## Poisson distributions 2A

**1 a** $P(X=3)=\dfrac{e^{-2.5}\times 2.5^3}{3!}$

$=0.213763... = 0.2138$ (4 d.p.)

**b** $P(X>1)=1-P(X\leqslant 1)=1-P(X=0)-P(X=1)$

$=1-\dfrac{e^{-2.5}\times 2.5^0}{0!}-\dfrac{e^{-2.5}\times 2.5^1}{1!}$

$=1-0.08208\ldots-0.20521\ldots=0.7127$ (4 d.p.)

**c** $P(1<X\leqslant 3)=P(X=2)+P(X=3)$

$=\dfrac{e^{-2.5}\times 2.5^2}{2!}+\dfrac{e^{-2.5}\times 2.5^3}{3!}$

$=0.25651\ldots+0.21376\ldots=0.4703$ (4 d.p.)

**2 a** $P(X=4)=\dfrac{e^{-3.1}\times 3.1^4}{4!}$

$=0.173347... = 0.1733$ (4 d.p.)

**b** $P(X\geqslant 2)=1-P(X<2)=1-P(X=0)-P(X=1)$

$=1-\dfrac{e^{-3.1}\times 3.1^0}{0!}-\dfrac{e^{-3.1}\times 3.1^1}{1!}$

$=1-0.045049\ldots-0.139652\ldots=0.8153$ (4 d.p.)

**c** $P(1\leqslant X\leqslant 4)=P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$=e^{-3.1}\left(\dfrac{3.1^1}{1!}+\dfrac{3.1^2}{2!}+\dfrac{3.1^3}{3!}+\dfrac{3.1^4}{4!}\right)$

$=0.045049...\times(3.1+4.805+4.96516...+3.84800...)=0.7531$ (4 d.p.)

**3 a** $P(X=2)=\dfrac{e^{-4.2}\times 4.2^2}{2!}$

$=0.13226... = 0.1323$ (4 d.p.)

**b** $P(X\leqslant 3)=P(X=0)+P(X=1)+P(X=2)+P(X=3)$

$=e^{-4.2}\left(\dfrac{4.2^0}{0!}+\dfrac{4.2^1}{1!}+\dfrac{4.2^2}{2!}+\dfrac{4.2^3}{3!}\right)$

$=0.0149955...\times(1+4.2+8.82+12.384)=0.3954$ (4 d.p.)

**c** $P(3\leqslant X\leqslant 5)=P(X=3)+P(X=4)+P(X=5)$

$=e^{-4.2}\left(\dfrac{4.2^3}{3!}+\dfrac{4.2^4}{4!}+\dfrac{4.2^5}{5!}\right)$

$=0.0149955...\times(12.384+12.9654+10.8909...)=0.5429$ (4 d.p.)

**4 a** $P(X=1)=\dfrac{e^{-0.84}\times 0.84^1}{1!}$

$=0.362638\ldots=0.3626$ (4 d.p.)

**b** $P(X \geqslant 1)=1-P(X<1)=1-P(X=0)$

$=1-\dfrac{e^{-0.84}\times 0.84^0}{0!}$

$=1-0.431710\ldots=0.5683$ (4 d.p.)

**c** $P(1<X \leqslant 3)=P(X=2)+P(X=3)$

$=e^{-0.84}\left(\dfrac{0.84^2}{2!}+\dfrac{0.84^3}{3!}\right)$

$=0.43171\ldots\times(0.3528+0.098784)=0.1950$ (4 d.p.)

**5** $P(X=2)=e^{-\lambda}\dfrac{\lambda^2}{2!}$ and $P(X=3)=e^{-\lambda}\dfrac{\lambda^3}{3!}$

If $P(X=2)=P(X=3)$ then $\dfrac{\lambda^2}{2!}=\dfrac{\lambda^3}{3!}$ so $\lambda=3$

**6** $P(X=4)=e^{-\lambda}\dfrac{\lambda^4}{4!}$ and $P(X=2)=e^{-\lambda}\dfrac{\lambda^2}{2!}$

If $P(X=4)=3\times P(X=2)$ then $\dfrac{\lambda^4}{4!}=3\times\dfrac{\lambda^2}{2!}$ so $\lambda^2=36$ and therefore $\lambda=6$

Reject the negative root because the Poisson parameter must be positive.