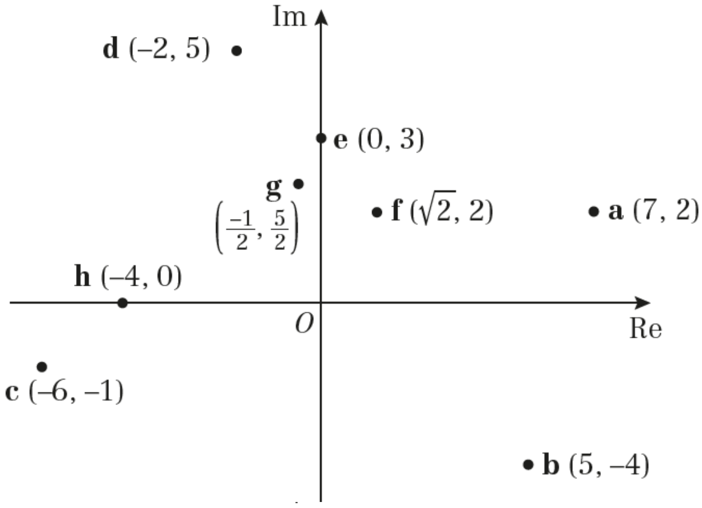
Argand diagrams 2A

1



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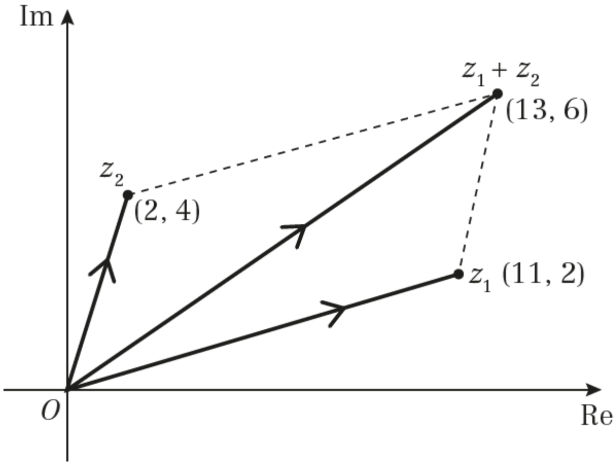

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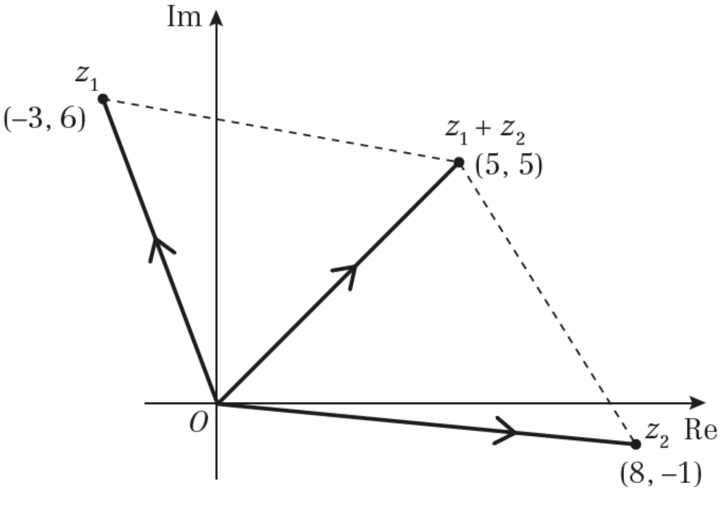

