Further kinematics Mixed exercise 8

1 $\mathbf{u} = 0,\ t = 5,\ \mathbf{v} = 6\mathbf{i} - 8\mathbf{j},\ \mathbf{a} = ?$

Using $\mathbf{v} = \mathbf{u} + \mathbf{a}t$,

$(6\mathbf{i} - 8) = \mathbf{a} \times 5,$

$$\mathbf{a} = \frac{1}{5}(6\mathbf{i} - 8\mathbf{j})$$

Using $\mathbf{F} = m\mathbf{a}$,

$$\mathbf{F} = 4 \times \frac{1}{5}(6\mathbf{i} - 8\mathbf{j})$$
$$= 4.8\mathbf{i} - 6.4\mathbf{j}$$

2 Using $\mathbf{F} = m\mathbf{a}$,

$(2\mathbf{i} - \mathbf{j}) = 2\mathbf{a},$

$$\mathbf{a} = \mathbf{i} - \frac{1}{2}\mathbf{j}$$

$\mathbf{u} = \mathbf{i} + 3\mathbf{j},\ t = 3,\ \mathbf{a} = \mathbf{i} - \frac{1}{2}\mathbf{j},\ \mathbf{s} = ?$

Using $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$,

$$\mathbf{s} = (\mathbf{i} + 3\mathbf{j}) \times 3 + \frac{1}{2}\left(\mathbf{i} - \frac{1}{2}\mathbf{j}\right) \times 3^2$$
$$= 3\mathbf{i} + 9\mathbf{j} + \frac{9}{2}\mathbf{i} - \frac{9}{4}\mathbf{j}$$
$$= \frac{15}{2}\mathbf{i} + \frac{27}{4}\mathbf{j}$$

$$\text{distance} = \sqrt{\left(\frac{15}{2}\right)^2 + \left(\frac{27}{4}\right)^2}$$
$$= \sqrt{56.25 + 45.5625}$$
$$= \sqrt{101.8125}$$
$$= 10.1\text{m (3 s.f.)}$$

3 **a** Using $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$,

$$\mathbf{r} = -500\mathbf{j} + (2\mathbf{i} + 3\mathbf{j}) \times t$$
$$= -500\mathbf{j} + 2t\mathbf{i} + 3t\mathbf{j}$$
$$= 2t\mathbf{i} + (-500 + 3t)\mathbf{j}$$

3 b 5 minutes $= 5 \times 60$ seconds

$= 300$ seconds

At 2.05 pm, the dinghy has position:

$\mathbf{r} = 2 \times 300\,\mathbf{i} + (-500 + 3 \times 300)\,\mathbf{j}$

$= 600\,\mathbf{i} + 400\,\mathbf{j}$

distance $= \sqrt{600^2 + 400^2}$

$= \sqrt{360\,000 + 160\,000}$

$= \sqrt{520\,000}$

$= 721\text{ m}$ (3 s.f.)

4 a Using $\mathbf{r}_A = \mathbf{r}_{A_0} + \mathbf{v}_A t$ for **A**,

$\mathbf{r}_A = (\mathbf{i} + 3\,\mathbf{j}) + (2\mathbf{i} - \mathbf{j}) \times t$

$= (1 + 2t)\mathbf{i} + (3 - t)\mathbf{j}$

Using $\mathbf{r}_B = \mathbf{r}_{B_0} + \mathbf{v}_B t$ for **B**,

$\mathbf{r}_B = (5\mathbf{i} - 2\,\mathbf{j}) + (-\mathbf{i} + 4\mathbf{j}) \times t$

$= (5 - t)\mathbf{i} + (-2 + 4t)\mathbf{j}$

b $\mathbf{r}_{\overline{AB}} = \mathbf{r}_B - \mathbf{r}_A$

$= \big((5-t)\mathbf{i} + (-2+4t)\mathbf{j}\big) - \big((1+2t)\mathbf{i} + (3-t)\mathbf{j}\big)$

$= (5 - t - 1 - 2t)\mathbf{i} + (-2 + 4t - 3 + t)\mathbf{j}$

$= (4 - 3t)\mathbf{i} + (-5 + 5t)\mathbf{j}$

c If A and B collide, the vector $\boldsymbol{AB}$ would be zero, so $4 - 3t = 0$ and $-5 + 5t = 0$, but these two equations are not consistent $(t = 1 \text{ and } t \neq 1)$, so vector $\boldsymbol{AB}$ can never be zero and A and B will not collide.

