Hypothesis testing 4A

1 $X \sim \text{Po}(\lambda)$

$\text{H}_0 : \lambda = 8 \quad \text{H}_1 : \lambda < 8$

Assume H_0, so that $X \sim \text{Po}(8)$

Significance level 5%, one-tailed test

From the tables $\text{P}(X \leqslant 3) = 0.0424$

$0.0424 < 0.05$

As the value $X = 3$ lies within the lowest 5% of the distribution, there is sufficient evidence to reject H_0.

2 $X \sim \text{Po}(\lambda)$

$\text{H}_0 : \lambda = 6.5 \quad \text{H}_1 : \lambda < 6.5$

Assume H_0, so that $X \sim \text{Po}(6.5)$

Significance level 5%, one-tailed test

From the tables $\text{P}(X \leqslant 2) = 0.0430$

$0.0430 < 0.05$

So there is sufficient evidence to reject H_0.

3 $X \sim \text{Po}(\lambda)$

$\text{H}_0 : \lambda = 5.5 \quad \text{H}_1 : \lambda > 5.5$

Assume H_0, so that $X \sim \text{Po}(5.5)$

Significance level 5%, one-tailed test

The observed value, 8, is greater than the mean so find $\text{P}(X \geqslant 8)$

From the tables $\text{P}(X \geqslant 8) = 1 - \text{P}(X \leqslant 7) = 1 - 0.8085 = 0.1915$

$0.1915 > 0.05$

There is insufficient evidence at the 5% level to reject H_0.

4 $X \sim \text{Po}(\lambda)$

$\text{H}_0 : \lambda = 5.5 \quad \text{H}_1 : \lambda \neq 5.5$

Assume H_0, so that $X \sim \text{Po}(5.5)$

Significance level 5%, this is a two-tailed test so the significance level in each tail is 2.5%

The observed value, 10, is greater than the mean so find $\text{P}(X \geqslant 10)$

From the tables $\text{P}(X \geqslant 10) = 1 - \text{P}(X \leqslant 9) = 1 - 0.9462 = 0.0538$

$0.0538 > 0.025$

There is insufficient evidence at the 5% level to reject H_0.

5 Let the random variable X denote the number of misprints found on a page of the paper.

$H_0: \lambda = 7.5 \quad H_1: \lambda > 7.5$

Assume H_0, so that $X \sim \text{Po}(7.5)$

Significance level 5%, one-tailed test

From the tables $P(X \geqslant 13) = 1 - P(X \leqslant 12) = 1 - 0.9574 = 0.0426$

$0.0426 < 0.05$

There is sufficient evidence to reject H_0, and conclude that the average number of misprints in the paper has increased.

6 Let the random variable X denote the rate of accidents that occur on the stretch of road per month.

$H_0: \lambda = 0.8 \quad H_1: \lambda > 0.8$

Assume H_0, so that $X \sim \text{Po}(0.8)$

Significance level 5%, one-tailed test

By calculator $P(X \geqslant 3) = 1 - P(X \leqslant 2) = 1 - 0.9526 = 0.0474$

$0.0474 < 0.05$

There is sufficient evidence to reject H_0, and conclude that the monthly rate of accidents on the stretch of road has increased.

7 Let the random variable X denote the number of times the coffee machine seizes up in a five-week period.

$H_0: \lambda = 5 \times 0.2 = 1 \quad H_1: \lambda > 1$

Assume H_0, so that $X \sim \text{Po}(1)$

Significance level 5%, one-tailed test

From the tables $P(X \geqslant 3) = 1 - P(X \leqslant 2) = 1 - 0.9197 = 0.0803$

$0.0803 > 0.05$

There is insufficient evidence at the 5% level to reject H_0 and there is therefore no evidence to suggest that the rate at which the coffee machine seizes up has increased.

8 Let the random variable X denote the number of houses sold in a four-week period.

$H_0: \lambda = 4 \times 2.25 = 9 \quad H_1: \lambda \neq 9$

Assume H_0, so that $X \sim \text{Po}(9)$

Significance level 5%, this is a two-tailed test so significance level in each tail is 2.5%

From the tables $P(X \leqslant 6) = 0.2068$

$0.2068 > 0.025$

There is insufficient evidence at the 5% level to reject H_0 and there is therefore no evidence to suggest that the rate of sales has changed.

