Route inspection 4A

1 a

vertex	A	B	C	D	E	F
valency	2	3	2	3	3	3

There are 4 nodes with odd valency so the graph is *neither* Eulerian nor Semi-Eulerian.

b

vertex	G	H	I	J	K
valency	3	4	3	2	4

There are precisely 2 nodes of odd degree (G and I) so the graph is *semi-Eulerian.*
A possible route starting at G and finishing at I is:
G – H – K – I – J – K – G – H – I

c

vertex	L	M	N	P	Q	R
valency	2	4	2	4	2	4

All vertices have even valency, so the graph is *Eulerian.*
A possible route starting and finishing at L is:
L – M – N – P – M – R – P – Q – R – L

2 a i

vertex	A	B	C	D	E	F	G	H
valency	4	2	4	2	2	4	2	2

ii

vertex	A	B	C	D	E	F	G
valency	4	4	2	4	2	4	4

b i A possible route is: A – B – C – A – F – C – E – G – H – F – D – A
ii A possible route is: A – C – F – A – B – E – G – B – D – G – F – D – A

3 a i

vertex	R	S	T	U	V	W
valency	2	2	3	3	2	2

Precisely 2 vertices of odd valency (T and U) so semi-Eulerian.

ii

vertex	H	I	J	K	L	M	N
valency	2	4	3	2	3	4	4

Precisely 2 nodes of odd degree (J and L) so semi-Eulerian.

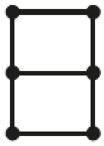
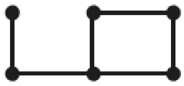
b i A possible route starting of T and finishing at U is:
T – R – S – U – W – V – T – U
ii A possible route starts at J and finishes at L:
J – K – L – M – J – I – M – N – I – H – N – L

4 a The number of odd nodes of any graph must be even so this is not possible as there are 3 odd nodes.

b i $2x+1+2x+4x-1+4x+6x=2E=18$
$\Rightarrow 18x=18$
$\Rightarrow x=1$

ii Semi-Eulerian since there are two odd nodes.

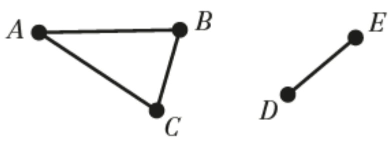
c Numerous possible answers e.g.:


or


5 a Not connected. There are no connections from *A*, *B* or *C* to *D* or *E*.

b Neither. To be Eulerian or semi-Eulerian the graph must be connected.

c



6 Adding up the numbers in each row, the orders of A, B, C, D, E are 2, 2, 2, 4, 4. Since they are all even the graph must be Eulerian.

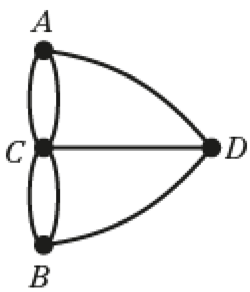
7 a n must be odd so that each vertex will have degree n - 1 which is even.

b

8 The example given in the question 1**a** is a counterexample. *ABEFCDA* is a Hamiltonian cycle, but the graph is not Eulerian.

Challenge

a

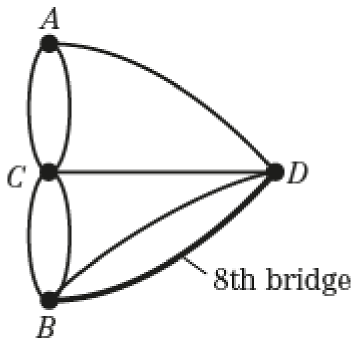


vertex	A	B	C	D
valency	3	3	5	3

There are more than two odd nodes, so the graph is *not* traversable.

Challenge

b

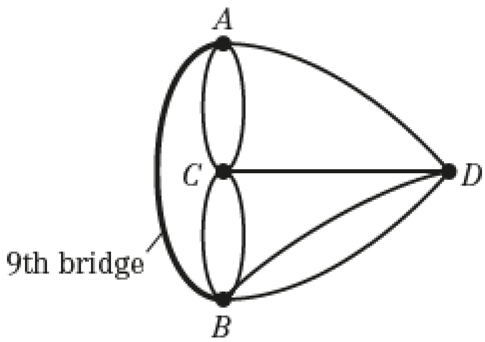


We will start at A and finish at C so these still need to have odd valency. We can only have two odd valencies so B and D must have even valencies (see table).

We need to change the valency of B and of D. So we build a bridge from B to D.

vertex	A	B	C	D
valency with 7 bridges	odd	*odd*	odd	*odd*
valency wanted	odd	*even*	odd	*even*

c

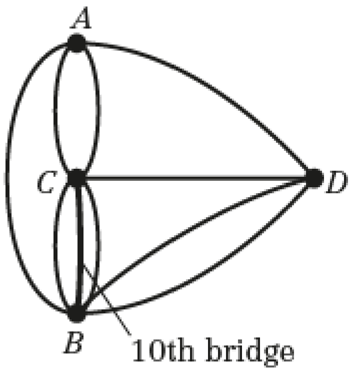


We will start at B and finish at C so these vertices need to be the two vertices with odd valency. We need A and D to have even valency (see table). We need to change the valency of node A and of node B. So we build a bridge from A to B.

vertex	A	B	C	D
valency with 8 bridges	*odd*	*even*	odd	even
valency wanted	*even*	*odd*	odd	even

Challenge

d



All vertices now need to have even valency.
This means we need to change the valencies of nodes B and C.
So the 10th bridge needs to be built from B to C.

vertex	A	B	C	D
valency with 9 bridges	even	*odd*	*odd*	even
valency wanted	even	*even*	*even*	even