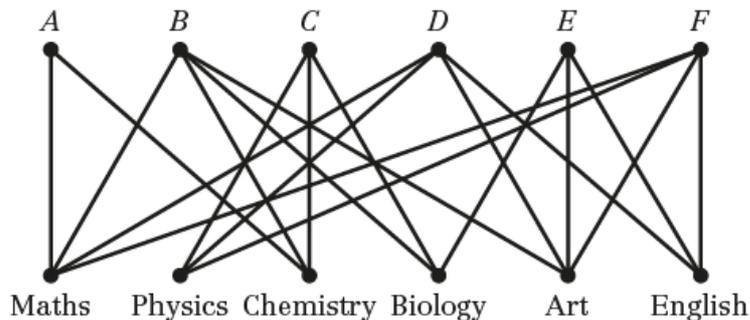


Graphs and networks 2A

- 1 a i a student
 ii friendship, i.e. that a pair of students are friends
- b Becky, Dhevan, Esma
- c The pairs which do not have mutual friends are: Esme and Govinda, Chris and Esme, Esme and Alex, Alex and Becky, Becky and Chris. Adding friendships: Alex and Chris, Esme and Fred solves the problem.

2 a



- b The subjects with the largest number of arcs (4) are Maths and Art, so these are the subjects studied by most students.
- 3 a i E.g. Marylebone to Oxford Circus to Victoria has one change at Oxford Circus.
 ii E.g. the same route as in part i passes through 3 stations and this is the minimum number.
- b i E.g. Kings Cross St Pancras - Green Park - Waterloo; 4 stops + change + 2 stops give 11 min.
 ii E.g. Holborn - Oxford Circus - Victoria - St James's Park; 2 stops + change + 2 stop + change + 1 stop give 12 min 40s.
 iii Victoria - Green Park - Baker Street; 1 stop + change + 2 stops give 7 min.
- c In our model the route via Oxford Circus: 3 stops + change + 4 stops give 12 min 20s, i.e. longer than the route via Green Park. E.g. it will take longer to change at some stations than others or some lines are faster than others.
- 4 a 40 min
- b Aberdeen - Cork (118min)
- c Dublin - it is the airport with most connections.
- 5 a *PTV*
- b The student is not correct *PTQRSV* is shorter (25km).

Challenge

- a** Minimum length is 3. We need to go once up, once to the right and once into page. However we can do it in arbitrary order, e.g. up - right - into page ($AEFG$) or into page - right - up ($ADCG$). Hence, by considering all permutations we get 6 different routes.
- b** First note that all routes must have odd length. Moreover we cannot use more than 7 arcs. Hence, all allowed routes have length 3, 5 or 7. There are 6 routes of length 3. We can use these to construct routes of length 5 or 7. For example we can extend $AEFG$ to $AEFBCG$ and to $AEFBCDHG$. We can realise that this way each route of length 3 gives rise to exactly one route of length 5 and exactly one route of length 7. Hence, we conclude that there are 18 allowed routes (6 of length 3, 6 of length 5 and 6 of length 7).