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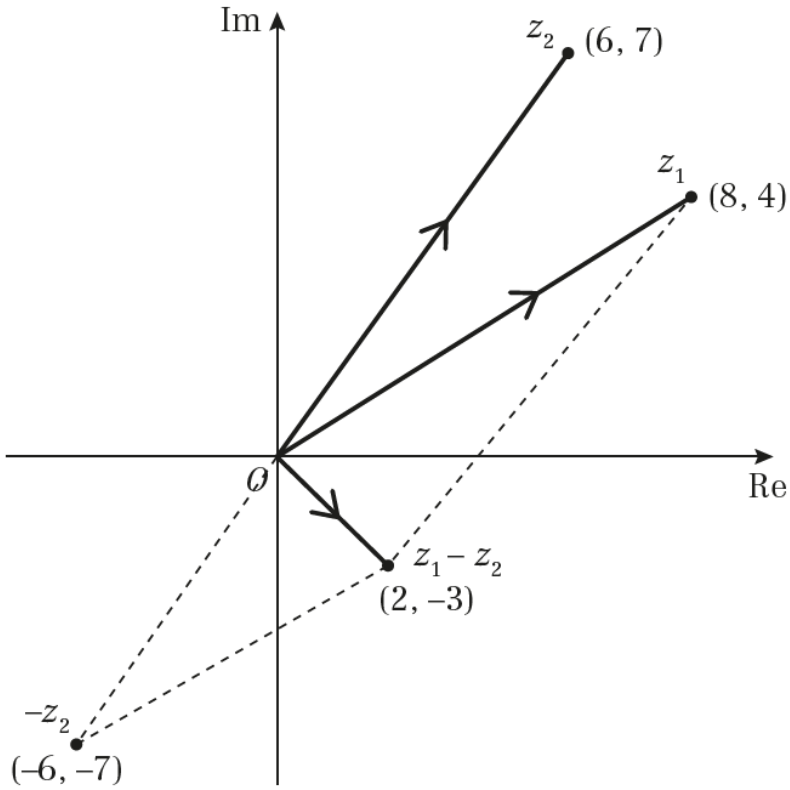

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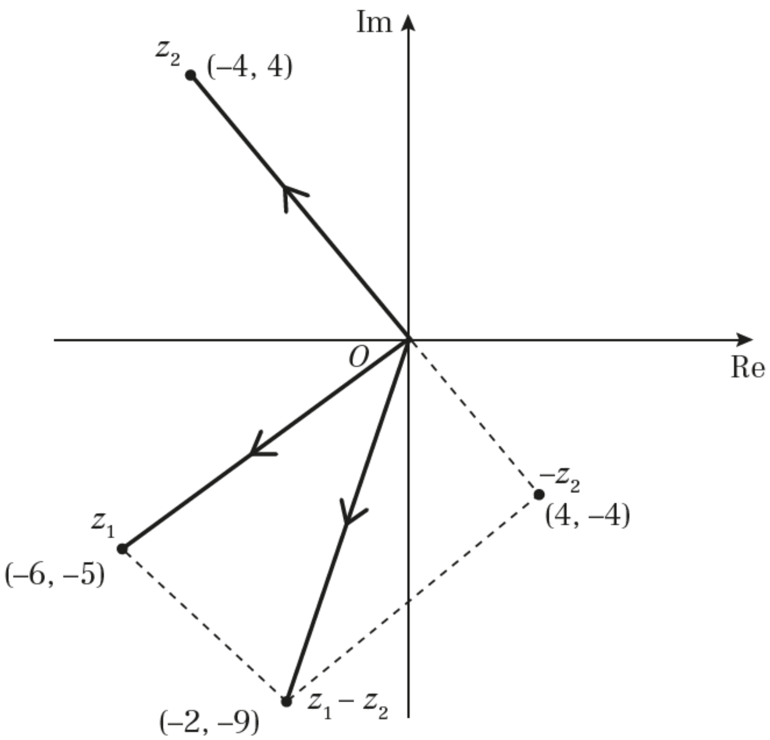


6 a
$$z_3 = z_1 + z_2$$
$$-3+2\text{i} = (7-5\text{i})+(a+b\text{i})$$
$$= (7+a)+(-5+b)\text{i}$$

