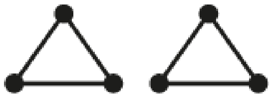
Route inspection Mixed exercise

1 a The graph is Eulerian as all vertices are even.

b The graph is neither as there are more than 2 odd nodes.

2 Any not connected graph with 6 even nodes, e.g.



If the graph is connected it will be Eulerian

3 a $3^{2x} - 700 + 3^{x+1} - 60 + 20 - x + x = 2 \times 35$

$\Rightarrow 3^{2x} + 3^{x+1} - 740 = 70$

$\Rightarrow \left(3^x\right)^3 + 3 \times 3^x - 810 = 0$

$\Rightarrow (3^x - 27)(3^x + 30) = 0$

$\Rightarrow 3^x = 27$

$\Rightarrow x = 3$

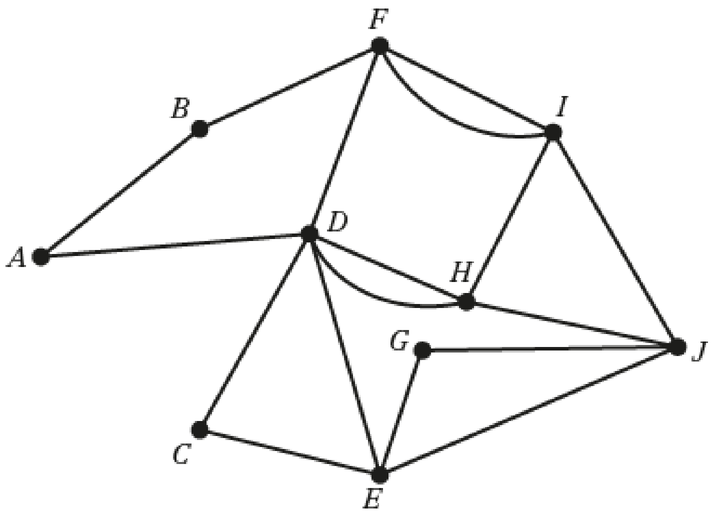
b The orders of the vertices are 29, 21, 17 and 3
The graph is neither Eulerian not semi-Eulerian since it has more than 2 odd vertices.

4 a

vertex	A	B	C	D	E	F	G	H	I	J
degree	2	2	2	5	4	3	2	3	3	4

b $DF + HI = 19 + 36 = 55$
$DH + FI = 22 + 27 = 49 \leftarrow$ least weight
$DI + HF = 46 + 41 = 87$
Repeat DH and FI
Add these to the network to get

Shortest routes
D to I is DFI
H to F is HDF



A possible route is
A – B – F – I – J – G – E – J – H – D – F – I – H – D – C – E – D – A

c $\text{length} = 725 + 49 = 774$

5 a The odd vertices are Q, R, T and V

$QR + TV = 104 + 189 = 293$

$QT + RV = 153 + 115 = 268$ ← Shortest route Q to T is QST

$QV + RT = 163 + 123 = 286$

The postman can repeat QT via S and RV so QS, ST and RV are repeated.

b The total length of the route is 1890 + 268 = 2176 m

c Only QV now needs to be repeated.

Total length = 1890 − 123 + 163 = 1930 m

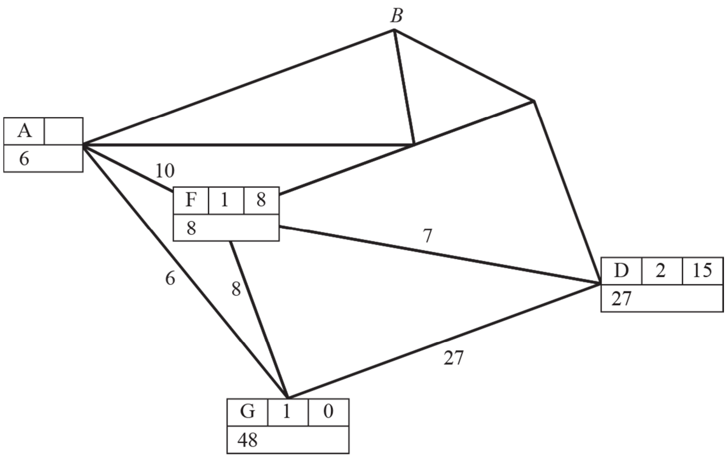
The route is now 246 m shorter.