d At 10 am, $t = 2$:

$\mathbf{r}_{\overline{AB}} = (4 - 3 \times 2)\,\mathbf{i} + (-5 + 5 \times 2)\,\mathbf{j}$

$= -2\,\mathbf{i} + 5\,\mathbf{j}$

Distance $= \sqrt{(-2)^2 + 5^2}$

$= \sqrt{29}$

$= 5.39\text{ km}$

5 Let x be the horizontal distance between O and S, and y be the vertical distance between O and S.
$\mathbf{u} = 8\mathbf{i} + 10\mathbf{j},\ \mathbf{a} = -9.8\mathbf{j},\ t = 6,\ \mathbf{s} = x\mathbf{i} + y\mathbf{j}$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$x\mathbf{i} + y\mathbf{j} = (8\mathbf{i} + 10\mathbf{j}) \times 6 + \frac{1}{2}(-9.8\mathbf{j}) \times 36$$

a Equating **i** components:
$x = 48$
The horizontal distance between O and S is 48 m.

b Equating **j** components:
$y = 60 - 4.9 \times 36$
$= -116$
The vertical distance between O and S is 116 m (3 s.f.).

6 $\mathbf{u} = (p\mathbf{i} + q\mathbf{j})\ \text{ms}^{-1},\ \mathbf{r}_0 = 0.8\mathbf{j}\ \text{m},\ \mathbf{a} = -9.8\mathbf{j}\ \text{ms}^{-2};\ t = 4\ \text{s},\ \mathbf{r} = 64\mathbf{i}\ \text{m}$

a Combining $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ and $\mathbf{s} = \mathbf{r} - \mathbf{r_0}$ gives

$$\mathbf{r} - \mathbf{r_0} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

Using vector notation:

$$\begin{pmatrix} 64 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0.8 \end{pmatrix} = 4\begin{pmatrix} p \\ q \end{pmatrix} + \frac{4^2}{2}\begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$$

$$\begin{pmatrix} 64 \\ -0.8 \end{pmatrix} = \begin{pmatrix} 4p \\ 4q - 78.4 \end{pmatrix}$$

Considering **i** components:
$64 = 4p$
$p = 16$
Considering **j** components:
$-0.8 = 4q - 78.4$
$4q = 78.4 - 0.8$
$q = 19.6 - 0.2 = 19.4$

The values of p and q are 16 and 19.4 respectively. $\mathbf{s} = \begin{pmatrix} 16 \\ 19.4 \end{pmatrix} t + \begin{pmatrix} 0 \\ -4.9 \end{pmatrix} t^2$

b $|\mathbf{u}| = \sqrt{16^2 + 19.4^2} = 25.146...$
The initial speed of the ball is 25.1 ms^{-1}.

c $\tan\alpha = \dfrac{q}{p} = \dfrac{19.4}{16}$

The exact value of $\tan\alpha$ is $\dfrac{97}{80}$

6 d Use $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ and $\mathbf{s} = \mathbf{r} - \mathbf{r}_0$ to find values of t for which $\mathbf{r} = x\mathbf{i} + 5\mathbf{j}$:

$$\begin{pmatrix} x \\ 5 \end{pmatrix} - \begin{pmatrix} x \\ 0.8 \end{pmatrix} = \begin{pmatrix} 16 \\ 19.4 \end{pmatrix} t + \begin{pmatrix} 0 \\ -4.9 \end{pmatrix} t^2$$

Considering **j** components:

$$5 - 0.8 = 19.4t - 4.9t^2$$

$$4.9t^2 - 19.4t + 4.2 = 0$$

Using the equation for the roots of a quadratic equation:

$$t = \frac{19.4 \pm \sqrt{19.4^2 - (4 \times 5 \times 4.2)}}{2 \times 4.9}$$

$t = 3.7243...$ or $t = 0.2348...$

The ball is above 5 m between these two times, i.e. for 3.7243… − 0.2348…= 3.50 s (3s.f.).

e To make the model more realistic, one should consider factors such as air resistance and how it is affected by the shape (especially the seam) and the spin of the ball.

