Complex numbers 1B

1 a
$$\begin{aligned} z^2+121&=0\\ z^2&=-121\\ z&=\pm\sqrt{-121}\\ &=\pm\sqrt{121}\sqrt{-1}\\ &=\pm 11\text{i} \end{aligned}$$

b
$$\begin{aligned} z^2+40&=0\\ z^2&=-40\\ z&=\pm\sqrt{-40}\\ &=\pm\sqrt{4}\sqrt{10}\sqrt{-1}\\ &=\pm\sqrt{4}\sqrt{10}\text{i}\\ &=\pm 2\text{i}\sqrt{10} \end{aligned}$$

c
$$\begin{aligned} 2z^2+120&=0\\ 2z^2&=-120\\ z^2&=-60\\ z&=\pm\sqrt{-60}\\ &=\pm\sqrt{4}\sqrt{15}\sqrt{-1}\\ &=\pm 2\text{i}\sqrt{15} \end{aligned}$$

d
$$\begin{aligned} 3z^2+150&=38-z^2\\ 4z^2&=-112\\ z^2&=-28\\ z&=\pm\sqrt{-28}\\ &=\pm\sqrt{4}\sqrt{7}\sqrt{-1}\\ &=\pm 2\text{i}\sqrt{7} \end{aligned}$$

e
$$\begin{aligned} z^2+30&=-3z^2-66\\ 4z^2&=-96\\ z^2&=-24\\ z&=\pm\sqrt{-24}\\ &=\pm\sqrt{4}\sqrt{6}\sqrt{-1}\\ &=\pm 2\text{i}\sqrt{6} \end{aligned}$$

1 f
$$\begin{aligned} 6z^2+1&=2z^2\\ 4z^2&=-1\\ z^2&=-\frac{1}{4}\\ z&=\pm\sqrt{\frac{-1}{4}}\\ &=\pm\sqrt{\frac{1}{4}}\sqrt{-1}\\ &=\pm\frac{1}{2}\text{i} \end{aligned}$$

2 a
$$\begin{aligned} (z-3)^2-9&=-16\\ (z-3)^2&=-7\\ z-3&=\pm\text{i}\sqrt{7}\\ z&=3\pm\text{i}\sqrt{7} \end{aligned}$$

b
$$\begin{aligned} 2(z-7)^2+30&=6\\ 2(z-7)^2&=-24\\ (z-7)^2&=-12\\ z-7&=\pm 2\text{i}\sqrt{3}\\ z&=7\pm 2\text{i}\sqrt{3} \end{aligned}$$

c
$$\begin{aligned} 16(z+1)^2+11&=2\\ 16(z+1)^2&=-9\\ (z+1)^2&=-\frac{9}{16}\\ z+1&=\pm\frac{3}{4}\text{i}\\ z&=-1\pm\frac{3}{4}\text{i} \end{aligned}$$

3 [Note that **3a**, **3b** & **3c** use the quadratic formula, and **3d**, **3e** & **3f** use completion of the square]

a $a=1,\ b=2,\ c=5$

$$z=\frac{-2\pm\sqrt{(4-20)}}{2}=\frac{-2\pm 4\mathrm{i}}{2}=-1\pm 2\mathrm{i}$$

b $a=1,\ b=-2,\ c=10$

$$z=\frac{2\pm\sqrt{(4-40)}}{2}=\frac{2\pm 6\mathrm{i}}{2}=1\pm 3\mathrm{i}$$

c $a=1,\ b=4,\ c=29$

$$z=\frac{-4\pm\sqrt{(16-116)}}{2}=\frac{-4\pm 10\mathrm{i}}{2}=-2\pm 5\mathrm{i}$$

d

$$z^2+10z+26=0$$
$$(z+5)^2-25+26=0$$
$$(z+5)^2=-1$$
$$z+5=\pm\mathrm{i}$$
$$z=-5\pm\mathrm{i}$$

e

$$z^2+5z+25=0$$
$$\left(z+\frac{5}{2}\right)^2-\frac{25}{4}+25=0$$
$$\left(z+\frac{5}{2}\right)^2=-\frac{75}{4}$$
$$z+\frac{5}{2}=\pm\frac{\mathrm{i}\sqrt{25\times 3}}{\sqrt{4}}$$
$$z=-\frac{5}{2}\pm\frac{5\mathrm{i}\sqrt{3}}{2}$$

3 f

$$z^2+3z+5=0$$
$$\left(z+\frac{3}{2}\right)^2-\frac{9}{4}+5=0$$
$$\left(z+\frac{3}{2}\right)^2=-\frac{11}{4}$$
$$z+\frac{3}{2}=\pm\frac{\mathrm{i}\sqrt{11}}{2}$$
$$z=-\frac{3}{2}\pm\frac{\mathrm{i}\sqrt{11}}{2}$$

4 [Note that **4a** uses completion of the square and **4b** & **4c** use the quadratic formula]

a

$$2z^2+5z+4=0$$
$$z^2+\frac{5}{2}z+2=0$$
$$\left(z+\frac{5}{4}\right)^2-\frac{25}{16}+2=0$$
$$\left(z+\frac{5}{4}\right)^2=-\frac{7}{16}$$
$$z+\frac{5}{4}=\pm\frac{\mathrm{i}\sqrt{7}}{4}$$
$$z=-\frac{5}{4}\pm\frac{\mathrm{i}\sqrt{7}}{4}$$

b Use the quadratic formula with $a=7$, $b=-3$ and $c=3$

$$z=\frac{-(-3)\pm\sqrt{(-3)^2-(4)(7)(3)}}{2(7)}=\frac{3\pm\sqrt{9-84}}{14}=\frac{3\pm\sqrt{-75}}{14}=\frac{3}{14}\pm\frac{5\sqrt{3}}{14}\mathrm{i}$$

4 c Use the quadratic formula with
$a = 5$, $b = -1$ and $c = 3$

$$z = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(5)(3)}}{2(5)}$$
$$= \frac{1 \pm \sqrt{1-60}}{10}$$
$$= \frac{1}{10} \pm \frac{\sqrt{59}}{10}\text{i}$$

5 Method 1: Completing the square

$$(z-4)^2 - 16 + 21 = 0$$
$$(z-4)^2 = -5$$
$$z - 4 = \pm\text{i}\sqrt{5}$$
$$z = 4 \pm \text{i}\sqrt{5}$$

So $z_1 = 4 + \text{i}\sqrt{5}$ and $z_2 = 4 - \text{i}\sqrt{5}$

Method 2: using the quadratic formula

$$z = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(21)}}{2(1)}$$
$$= \frac{8 \pm \sqrt{-20}}{2}$$
$$= \frac{8 \pm \text{i}\sqrt{4}\sqrt{5}}{2}$$
$$= 4 \pm \text{i}\sqrt{5}$$

6 $z^2 + bz + 11 = 0$ has two distinct complex roots when

$$b^2 - 4(1)(11) < 0$$
$$b^2 < 44$$
$$-\sqrt{44} < b < \sqrt{44}$$
or $-2\sqrt{11} < b < 2\sqrt{11}$