Linear Programming Mixed exercise

1 a Flour: $200x + 200y \leqslant 2800$

so $x + y \leqslant 14$

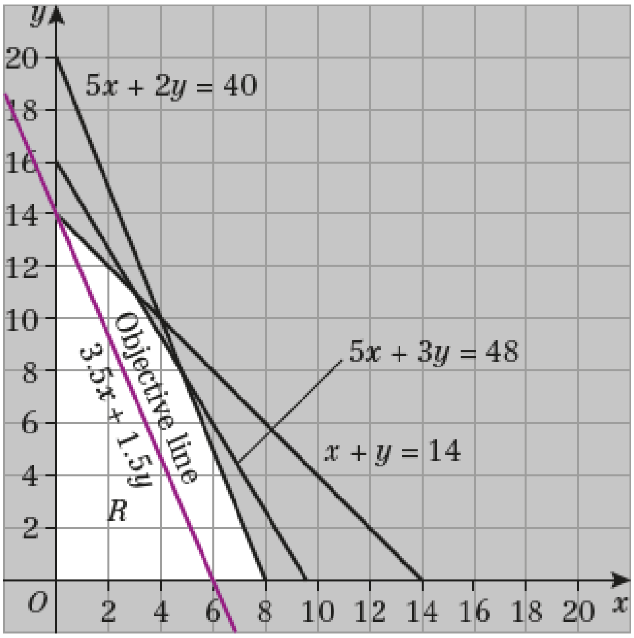
Fruit: $125x + 50y \leqslant 1000$

so $5x + 2y \leqslant 40$

b Cooking time $50x + 30y \leqslant 480$

so $5x + 3y \leqslant 48$

c



d $P = 3.5x + 1.5y$

e Integer solution required (6, 5)

f $P_{\text{max}} = £28.50$

2 a Storage: $0.08x + 0.08y \leqslant 6.4$

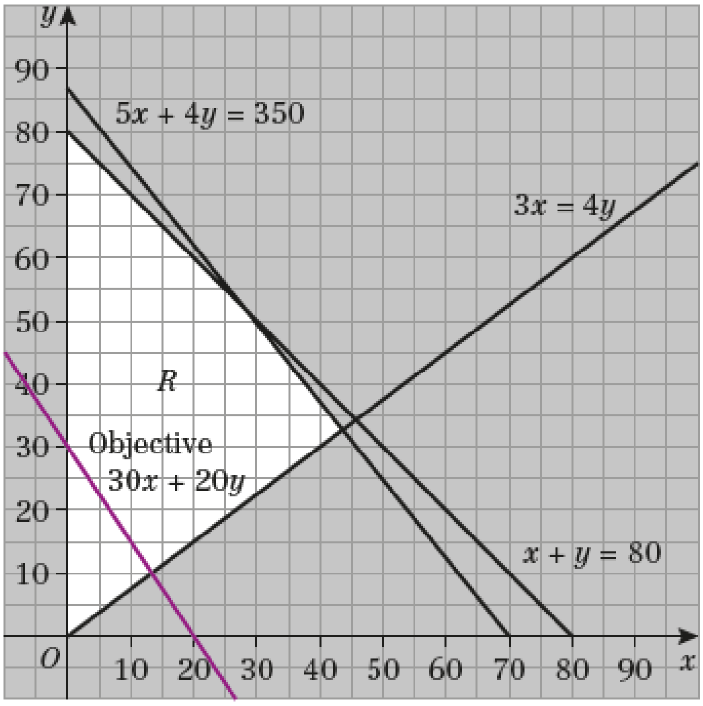
so $x + y \leqslant 80$

b Cost: $6x + 4.8y \leqslant 420$ so $5x + 4y \leqslant 350$

c Display $30x \leqslant 2 \times 20y$

$3x \leqslant 4y$

2 d



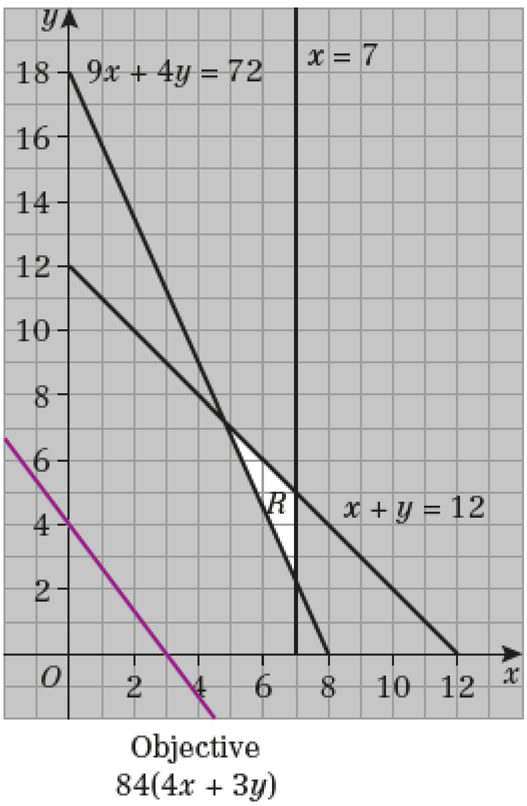
e Integer solution required (43, 33)
He should buy 43 CD storage units and 33 cassette storage units.