7 a $\mathbf{r} = \int \mathbf{v}\,\mathrm{d}t = \int t\left(2t^2 + 14\right)^{\frac{1}{2}} \mathrm{d}t$

$$= \frac{2}{3 \times 2 \times 2}\left(2t^2 + 14\right)^{\frac{3}{2}} + c$$

$$= \frac{1}{6}\left(2t^2 + 14\right)^{\frac{3}{2}} + c$$

$\mathbf{r} = 0$ when $t = 0$, hence

$$0 = \frac{1}{6}(0 + 14)^{\frac{3}{2}} + c$$

$$c = -8.73$$

$$\Rightarrow \mathbf{r} = \frac{1}{6}\left(2t^2 + 14\right)^{\frac{3}{2}} - 8.73$$

At $t = 5$ s, $\mathbf{r} = \frac{1}{6}(50 + 14)^{\frac{3}{2}} + 8.73 = 76.6$

At $t = 5$ s, the displacement of P from O is 76.6 m (3s.f.).

b $\mathbf{v} = \frac{1000}{t^2}$ ms^{-1}, $t = 5$ s, $\mathbf{r} = 76.6$ m; $t = 6$ s, $\mathbf{r} = ?$

$\mathbf{r} = \int \mathbf{v}\,\mathrm{d}t = \int 1000t^{-2}\mathrm{d}t$

$$= -\frac{1000}{t} + c$$

Using that fact that at $t = 5$ s the position of the particle will be as given in part **a**:

$$76.6 = \frac{-1000}{5} + c$$

$$c = 76.6 + 200 = 276.6$$

$$\Rightarrow \mathbf{r} = \frac{-1000}{t} + 276.6$$

At $t = 6$ s,

$$\mathbf{r} = \frac{-1000}{6} + 276.6 = 109.9$$

At $t = 6$ s, the displacement of P from O is 110 m (3s.f.).

8 a $x = 2t + k(t+1)^{-1}$

$v = \frac{dx}{dt} = 2 - k(t+1)^{-2} = 2 - \frac{k}{(t+1)^2}$

When $t = 0$, $v = 6$

$6 = 2 - \frac{k}{1^2} \Rightarrow k = -4$

b With $k = -4$

$x = 2t - \frac{4}{t+1}$

When $t = 0$, $x = 0 - \frac{4}{0+1} = -4$

The distance of P from O when $t = 0$ is 4 m.

c $v = 2 - 4(t+1)^{-2}$

$a = \frac{dv}{dt} = 8(t+1)^{-3} = \frac{8}{(t+1)^3}$

When $t = 3$

$a = \frac{8}{4^3} = \frac{1}{8}$

$F = ma$

$= 0.4 \times \frac{1}{8} = 0.05$

The magnitude of **F** when $t = 3$ is 0.05.

9 a When $t = \frac{1}{2}$

$x = 0.6\cos\left(\frac{\pi}{3} \times \frac{1}{2}\right)$

$= 0.6\cos\frac{\pi}{6}$

$= 0.6 \times \frac{\sqrt{3}}{2} = 0.3\sqrt{3}$

The distance of B from O when $t = \frac{1}{2}$ is $0.3\sqrt{3}$ m.

b $v = \frac{dx}{dt} = -0.6 \times \frac{\pi}{3}\sin\left(\frac{\pi t}{3}\right)$

The smallest value at which $v = 0$ is given by

$\frac{\pi t}{3} = \pi \Rightarrow t = 3$ s.

9 c $a = \frac{dv}{dt} = -0.6\left(\frac{\pi}{3}\right)^2 \cos\left(\frac{\pi t}{3}\right)$

When $t = 1$

$a = -0.6\left(\frac{\pi}{3}\right)^2 \cos\left(\frac{\pi}{3}\right) = -0.3289...$

The magnitude of the acceleration of B when $t = 1$ is $0.329\,\text{m s}^{-2}$ (3 s.f.).

10 a $v = \frac{dx}{dt} = 4e^{-0.5t} - 2te^{-0.5t}$

$a = \frac{dv}{dt} = -2e^{-0.5t} - 2e^{-0.5t} + te^{-0.5t} = (t-4)e^{-0.5t}$

When $t = \ln 4$

$a = (\ln 4 - 4)e^{-0.5\ln 4}$

$= (\ln 2^2 - 4)e^{\ln 4^{-\frac{1}{2}}}$

$= (2\ln 2 - 4)e^{\ln\frac{1}{2}}$

$= \frac{1}{2}(2\ln 2 - 4)$

$= \ln 2 - 2$

The acceleration of S when $t = \ln 4$ is $(\ln 2 - 2)\,\text{m s}^{-2}$ in the direction of x increasing.

b For a maximum of x, $\frac{dx}{dt} = v = 0$

$v = (4 - 2t)e^{-0.5t} = 0 \Rightarrow t = 2$

When $t = 2$

$x = 4 \times 2e^{-0.5\times 2} = 8e^{-1}$

The greatest distance of S from O is $\frac{8}{e}$ m.