Equate real coefficients: $-3 = 7+a$

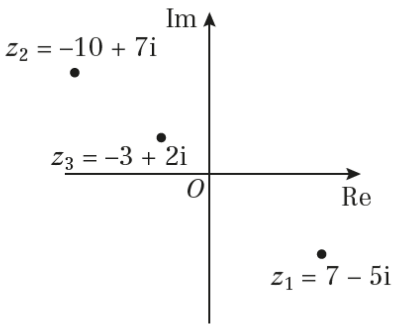
$a = -10$

Equate imaginary coefficients: $2 = -5+b$

$b = 7$

Hence $z_2 = -10+7\text{i}$

b



7 a
$$z_3 = z_1 + z_2$$
$$-8+5\text{i} = (p+q\text{i})+(9-5\text{i})$$

Equate real coefficients: $-8 = p+9$

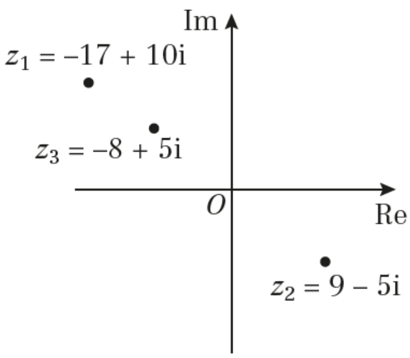
$p = -17$

Equate imaginary coefficients: $5 = -5+q$

$q = 10$

$z_1 = -17+10\text{i}$

b



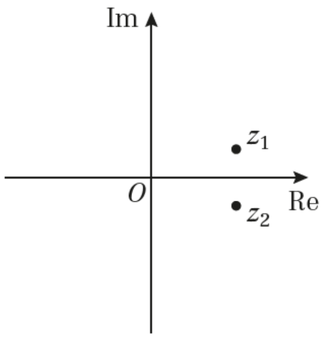
8 a $z^2 - 6z + 10 = 0$

Solve by completing the square:

$$(z-3)^2 - 9 + 10 = 0$$
$$(z-3)^2 = -1$$
$$z - 3 = \pm\mathrm{i}$$
$$z = 3 \pm \mathrm{i}$$

So $z_1 = 3 + \mathrm{i}$ and $z_2 = 3 - \mathrm{i}$.

b



9 a $\mathrm{f}(z) = 2z^3 - 19z^2 + 64z - 60$

$$\begin{aligned}\mathrm{f}\left(\frac{3}{2}\right) &= 2\left(\frac{3}{2}\right)^3 - 19\left(\frac{3}{2}\right)^2 + 64\left(\frac{3}{2}\right) - 60\\ &= 2\left(\frac{27}{8}\right) - 19\left(\frac{9}{4}\right) + 64\left(\frac{3}{2}\right) - 60\\ &= \frac{54}{8} - \frac{171}{4} + \frac{192}{2} - 60\\ &= \frac{27}{4} - \frac{171}{4} + \frac{384}{4} - \frac{240}{4}\\ &= 0\end{aligned}$$

b If $\mathrm{f}\left(\frac{3}{2}\right) = 0$, then $2z - 3$ is a factor of $\mathrm{f}(z)$.

Use division to find the other factors:

$$\begin{array}{r} z^2 - 8z + 20 \\ 2z-3 \overline{\big) 2z^3 - 19z^2 + 64z - 60} \\ 2z^3 - \ \ 3z^2 \qquad\qquad\quad \\ -16z^2 + \ 64z \qquad\ \ \\ -16z^2 + \ 24z \qquad\ \ \\ 40z - 60 \\ 40z - 60 \\ 0 \end{array}$$

