Hypothesis testing 4C

1 $X \sim \text{Geo}(p)$

$H_0 : p = 0.25,\ H_1 : p < 0.25$

Assume H_0, so that $X \sim \text{Geo}(0.25)$

Significance level 5%

$P(X \geqslant 9) = (1-0.25)^8 = 0.1001$ (4 d.p.)

$0.1001 > 0.05$

There is insufficient evidence to reject H_0.

2 $X \sim \text{Geo}(p)$

$H_0 : p = 0.6,\ H_1 : p < 0.6$

Assume H_0, so that $X \sim \text{Geo}(0.6)$

Significance level 5%

$P(X \geqslant 6) = (1-0.6)^5 = 0.0102$ (4 d.p.)

$0.0102 < 0.05$

This is significant, and there is sufficient evidence to reject H_0.

3 $X \sim \text{Geo}(p)$

$H_0 : p = 0.01,\ H_1 : p > 0.01$

Assume H_0, so that $X \sim \text{Geo}(0.01)$

Significance level 5%

$P(X \leqslant 3) = 1-(1-0.01)^3 = 0.0297$ (4 d.p.)

$0.0297 < 0.05$

This is significant, and there is sufficient evidence to reject H_0.

4 $X \sim \text{Geo}(p)$

$H_0 : p = 0.15,\ H_1 : p < 0.15$

Assume H_0, so that $X \sim \text{Geo}(0.15)$

Significance level 5%

$P(X \geqslant 18) = (1-0.15)^{17} = 0.0631$ (4 d.p.)

$0.0631 > 0.05$

There is insufficient evidence to reject H_0.

5 $X \sim \text{Geo}(p)$

$H_0 : p = 0.02,\ H_1 : p > 0.02$

Assume H_0, so that $X \sim \text{Geo}(0.02)$

Significance level 5%

$P(X \leqslant 2) = 1-(1-0.02)^2 = 0.0396$ (4 d.p.)

$0.0396 < 0.05$

This is significant, and there is sufficient evidence to reject H_0.

6 Let the random variable X denote the number of throws before getting a 6.

$H_0: p = \frac{1}{6}$, $H_1: p < \frac{1}{6}$

Assume H_0, so that $X \sim \text{Geo}\left(\frac{1}{6}\right)$

Significance level 5%

$P(X \geqslant 20) = \left(1 - \frac{1}{6}\right)^{19} = 0.0313$ (4 d.p.)

$0.0313 < 0.05$

There is sufficient evidence to reject H_0 and conclude that dice is biased and the probability of getting a 6 is less than $\frac{1}{6}$.

7 Let the random variable X denote the number of letters drawn before getting an A.

$H_0: p = \frac{1}{5}$, $H_1: p < \frac{1}{5}$

Assume H_0, so that $X \sim \text{Geo}(0.2)$

Significance level 5%

$P(X \geqslant 15) = (1 - 0.2)^{14} = 0.0440$ (4 d.p.)

$0.0440 < 0.05$

There is sufficient evidence to reject H_0 and conclude that the probability of getting an A is less than $\frac{1}{5}$.

8 **a** Use a geometric distribution model. Let the random variable X denote the number of free kicks taken by Lucy before she scores a goal, so $X \sim \text{Geo}(0.25)$.

b $P(X = 5) = 0.25(1 - 0.25)^4 = 0.0791$ (4 d.p.)

c $H_0: p = 0.25$ $H_1: p < 0.25$

Assume H_0, so that $X \sim \text{Geo}(0.25)$

Significance level 5%

$P(X \geqslant 10) = (1 - 0.25)^9 = 0.0751$ (4 d.p.)

$0.0751 > 0.05$

There is insufficient evidence to reject H_0 and no reason to conclude that the probability of Lucy scoring from a free kick is now less than $\frac{1}{4}$.

9 Let the random variable X denote the number of scratch cards bought getting a win.

$H_0: p = 0.25$, $H_1: p < 0.25$

Assume H_0, so that $X \sim \text{Geo}(0.25)$

Significance level 5%

$P(X \geqslant 12) = (1 - 0.25)^{11} = 0.0422$ (4 d.p.)

$0.0422 < 0.05$

There is sufficient evidence to reject H_0 and conclude that the student's suspicion is correct and the probability of winning is less than 1 in 4.

10 Let the random variable X denote the number of people questioned up to an including the first person to own a *Wisetalk* phone.

$H_0 : p = 0.22,\ H_1 : p < 0.22$

Assume H_0, so that $X \sim \text{Geo}(0.22)$

Significance level 5%

$P(X \geqslant 14) = (1-0.22)^{13} = 0.0396$ (4 d.p.)

$0.0396 < 0.05$

There is sufficient evidence to reject H_0 and conclude that *Wisetalk* are overstating the percentage.

11 Let the random variable X denote the number of penalties taken by Marie before she scores a goal.

$H_0 : p = 0.3,\ H_1 : p < 0.3$

Assume H_0, so that $X \sim \text{Geo}(0.3)$

Significance level 5%

$P(X \geqslant 10) = (1-0.3)^{9} = 0.0404$ (4 d.p.)

$0.0404 < 0.05$

There is sufficient evidence to reject H_0 and conclude that Marie is overstating her ability.

12 a Use a geometric distribution model, $X \sim \text{Geo}\left(\frac{1}{6}\right)$. For the model to be valid, the probability of seeing a robin on any day must be constant, and the probability of seeing a robin on one day is independent of seeing a robin on another day.

b i $P(X = 3) = \frac{1}{6}\left(1-\frac{1}{6}\right)^2 = 0.1157$ (4 d.p.)

ii $P(X > 4) = P(X \geqslant 5) = \left(1-\frac{1}{6}\right)^4 = 0.4823$ (4 d.p.)

c $H_0 : p = 0.25 \quad H_1 : p < 0.25$

Assume H_0, so that $X \sim \text{Geo}(0.25)$

Significance level 5%

$P(X \geqslant 12) = (1-0.25)^{11} = 0.0422$ (4 d.p.)

$0.0422 < 0.05$

There is sufficient evidence to reject H_0 and conclude that Imelda is overstating the probability of seeing a magpie on any given day.