The simplex algorithm 7A

1 Let x_1, x_2 and x_3 be the number of round, square and rectangular boxes respectively.

Maximise $P = 12x_1 + 10x_2 + 11x_3$

Subject to:

$$4x_1 + 2x_2 + 3x_3 + r = 360$$
$$2x_1 + 3x_2 + 3x_3 + s = 360$$
$$x_1, x_2, x_3, r, s \geqslant 0$$

2 Let x_A, x_B, x_C and x_D be the number of type A, B, C and D backpacks made.

Maximise $P = 8x_A + 7x_B + 6x_C + 9x_D$

Subject to:

$$2.5x_A + 3x_B + 2x_C + 4x_D + r = 1400$$
$$10x_A + 12x_B + 8x_C + 15x_D + s = 9000$$
$$5x_A + 7x_B + 4x_C + 9x_D + t = 4800$$
$$x_A + x_B + x_C + x_D + u = 500$$
$$x_A, x_B, x_C, x_D, r, s, t, u \geqslant 0$$

3 Let x_A, x_C and x_S be the number of adults, children and senior members.

Maximise $P = 40x_A + 10x_C + 20x_S$

Subject to:

$$x_A + x_C + x_S + r = 100$$
$$-x_A + x_C - x_S + s = 0$$
$$-2x_A + x_C + x_S + t = 0$$
$$x_A, x_C, x_S, r, s, t \geqslant 0$$

$x_C \leqslant \frac{1}{2}(x_A + x_C + x_S)$

$\frac{1}{2}x_C \leqslant \frac{1}{2}x_A + \frac{1}{2}x_S$

$x_c \leqslant x_A + x_S$

$x_c - x_A - x_S \leqslant 0$

$x_A \geqslant \frac{1}{3}(x_A + x_C + x_S)$

$3x_A \geqslant x_A + x_C + x_S$

$2x_A \geqslant x_C + x_S$

$x_C + x_S - 2x_A \leqslant 0$

4 Let x_r, x_f and x_m be the number of batches of rock cakes, fairy cakes and muffins made.

Maximise $T = 10x_r + 18x_f + 12x_m$

Subject to:

$$220x_r + 100x_f + 250x_m + r = 3000$$
$$100x_r + 100x_f + 50x_m + s = 2000$$
$$50x_r + 100x_f + 75x_m + t = 1500$$
$$x_r, x_f, x_m, r, s, t, \geqslant 0$$

5 x_s - number of small boxes, x_m - number of medium boxes, x_l - number of large boxes

Minimise cost, i.e. minimise $C = 0.3x_s + 0.5x_m + 0.8x_l$ (in pounds) subject to the following constraints:

at least 28 m^3 $\Rightarrow 0.1x_s + 0.3x_m + 0.7x_l \geqslant 28 \Rightarrow x_s + 3x_m + 7x_l \geqslant 280$
at least 600 kg $\Rightarrow 3x_s + 8x_m + 18x_l \geqslant 600$
at least half should be small $\Rightarrow x_s \geqslant 0.5(x_s + x_m + x_l) \Rightarrow -x_s + x_m + x_l \leqslant 0$
at least twice as many medium as large $x_m \geqslant 2x_l \Rightarrow -x_m + 2x_l \leqslant 0$
non-negativity $\Rightarrow x_s, x_m, x_l \geqslant 0$