}r - \frac{3}{5}s$, so increasing z, r or s would decrease P.

7 *For Q4*

b $P + \frac{11}{2}x_2 + \frac{1}{2}x_4 + \frac{1}{2}s + u = \frac{211}{2}$

$4x_2 - 3x_4 + r - 2s + u = 33$

$-\frac{1}{4}x_2 + x_3 + \frac{5}{4}x_4 + \frac{3}{4}s - \frac{1}{2}u = \frac{57}{4}$

$\frac{11}{4}x_2 - \frac{3}{4}x_4 - \frac{5}{4}s + t + \frac{1}{2}u = \frac{109}{4}$

$x_1 + \frac{3}{4}x_2 + \frac{1}{4}x_4 - \frac{1}{4}s + \frac{1}{2}u = \frac{77}{4}$

c $P = \frac{211}{2} - \frac{11}{2}x_2 - \frac{1}{2}x_4 - \frac{1}{2}s - u$, so increasing x_2, x_4, s or u would decrease P.

7 *For Q5*

b $P - 5x - 12y - 2t = -336$

$-\frac{1}{2}x + \frac{3}{2}y + r - \frac{3}{2}t = 420$

$\frac{3}{2}x - \frac{7}{2}y + s - \frac{3}{2}t = 84$

$\frac{1}{16}x + \frac{3}{16}y + z + \frac{1}{16}t = \frac{21}{2}$

c $P = -366 + 5x + 12y + 2t$, so increasing x, y or t would decrease P.

7 *For Q6*

b $P - 6x - 4z - r = -2$

$2x + y + \frac{1}{2}z + \frac{1}{2}r = 1$

$-6x - 2r + s = 4$

$-3x + \frac{5}{2}z - \frac{3}{2}r + t = 1$

c $P = -2 + 6x + 4z + r$, so increasing x, z or r would increase P.

8 a x, y are not basic variables, so $x = 0, y = 0$

first row: $z = \frac{3}{2}$

b Reading off from the last row: $P + 3x + 4y + r = 3$

9 a Reading off from the last row: $P - x + 2y - 2z = 0$

b

Basic variable	x	y	z	r	s	t	Value	θ values
r	4	1	2	1	0	0	2	1
s	1	2	1	0	1	0	8	8
t	2	4	3	0	0	1	4	$\frac{4}{3}$
P	–1	2	–2	0	0	0	0	

Basic variable	x	y	z	r	s	t	Value	Row operations
z	2	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	1	R1 ÷ 2
s	–1	$1\frac{1}{2}$	0	$-\frac{1}{2}$	1	0	7	R2 – R1
t	–4	$2\frac{1}{2}$	0	$-1\frac{1}{2}$	0	1	1	R3 – 3R1
P	3	3	0	1	0	0	2	R2 + 2R1

c $P = 2, x = 0, \text{y} = 0, z = 1, r = 0, s = 7, t = 1$