3 a i Total number of people $54x+24y \geqslant 408+24=432$ so $9x+4y \geqslant 72$

ii Number of adults is 24, at least 2 per coach, so $x+y \leqslant 12$

iii Number of large coaches, $x \leqslant 7$

3 b



3 c Minimise $C = 336x + 252y$
$= 84(4x + 3y)$

d Objective line passes through (0, 4) (3, 0)

e Integer coordinates needed (7, 3) so hire 7 large coaches and 3 small coaches
cost = £3108

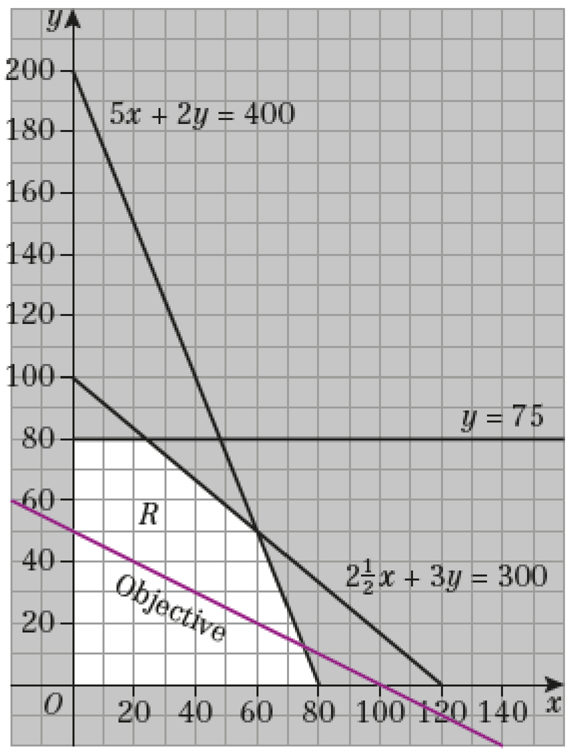
4 a $4x + 5y \leqslant 47$
$y \geqslant 2x - 8$
$4y - x - 8 \leqslant 0$
$x, y \geqslant 0$

b Solving simultaneous equations
$y = 2x - 8$
$4x + 5y = 47$
$\left(6\frac{3}{14}, 4\frac{3}{7}\right)$

c i For example where x and y
- types of car to be hired
- number of people, etc

ii (6, 4)

5



a $2\frac{1}{2}x + 3y \leqslant 300 \ (5x + 6y \leqslant 600)$
$5x + 2y \leqslant 400$
$2y \leqslant 150 \ (y \leqslant 75)$

5 b Maximise $P = 2x + 4y$

c (30, 75) $P = 360$

d The optimal point is at the intersection of $y = 75$ and $2\frac{1}{2}x + 3y = 300$
So the constraint $5x + 2y \leqslant 400$ is not at its limit.
At (30, 75) $5x + 2y = 300$ so 100 minutes are unused.

Challenge

a Either solve matrix equation

$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & -1 \\ 0 & 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 160 \\ 25 \\ 100 \end{pmatrix}$$

or simply manipulate the equations

$x + y + 2z = 160$ **(1)**

$x - z = 25$ **(2)**

$y + 2z = 100$ **(3)**

e.g.

(1) − (3) $\Rightarrow x + y + 2z - y - 2z = 160 - 100 \Rightarrow x = 60$

from **(2)** $\Rightarrow z = x - 25 = 35$

from **(3)** $\Rightarrow y = 100 - 2z = 30$

Thus the solution is $(60, 30, 35)$

b Constraints define 5 different planes in space. Each vertex of the feasible region lies at an intersections of 3 of them. We do not know a priori which intersections we should consider. However, we can hypothesise that the tetrahedron lies strictly in the region $x, y \geqslant 0$
Compute the vertices by considering simultaneous equations as in part **a**.

$x + y + 2z = 160, x - z = 25, y + 2z = 100 \Rightarrow x = 60, y = 30, z = 35$

$x - z = 25, y + 2z = 100, z = 15 \Rightarrow x = 40, y = 70, z = 15$

$x + y + 2z = 160, y + 2z = 100, z = 15 \Rightarrow x = 60, y = 70, z = 15$

$x + y + 2z = 160, x - z = 25, z = 15 \Rightarrow x = 40, y = 90, z = 15$

All of them lie strictly in the region $x, y \geqslant 0$ so our hypothesis is true.

Challenge

c We use the vertices testing method

Point	Value of *P*
$x = 60, y = 30, z = 35$	175
$x = 40, y = 70, z = 15$	275
$x = 60, y = 70, z = 15$	315
$x = 40, y = 90, z = 15$	335

Optimal value of P is 335, attained at point $(40, 90, 15)$