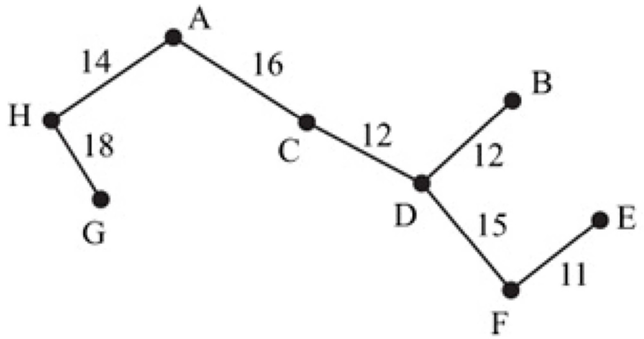
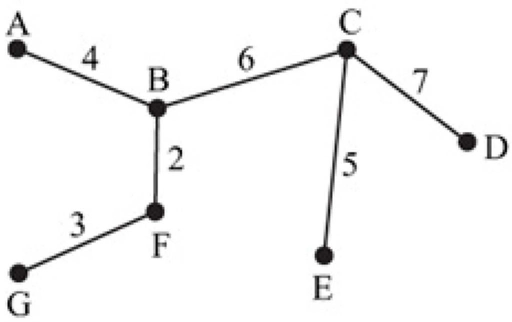
Algorithms on graphs 3B

1 **a** Arcs must be chosen in this order:
AH, AC, CD, BD, DF, FE, GH



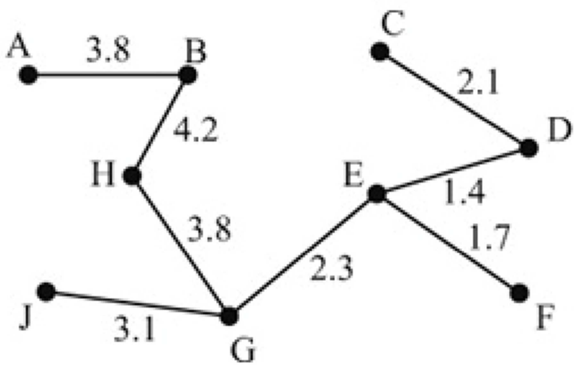
weight: 98

b Arcs must be chosen in this order:
AB, BF, FG, BC, CE, CD



weight: 27

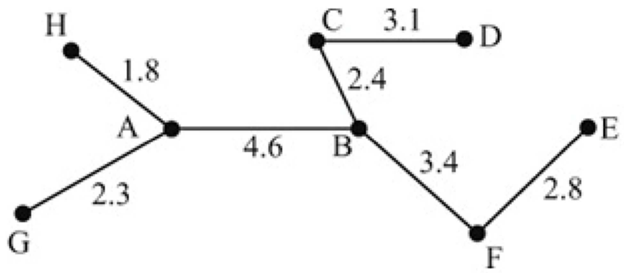
c Arcs must be chosen in this order:
AB, BH, GH, GE, DE, EF, CD, GJ



weight: 22.4

2 The first difference is that Prim's algorithm considers vertices and Kruskal's algorithm considers edges.
Another difference is that Prim's algorithm always grows in a connected fashion, whereas Kruskal's algorithm can be chaotic.

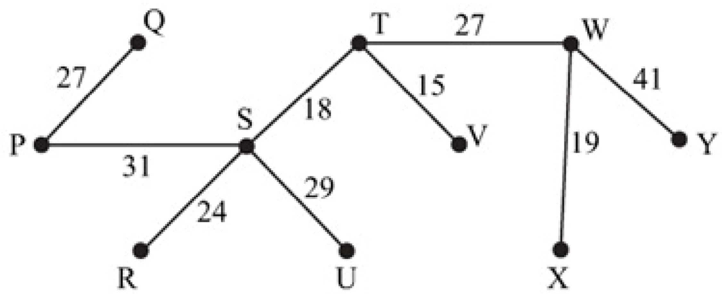
3 a Arcs must be chosen in this order:
AH, AG, AB, BC, CD, BF, FE



b $\text{cost} = 20.4 \times 850$
$= £17\,340$

4 a Arcs must be chosen in this order:
PQ, PS, ST, TV, RS, TW, WX, SU, WY

b

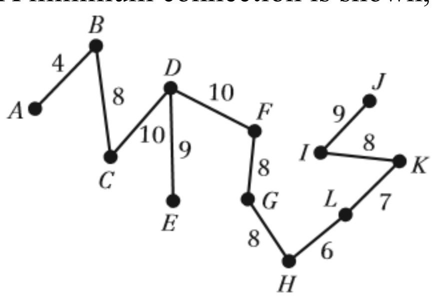


Cost: £231 000

c In the original situation vertex X gets connected to the tree before vertices U and Y but after all other vertices have already been added.
This means that at this stage we have to consider edges linking the network only to these three vertices: U, X and Y. These are:
SU 29, RU 37, TU 33, VU 34, WX 34, VX 28 and WY 41
Following Prim's algorithm, we pick VX and add 28 to the total weight of the spanning tree. We then add SU and WY, exactly the same as in the previous case.
So the only change in the spanning tree was adding VX with weight 28 instead of WX with weight 19. This means that the total weight of this network changed as follows:
£231,000 – £19,000 + £28,000 = £240,000

5 a Prim's algorithm identifies the next node to link to the existing tree. Linking a new node cannot form a cycle.

b A minimum connection is shown,



Order of connection $AB(4)$, $BC(8)$, $CD(10)$, $DE(9)$, $DF(10)$, $FG(8)$, $GH(8)$, $HL(6)$, $KL(7)$, $IK(8)$, $IJ(9)$. The total weight = 87

c This minimum spanning tree is not unique; there are three minimum connectors with total weight 87.