Geometric and negative binomial distributions 3A

1 a $P(X=10)=0.15(1-0.15)^9=0.0347$ (4 d.p.)

b $P(X \leqslant 7)=1-(1-0.15)^7=0.6794$ (4 d.p.)

c $P(3 \leqslant X \leqslant 12)=P(X \leqslant 12)-P(X \leqslant 2)$

$$=1-(1-0.15)^{12}-\left(1-(1-0.15)^2\right)$$

$$=0.8578-0.2775=0.5803 \text{ (4 d.p.)}$$

2 a $P(Y=6)=0.23(0.77)^5=0.0623$ (4 d.p.)

b $P(Y \geqslant 4)=0.77^3=0.4565$ (4 d.p.)

c $P(2<Y<8)=P(Y>2)-P(Y>7)$ Use $P(X>x)=(1-p)^x$

$$=0.77^2-0.77^7$$

$$=0.5929-0.1605=0.4324 \text{ (4 d.p.)}$$

3 a $P(X=4)=\left(\frac{5}{6}\right)^3\times\frac{1}{6}=\frac{125}{1296}=0.0965$ (4 d.p.)

b $P(X \leqslant 3)=1-\left(\frac{5}{6}\right)^3=\frac{91}{216}=0.4213$ (4 d.p.)

c $P(X \geqslant 5)=\left(\frac{5}{6}\right)^4=\frac{625}{1296}=0.4823$ (4 d.p.)

d $P(2 \leqslant X \leqslant 6)=P(X \leqslant 6)-P(X \leqslant 1)$

$$=\left(1-\left(\frac{5}{6}\right)^6\right)-\left(1-\left(\frac{5}{6}\right)\right)=0.6651-0.1667=0.4984 \text{ (4 d.p.)}$$

4 Let X denote the number of attempts needed to pass.

a i $P(X=3)=0.3(0.7)^2=0.147$

ii $P(X \geqslant 4)=0.7^3=0.343$

b The attempts are independent. The probability of passing remains the same on each attempt.

5 Let X denote the number of attempts needed to roll a 4.

a i $P(X=1)=\frac{1}{4}\left(\frac{3}{4}\right)^0=\frac{1}{4}=0.25$

ii $P(X=5)=\frac{1}{4}\left(\frac{3}{4}\right)^4=\frac{81}{1024}=0.0791$ (4 d.p.)

iii $P(X \leqslant 4)=1-\left(\frac{3}{4}\right)^4=\frac{175}{256}=0.6836$ (4 d.p.)

b The probability of rolling a 4 remains the same on each attempt.

6 a Let X denote the number of attempts needed complete the task.

$$P(X = x) = 0.45(0.55)^{x-1} = 0.136125$$
$$\Rightarrow 0.55^{x-1} = 0.3025$$
$$\Rightarrow x - 1 = \frac{\log 0.3025}{\log 0.55}$$
$$\Rightarrow x = 1 + 2 = 3$$

b $P(X \leqslant 4) = 1 - 0.55^4 = 0.9085$ (4 d.p.)

7 a $P(X = x) = 0.032(0.968)^{x-1} = 0.0203$

$$\Rightarrow 0.968^{x-1} = 0.634375$$
$$\Rightarrow x = 1 + \frac{\log 0.634375}{\log 0.968} = 14.9936$$

So $x = 15$

b $P(X \leqslant x) = 1 - 0.968^x < 0.1$

$$0.968^x > 0.9$$
$$\Rightarrow x < \frac{\log 0.9}{\log 0.968} = 3.2396$$

So $x = 3$

c $P(X \geqslant x) = 0.968^{x-1} < 0.05$

$$\Rightarrow x > 1 + \frac{\log 0.05}{\log 0.968} = 93.1106$$

So $x = 94$

8 Let X denote the number of calls up to and including the first report of a hardware problem.

a $P(X = 7) = 0.1(0.9)^6 = 0.0531$ (4 d.p.)

b $P(X > 5) = 0.9^5 = 0.5905$ (4 d.p.)

9 Let X denote the number of people asked up to and including the first person to like squid pizza.

a $P(X = 10) = 0.05(0.95)^9 = 0.0315$ (4 d.p.)

b $P(X \geqslant 15) = 0.95^{14} = 0.4877$ (4 d.p.)

10 Let X denote the number of games required to reach a decision.

a There are $2^3 = 8$ possible outcomes, since each player can either roll an even or an odd score. The only outcomes that wouldn't result in a decision are OOO (odd, odd, odd) and EEE (even, even, even). So the probability of a decision being made in any game is $\frac{8-2}{8} = \frac{3}{4}$

$$P(X = 3) = \frac{3}{4}\left(\frac{1}{4}\right)^2 = \frac{3}{64} = 0.0469 \text{ (4 d.p.)}$$

b $P(X \geqslant 4) = \left(\frac{1}{4}\right)^3 = \frac{1}{64} = 0.0156$ (4 d.p.)

11 a Use this model $X \sim \text{Po}(4)$, and find the required value from the tables in the textbook

$$\begin{aligned}P(X \geqslant 5) &= 1 - P(X \leqslant 4)\\ &= 1 - 0.6288 = 0.3712\end{aligned}$$

b Let Y denote the number of hours that pass before at least 5 customers have come into the pharmacy in an hour. So from part **a**, $Y \sim \text{Geo}(0.3712)$

$P(Y = 5) = 0.3712(0.6288)^4 = 0.0580$ (4 d.p.)

c $P(Y > 8) = 0.6288^8 = 0.0244$ (4 d.p.)