Route inspection 4C

1 There are 6 odd nodes B, C, D, E, G and H.
B and G remain as odd nodes.
The minimum path lengths for the pairings are:
$CD + EH = 3 + 14 = 17$
$CE + DH = 4 + 13 = 17$
$CH + DE = 10 + 1 = 11 \leftarrow$ least weight
The edges that must be traversed twice are CH and DE.
Length of route = 96 + 11 = 107 km

2 a There are 6 odd nodes B, C, D, E, F and G.
Starting at C so always remains odd
Case (i): Finishing at E
The pairings are:
$BD + FG = 21 + 7 = 28$
$BF + DG = 16 + 19 = 35$
$BG + DF = 9 + 12 = 21 \leftarrow$ least weight
Case (ii): Finishing at G
The pairings are:
$BD + EF = 21 + 8 = 29$
$BE + DF = 24 + 12 = 36$
$BF + DE = 16 + 11 = 27$
The edges that must be traversed twice are BG and DF.

b The length of the route is
106 + 21 = 127 km

c Now the only odd nodes will be C, E, F and G. Finishing at E, FG would need to be added. Finishing at G, EF would need to be added. So since FG is shorter, it finishes at E.

$$106 + 7 + BD = 127 + \frac{1}{2}BD$$

$$\Rightarrow \frac{1}{2}BD = 14$$

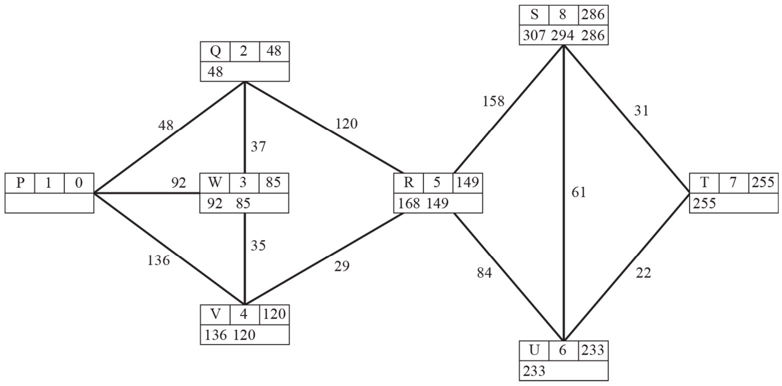
$$\Rightarrow BD = 28$$

3 a The odd nodes are A, B, C, D, E and F. But we already know EF is traversed twice. Also note that GA+GB< AB and GA+FG<FA so we use GA twice. That leaves GB and CD. So we use GB, CD and GA twice.

b 630 + 2220 = 2850 m

3 c Instead of 630+2220 as in **b**, we subtract BG from both numbers giving 570+1160 = 2730.

4 a Minimum weight of Q is 48
Minimum weight of W is 48+37 = 85
Minimum weight of V is 85+35=120
Minimum weight of R is 120+29 = 149
We definitely need to go via R
Minimum weight of U is 149+84 = 233
Minimum weight of T is 233+22 = 255
Minimum weight of S is 255+31 = 286
So shortest path is PQWVRUTS and length is 286m.



b Since the path represents the shortest route from P to S, passing through every other vertex along the way, the shortest route between any pair of vertices must be contained within it. In particular, the shortest route between any pair of odd vertices must be contained within it.

c The odd nodes are P, Q, V, W, U and S.
SU must be reused. That leaves either PQ+VW = 83 or QW+PV =173
So PQ, VW, UT and TS need to be used twice. UT + TS = 53, which is shorter than US (61). So the length of the shortest route is 853 + 48 + 35 + 22 + 31 = 989 m