So $2z^3 - 19z^2 + 64z - 60 = (2z - 3)(z^2 - 8z + 20)$

9 b Either $2z-3=0 \Rightarrow z=\frac{3}{2}$

or $z^2-8z+20=0$

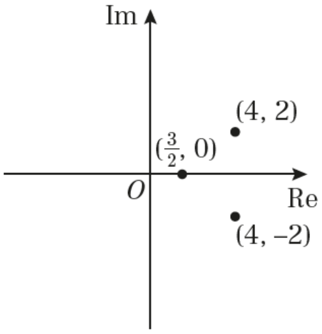
Solve by completing the square:

$$(z-4)^2-16+20=0$$
$$(z-4)^2=-4$$
$$z-4=\pm 2\text{i}$$
$$z=4\pm 2\text{i}$$

So the roots of $\text{f}(z)=0$

are $\frac{3}{2}$, $4+2\text{i}$ and $4-2\text{i}$

c



Challenge

a

$$z^6=1$$
$$z^6-1=0$$
$$(z^3-1)(z^3+1)=0 \quad (*)$$

Let $\text{f}(z)=z^3-1$.

Since $\text{f}(1)=0$, then $z-1$ is a factor of $\text{f}(z)$.

Hence $\text{f}(z)=(z-1)(z^2+bz+c)$

Equate coefficients of z^2: $0=b-1$

$b=1$

Equate constants: $-1=-c$

$c=1$

So $\text{f}(z)=(z-1)(z^2+z+1)$

Let $\text{g}(z)=z^3+1$.

Since $\text{g}(-1)=0$, then $z+1$ is a factor of $\text{g}(z)$

Hence $\text{g}(z)=(z+1)(z^2+pz+q)$

Equate coefficients of z^2: $0=1+p$

$p=-1$

Equate constants: $q=1$

Challenge

a So $g(z)=(z+1)(z^2-z+1)$

By (*), $0=f(z)g(z)$

$$0=(z-1)(z^2+z+1)(z+1)(z^2-z+1)$$

Either $z-1=0 \Rightarrow z=1$

or

$$z^2+z+1=0$$
$$\left(z+\frac{1}{2}\right)^2-\frac{1}{4}+1=0$$
$$\left(z+\frac{1}{2}\right)^2=-\frac{3}{4}$$
$$z+\frac{1}{2}=\pm\frac{\sqrt{3}}{2}\text{i}$$
$$z=-\frac{1}{2}\pm\frac{\sqrt{3}}{2}\text{i}$$

or $z+1=0 \Rightarrow z=-1$

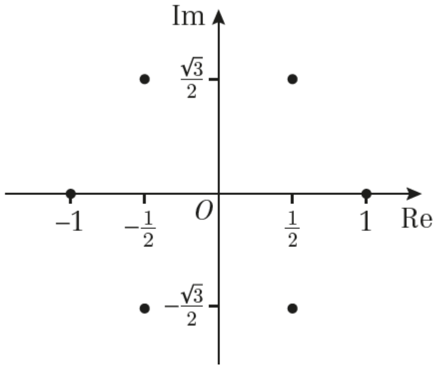
or

$$z^2-z+1=0$$
$$\left(z-\frac{1}{2}\right)^2-\frac{1}{4}+1=0$$
$$\left(z-\frac{1}{2}\right)^2=-\frac{3}{4}$$
$$z-\frac{1}{2}=\pm\frac{\sqrt{3}}{2}\text{i}$$
$$z=\frac{1}{2}\pm\frac{\sqrt{3}}{2}\text{i}$$

So the roots of $z^6=1$

are -1, 1, $-\frac{1}{2}+\frac{\sqrt{3}}{2}\text{i}$, $-\frac{1}{2}-\frac{\sqrt{3}}{2}\text{i}$, $\frac{1}{2}+\frac{\sqrt{3}}{2}\text{i}$ and $\frac{1}{2}-\frac{\sqrt{3}}{2}\text{i}$

b



Challenge

c $(0,1)$ and $(0,-1)$ are on the unit circle.

Use Pythagoras' Theorem to check $\pm\frac{1}{2}\pm\frac{\sqrt{3}}{2}\mathrm{i}$ also lie on a circle with centre $(0,0)$ and radius 1.

$\left(\pm\frac{1}{2}\right)^2+\left(\pm\frac{\sqrt{3}}{2}\right)^2=\frac{1}{4}+\frac{3}{4}=1$

So all points lie on a circle with centre $(0,0)$ and radius 1.