11 a $\mathbf{v}_P = \dot{\mathbf{r}}_P = 6t\mathbf{i} + 2\mathbf{j}$

$\mathbf{v}_Q = \dot{\mathbf{r}}_Q = \mathbf{i} + 3t\mathbf{j}$

$\frac{d}{dt}\left((t+6)\mathbf{i}\right) = 1\mathbf{i} = \mathbf{i}$

The velocity of P at time t seconds is $(6t\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$ and the velocity of Q is $(\mathbf{i} + 3t\mathbf{j})\,\text{m s}^{-1}$

b When $t = 2$

$\mathbf{v}_P = 12\mathbf{i} + 2\mathbf{j}$

$|\mathbf{v}_P|^2 = 12^2 + 2^2 = 148 \Rightarrow \mathbf{v}_P = \sqrt{148} = 12.165...$

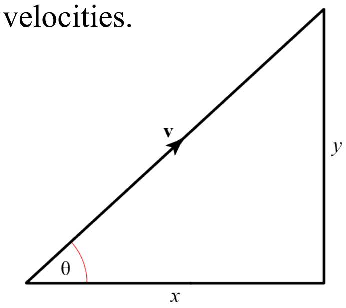
The speed of P when $t = 2$ is $12.2\,\text{m s}^{-1}$ (3 s.f.).

11 c When P is moving parallel to Q

$$\frac{2}{6t}=\frac{3t}{1}$$
$$\Rightarrow 18t^2=2$$
$$\Rightarrow t^2=\frac{1}{9}$$
$$t \geqslant 0,\ t=\frac{1}{3}$$

When the particles are moving parallel to each other, the angle each makes with **i** is the same.

If $\mathbf{v}=x\mathbf{i}+y\mathbf{j}$, $\tan\theta=\frac{y}{x}$ must be the same for both velocities.



d **i**-components

$$3t^2+4=t+6$$
$$3t^2-t-2=0$$
$$(t-1)(3t+2)=0$$
$$t=1,\ -\frac{2}{3}$$

j-components

$$2t-\frac{1}{2}=\frac{3t^2}{2}$$
$$3t^2-4t+1=0$$
$$(t-1)(3t-1)=0$$
$$t=1,\ \frac{1}{3}$$

For the particles to collide, both the **i** and **j** components of their position vectors must be the same for the same value of t. The appropriate method is to equate the **i** components and solve the resulting quadratic, and then do the same for **j** components. If one of the roots of the quadratics is the same, then the particles collide.

1 is a common root of the equations and, hence, P and Q collide at the point with position vector $\left(7\mathbf{i}+\frac{3}{2}\mathbf{j}\right)$m.

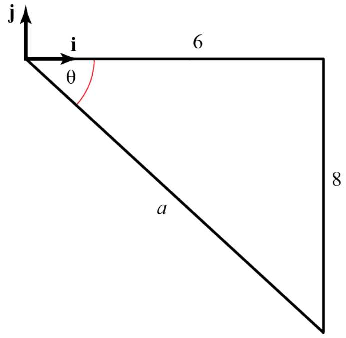
$t=1$ can be substituted into either $\mathbf{r}_P$ or $\mathbf{r}_Q$ to find the position vector of the point where the particles collide.

12 a $\mathbf{v}=\dot{\mathbf{r}}=6t\mathbf{i}-8t\mathbf{j}$

$\mathbf{a}=\dot{\mathbf{v}}=6\mathbf{i}-8\mathbf{j}$

Acceleration does not depend on t, hence the acceleration is constant.