6 a Minimum weight of A =6

Minimum weight of F =8

Minimum weight of E =13

So shortest route is GFD =15



b The odd vertices are G, B, C and D

$GB + CD = 16 + 3 = 19$ ← least weight

$GC + BD = 18 + 10 = 28$

$GD + BC = 15 + 7 = 22$

GA, AB and CD should be traversed twice.

Total length = 118 + 19 = 137 m

c GB and CD will not need to be repeated as they are now even

BD with weight 10 will be repeated.

So $x + 10 = 19 \Rightarrow x = 9$

7 a

vertex	A	B	C	D	E	F	G	H	I
degree	2	3	4	3	4	2	6	3	3

Odd valencies at B, D, H and I

7 b Considering all possible complete pairings and their weight

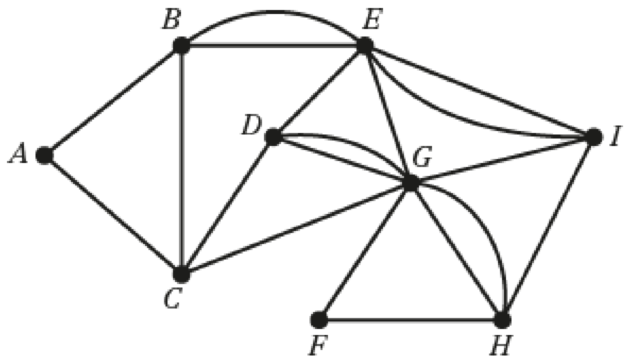
$BD + HI = 7.2 + 3.4 = 10.6$

$BH + DI = 7.6 + 4 = 11.6$

$BI + DH = 5.6 + 4.3 = 9.9 \leftarrow$ least weight

Repeat BE, EI and DG, GH.

Addings these arcs to the network gives



A possible route is:
A – B – E – I – H – G – I – E – B – C – D – G – D – E – G – H – F – G – C – A

c $\text{length} = 51.4 + 9.9 = 61.3\text{ km}$

d If BD is included B and D now have even valency.
Only H and I have odd valency.
So the shortest path from H to I needs to be repeated.

$$\text{Length of new route} = 51.4 + BD + \text{path from H to I}$$
$$= 51.4 + 6.4 + 3.4$$
$$= 61.2\text{ km}$$

This is (slightly) shorter than the previous route so choose to grit BD since it saves 0.1 km.

8 a Odd valencies B, C, E, H
Considering all possible complete pairings and their weight

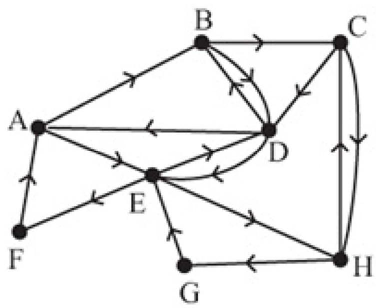
$BC + EH = 68 + 150 = 218$

$BE + CH = 95 + 73 = 168 \leftarrow$ least weight

$BH + CE = 141 + 85 = 226$

Shortest routes
BE is BDE
BH is BCH, CE is CDE

Repeat BD, DE and CH
Adding these arcs to the network gives



A possible route is:
A – B – D – B – C – H – C – D – E – D – A – E – H – G – E – F – A

$$\text{length} = 1011 + 168$$
$$= 1179\text{ m}$$

8 b This would make B the start and C the finish.
We would have to repeat the shortest path between E and H only.
New route $= 1011 + 150 = 1161\text{m}$

$1161 < 1179$
So this would decrease the total distance by 18 m.

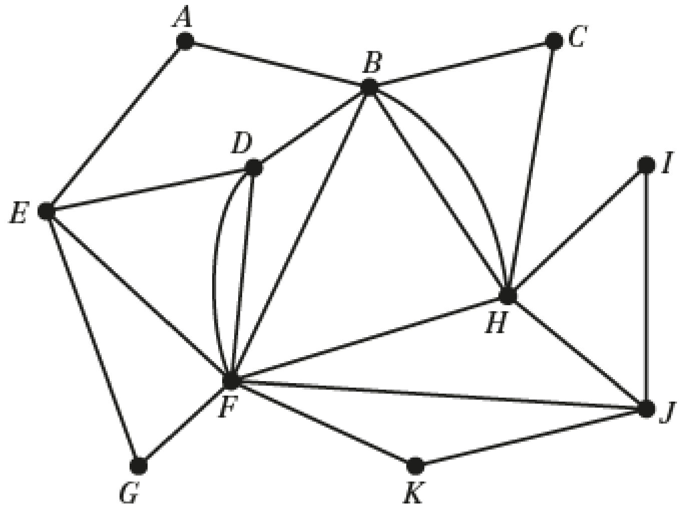
9 a The route inspection algorithm.

b Odd vertices B, D, F, H
Considering all complete pairings
$BD + FH = 14 + 15 = 29$ ← The shortest route DH is DBH.

