Hypothesis testing 4D

1 a $H_0 : p = 0.3 \quad H_1 : p < 0.3$

Assume H_0, so that $X \sim \text{Geo}(0.3)$

Significance level 5%

Require $P(X \geqslant c) < 0.05$

So $(1-0.3)^{c-1} < 0.05$

$(c-1)\log 0.7 < \log 0.05$

$c - 1 > \dfrac{\log 0.05}{\log 0.7}$

$c > 9.399$

So the critical region is $X \geqslant 10$

b Actual significance level $= P(X \geqslant 10) = (1-0.3)^9 = 0.0404$ (4 d.p.)

2 a $H_0 : p = 0.35 \quad H_1 : p < 0.35$

Assume H_0, so that $X \sim \text{Geo}(0.35)$

Significance level 5%

Require $P(X \geqslant c) < 0.05$

So $(1-0.35)^{c-1} < 0.05$

$(c-1)\log 0.65 < \log 0.05$

$c - 1 > \dfrac{\log 0.05}{\log 0.65}$

$c > 7.954$

So the critical region is $X \geqslant 8$

b Actual significance level $= P(X \geqslant 8) = (1-0.35)^7 = 0.0490$ (4 d.p.)

3 a $H_0 : p = 0.05 \quad H_1 : p > 0.05$

Assume H_0, so that $X \sim \text{Geo}(0.05)$

Significance level 10%

Require $P(X \geqslant c) < 0.1$

So $1-(1-0.05)^c < 0.1$

$0.95^c > 0.9$

$c\log 0.95 > \log 0.9$

$c < \dfrac{\log 0.9}{\log 0.95}$

$c < 2.054$

So the critical region is $X \leqslant 2$

b Actual significance level $= P(X \leqslant 2) = 1-(1-0.05)^2 = 0.0975$

4 a Let the random variable X denote the number of days Arun has to wait before winning a ticket.

$H_0 : p = 0.23 \quad H_1 : p < 0.23$

Assume H_0, so that $X \sim \text{Geo}(0.23)$

Significance level 5%

Require $P(X \geqslant c) < 0.05$

So $(1-0.23)^{c-1} < 0.05$

$(c-1)\log 0.77 < \log 0.05$

$c-1 > \dfrac{\log 0.05}{\log 0.77}$

$c > 12.462$

So the critical region is $X \geqslant 13$

b Probability of incorrectly rejecting $H_0 = P(X \geqslant 13) = (1-0.23)^{12} = 0.0434$ (4 d.p.)

c As $X = 11$ is not in the critical region, there is insufficient evidence to reject H_0.

5 a Let the random variable X denote the number of darts Dot throws until she hits a bullseye.

$H_0 : p = \frac{1}{3} \quad H_1 : p < \frac{1}{3}$

Assume H_0, so that $X \sim \text{Geo}\left(\frac{1}{3}\right)$

Significance level 5%

Require $P(X \geqslant c) < 0.05$

So $\left(\frac{2}{3}\right)^{c-1} < 0.05$

$(c-1)\log\left(\frac{2}{3}\right) < \log 0.05$

$c-1 > \dfrac{\log 0.05}{\log\left(\frac{2}{3}\right)}$

$c > 8.388$

So the critical region is $X \geqslant 9$

b Actual significance level $= P(X \geqslant 9) = \left(1-\frac{1}{3}\right)^8 = 0.0390$ (4 d.p.)

6 Let the random variable X denote the number of days before Rita next has a tremor.

$H_0 : p = 0.6 \quad H_1 : p < 0.6$

Assume H_0, so that $X \sim \text{Geo}(0.6)$

Significance level 5%

Require $P(X \geqslant c) < 0.05$

So $(1-0.6)^{c-1} < 0.05$

$(c-1)\log 0.4 < \log 0.05$

$c - 1 > \dfrac{\log 0.05}{\log 0.4}$

$c > 4.269$

So the critical region is $X \geqslant 5$

Challenge

a $H_0 : p = 0.009 \quad H_1 : p \neq 0.009$

Assume H_0, so that $X \sim \text{Geo}(0.009)$

Significance level 5%

If $X = c_1$ is the lower boundary of the upper critical region, require $P(X \geqslant c_1)$ to be as close as possible to 2.5%. First, find $P(X \geqslant a) < 0.025$

So $(1-0.009)^{a-1} < 0.025$

$a - 1 > \dfrac{\log 0.025}{\log 0.991} \Rightarrow a > 409.028$

So c_1 could be 409 or 410; $P(X \geqslant 409) = 0.025006$ and $P(X \geqslant 410) = 0.024781$, and as $P(X \geqslant 409)$ is closer to 0.025, the upper critical region is $X \geqslant 409$

If c_2 is the upper boundary of the lower critical region, require $P(X \leqslant c_2)$ to be as close as possible to 2.5%. First, find $P(X \leqslant b) < 0.025$

So $1-(1-0.009)^b < 0.025$

$0.991^b > 0.975$

$b < \dfrac{\log 0.975}{\log 0.991} \Rightarrow b < 2.8$

So c_2 could be 2 or 3; $P(X \leqslant 2) = 0.0179$ and $P(X \leqslant 3) = 0.02676$ and as $P(X \leqslant 3)$ is closer to 0.025, the lower critical region is $X \leqslant 3$

So the critical region is $X \leqslant 3$ or $X \geqslant 409$

b Probability of incorrectly rejecting $H_0 = P(X \leqslant 3) + P(X \geqslant 409)$

$= 0.02676 + 0.02500 = 0.0518$ (4 d.p.)

c As $X = 5$ is not in the critical region, there is insufficient evidence to reject H_0.