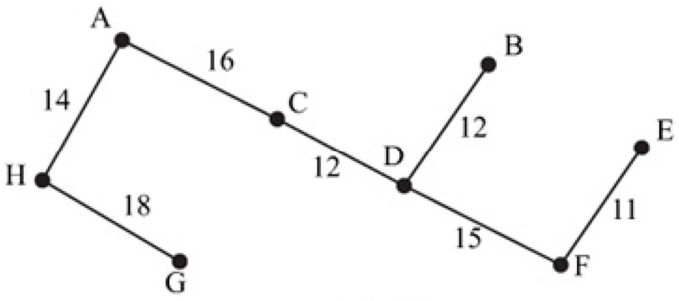
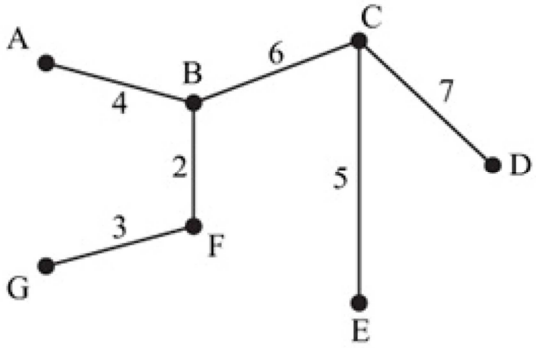
Algorithms on graphs 3A

1 a EF (11) add to tree
BD (12) add to tree
CD (12) add to tree
AH (14) add to tree
DF (15) add to tree
AC (16) add to tree
BC (17) reject
GH (18) add to tree
BE (18)
CH (20)
CG (21) reject all remaining arcs.
FG (24)
AB (25)



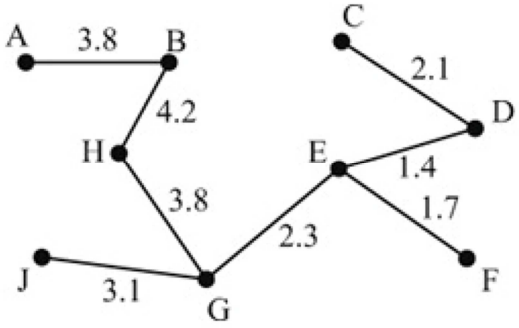
weight: 98

b BF (2) add to tree
FG (3) add to tree
AB (4) add to tree
BG (4) reject
AG (5) reject
CE (5) add to tree
BC (6) add to tree
EG (6) reject
AC (7) reject
CD (7) add to tree
EF (7)
DE (8) reject all remaining arcs.



weight: 27

c DE (1.4) add to tree
EF(1.7) add to tree
CD (2.1) add to tree
DF (2.1) reject
CE (2.2) reject
EG (2.3) add to tree
GJ (3.1) add to tree
FG (3.2) reject
AB (3.8) add to tree
GH (3.8) add to tree
GH (4.1) reject
HJ (4.1) reject
BH (4.2) add to tree
AH (4.3) reject

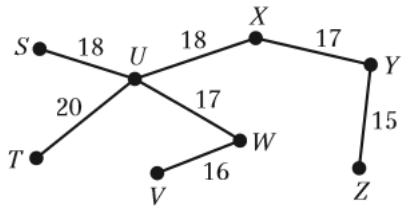


weight: 22.4

2 a i A tree is a connected graph with no cycles.
ii A minimum spanning tree is a tree of minimum total weight that connects all of the nodes.

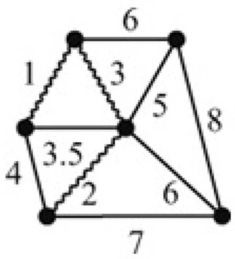
b By inspection the order of the arcs is
$\underline{YZ(15)}, \underline{VW(16)}, \underline{XY(17)}, \underline{UW(17)}, \underline{UX(18)}, WX(18), \underline{SU(18)}, WZ(18), UV(19), \underline{TU(20)}, ST(22), TV(23)$
Underlined arcs are in the minimum spanning tree. Total weight = 121

2 c



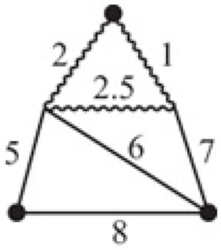
d This minimum spanning tree is not unique. For example, UX can be replaced with WX.

3 a For example;



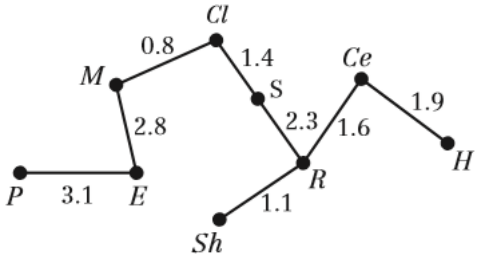
The three shortest edges form a cycle.

b For example;



The three shortest edges do not form a cycle.

4 a Add arcs in the order MCl (0.8), ShR (1.1), ClS(1.4), CeR(1.6), CeH(1.9), SR(2.3), ME(2.8), PE(3.1)



All vertices are connected so this is a minimum spanning tree.

b $0.8+1.1+1.4+1.6+1.9+2.3+2.8+3.1=15\text{ km}$