$BF + DH = 10 + 26 = 36$

$BH + DF = 12 + 16 = 28 \leftarrow$ least weight

Repeat BH and DF
Adding these arcs to the network gives



A possible route is:
A – B – H – C – B – H – I – J – H – F – J – K – F – B – D – E – F – G – E – A

c length of route $= 249 + 28 = 277$

d i We will still have to repeat the shortest path between a pair of the odd nodes.
We will choose the pair that requires the shortest path.
The shortest path of the six is BF (10)
We will use D and H as the start and finish nodes.

ii $249 + 10 = 259$

e Each edge, having two ends, contributes two to the sum of valencies for the network.
Therefore the sum $= 2 \times$ number of edges
The sum is even so any odd valencies must occur in pairs.

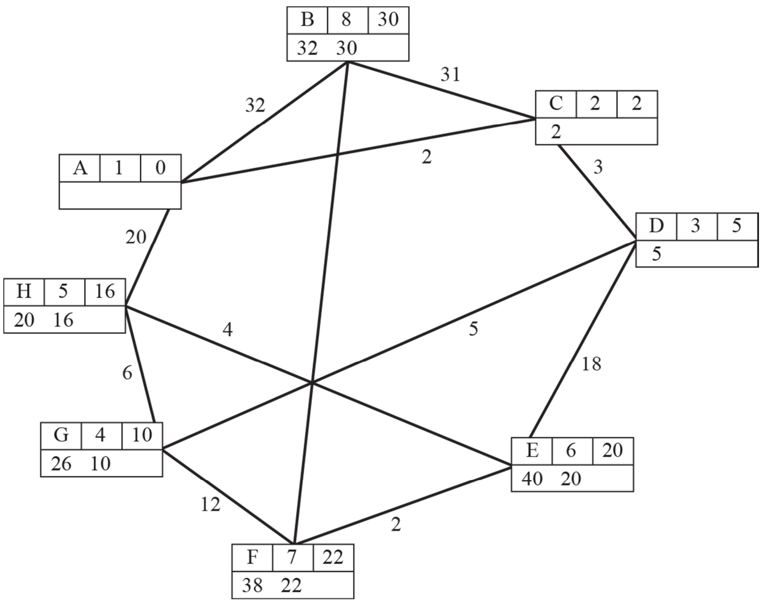
10 a Odd nodes are A, B, D, E, F and G
Starting at B so can leave as odd
Case (i): Land at D
$AE + FG = 19 + 10 = 29$
$AF + EG = 7 + 22 = 29$
$AG + EF = 6 + 12 = 18$ ← least weight
Case (ii) Land at F
$AD + EG = 26 + 22 = 48$
$AE + DG = 19 + 20 = 39$
$AG + ED = 6 + 14 = 20$
Better to use landing strip at D

b 168 + 18 = 186 miles

c Odd nodes unchanged.
Case (i): Land at D
$AE + FG = 40 + 13 = 53$
$AF + EG = 7 + 34 = 41$
$AG + EF = 6 + 47 = 53$
Case (ii) Land at F
$AD + EG = 26 + 34 = 60$
$AE + DG = 40 + 20 = 60$
$AG + ED = 6 + 14 = 20$ ← least weight
Now better to land at F
$168 - (10 + 25 + 12) + 20 = 141$ miles

Challenge

a Minimum weight of C is 2 (AC)
Minimum weight of D is 5 (ACD)
Minimum weight of G is 10 (ACDG)
Minimum weight of F is 22 (ACDGHEFB)
So shortest route from A to B is 30 with ACDGHEFB



b All vertices are odd so using the second answer in A, we repeat AC, DG, HE and FB
So minimum time = 143 +2+5+4+8 = 162 minutes.