9 Let the random variable X denote the number of accidents at the crossroads in a six-week period.

$H_0: \lambda = 6 \times 1.25 = 7.5 \quad H_1: \lambda < 7.5$

Assume H_0, so that $X \sim \text{Po}(7.5)$

Significance level 5%, one-tailed test

From the tables $P(X \leqslant 4) = 0.1321$

$0.1321 > 0.05$

There is insufficient evidence at the 5% level to reject H_0 and there is therefore no evidence to suggest that the accident rate at the crossroads has decreased.

10 Let the random variable X denote the number of flaws found in 150 m of cloth.

$H_0 : \lambda = 3 \times 2.3 = 6.9 \quad H_1 : \lambda \neq 6.9$

Assume H_0, so that $X \sim \text{Po}(6.9)$

Significance level 5%, this is a two-tailed test so significance level in each tail is 2.5%

By calculator $\text{P}(X \leqslant 3) = 0.0872$

$0.0872 > 0.025$

There is insufficient evidence at the 5% level to reject H_0 and there is therefore no evidence to suggest that the average number of flaws in the cloth has changed.

11 a Let the random variable X denote the number of vehicle breakdowns in a 20-day period, so $X \sim \text{Po}(20 \times 0.3)$, i.e. $X \sim \text{Po}(6)$

$$\text{P}(X = 5) = \frac{e^{-6} 6^5}{5!} = 0.1606 \text{ (4 d.p.)}$$

b From the tables $\text{P}(X \leqslant 8) = 0.8472$

c Let the random variable Y denote the number of vehicle breakdowns in a 30-day period.

$H_0 : \lambda = 30 \times 0.3 = 9 \quad H_1 : \lambda < 9$

Assume H_0, so that $Y \sim \text{Po}(9)$

Significance level 5%, one-tailed test

From the tables $\text{P}(Y \leqslant 5) = 0.1157$

$0.1157 > 0.05$

There is insufficient evidence at the 5% level to reject H_0 and there is therefore no evidence to suggest that the mean number of breakdowns has decreased.

12 Let the random variable X denote the number of patients with the particular condition seen by the doctor in a four-week period.

$H_0 : \lambda = 4 \times 2.25 = 9 \quad H_1 : \lambda < 9$

Assume H_0, so that $X \sim \text{Po}(9)$

Significance level 5%, one-tailed test

From the tables $\text{P}(X \leqslant 4) = 0.0550$

$0.0550 > 0.05$

There is insufficient evidence at the 5% level to reject H_0 and there is therefore no evidence to suggest a reduction in number of patients with the condition being seen by the doctor.

13 Let the random variable X denote the number of times the machine breaks down in a six-week period.

$H_0 : \lambda = 6 \times 1.5 = 9 \quad H_1 : \lambda \neq 9$

Assume H_0, so that $X \sim \text{Po}(9)$

Significance level 5%, this is a two-tailed test so significance level in each tail is 2.5%

From the tables $\text{P}(X \geqslant 13) = 1 - \text{P}(X \leqslant 12) = 1 - 0.8758 = 0.1242$

$0.1242 > 0.025$

There is insufficient evidence at the 5% level to reject H_0 and there is therefore no evidence to suggest that the rate of breakdowns has changed.

14 a Let the random variable X denote the number of defective components in a batch of 1000, so $X \sim B(1000, 0.01)$. Using a Poisson approximation $X \sim P(1000 \times 0.01)$, i.e. $X \sim P(10)$

i $P(X = 9) = \dfrac{e^{-10}10^9}{9!} = 0.1251$ (4 d.p.)

ii From the tables $P(X \leqslant 7) = 0.2202$

b The approximation is suitable because n (= 1000) is large and p (= 0.01) is small.

c Let the random variable X denote the number of defective components in a batch of 1000.

$H_0 : \lambda = 10 \quad H_1 : \lambda < 10$

Assume H_0, so that $X \sim Po(10)$

Significance level 5%, one-tailed test

From the tables $P(X \leqslant 5) = 0.0671$

$0.0671 > 0.05$

There is insufficient evidence at the 5% level to reject H_0 and there is therefore no evidence to suggest the servicing has reduced the number of defective components.