12 b



$|\mathbf{a}|^2 = 6^2 + (-8)^2 = 100$

$\Rightarrow |\mathbf{a}| = 10$

The magnitude of the acceleration is $10\,\mathrm{m\,s^{-2}}$

$\tan\theta = \frac{8}{6} \Rightarrow \theta = 53.1°$

The angle the acceleration makes with **j** is $90° + 53.1° = 143.1°$ (nearest 0.1°)

13 a $\mathbf{v} = \dot{\mathbf{r}} = -6\sin 3t\,\mathbf{i} - 6\cos 3t\,\mathbf{j}$

When $t = \frac{\pi}{6}$

$\mathbf{v} = -6\sin\frac{\pi}{2}\mathbf{i} - 6\cos\frac{\pi}{2}\mathbf{j}$

$= -6\mathbf{i} - 0$

The velocity of P when $t = \frac{\pi}{6}$ is $-6\mathbf{i}$ m s^{-1}

b $\mathbf{a} = \dot{\mathbf{v}} = -18\cos 3t\,\mathbf{i} + 18\sin 3t\,\mathbf{j}$

$|\mathbf{a}|^2 = (-18\cos 3t)^2 + (18\sin 3t)^2$

$= 18^2(\cos^2 3t + \sin^2 3t) = 18^2$

$|\mathbf{a}| = 18$

The magnitude of the acceleration is $18\,\mathrm{m\,s^{-2}}$, which is constant.

14 a $\mathbf{a} = \dot{\mathbf{v}} = 4c\,\mathbf{i} + 2(7-c)t\,\mathbf{j}$

$\mathbf{F} = m\mathbf{a}$

$= 0.5(4c\,\mathbf{i} + 2(7-c)t\,\mathbf{j})$

$= 2c\,\mathbf{i} + (7-c)t\,\mathbf{j}$, as required

b $t = 5 \Rightarrow \mathbf{F} = 2c\,\mathbf{i} + 5(7-c)\,\mathbf{j}$

$|\mathbf{F}|^2 = 4c^2 + 25(7-c)^2 = 17^2$

$4c^2 + 1225 - 350c + 25c^2 = 289$

$29c^2 - 350c + 936 = 0$

$(c-4)(29c-234) = 0$

$c = 4, \ \frac{234}{29} \approx 8.07$

15 a $\mathbf{v}=\int \mathbf{a}\,\mathrm{d}t=\int((8t^3-6t)\mathbf{i}+(8t-3)\mathbf{j})\mathrm{d}t$

$=(2t^4-3t^2)\mathbf{i}+(4t^2-3t)\mathbf{j}+C$

When $t=2$, $\mathbf{v}=16\mathbf{i}+3\mathbf{j}$

$16\mathbf{i}+3\mathbf{j}=20\mathbf{i}+10\mathbf{j}+C\Rightarrow C=-4\mathbf{i}-7\mathbf{j}$

$\mathbf{v}=(2t^4-3t^2-4)\mathbf{i}+(4t^2-3t-7)\mathbf{j}$

The velocity of P after t seconds is $\left((2t^4-3t^2-4)\mathbf{i}+(4t^2-3t-7)\mathbf{j}\right)\mathrm{m\,s^{-1}}$

b When P is moving parallel to **i**, the **j** component of the velocity is zero.

$4t^2-3t-7=0$

$(t+1)(4t-7)=0$

$t\geqslant 0\Rightarrow t=\frac{7}{4}$

16 $\mathbf{a}=\left(4t\mathbf{i}+5t^{-\frac{1}{2}}\mathbf{j}\right)$ ms^{-2}, $t=0$ s, $\mathbf{v}=10\mathbf{i}$ ms^{-1}; $t=5$ s, $|\mathbf{v}|=?$

$\mathbf{v}=\int \mathbf{a}\,\mathrm{d}t=\int\left(4t\mathbf{i}+5t^{-\frac{1}{2}}\mathbf{j}\right)\mathrm{d}t$

$=\frac{4t^2}{2}\mathbf{i}+\frac{5}{\frac{1}{2}}t^{\frac{1}{2}}\mathbf{j}+c$

$=2t^2\mathbf{i}+10t^{\frac{1}{2}}\mathbf{j}+c$

When $t=0$ s, $\mathbf{v}=10\,\mathbf{i}$ ms^{-1}

$10\mathbf{i}=0\mathbf{i}-0\mathbf{j}+c$

$c=10\mathbf{i}$

$\Rightarrow \mathbf{v}=(2t^2+10)\mathbf{i}+10t^{\frac{1}{2}}\mathbf{j}$

At $t=5$ s,

$\mathbf{v}=(50+10)\mathbf{i}+10\sqrt{5}\mathbf{j}$

$|\mathbf{v}|=\sqrt{60^2+\left(10\sqrt{5}\right)^2}=\sqrt{4100}$

$|\mathbf{v}|=10\sqrt{41}$

At $t=5$ s, the speed of the ball is $10\sqrt{41}$ ms^{-1}.

17 a $\mathbf{v}=\int \mathbf{a}\,\mathrm{d}t=\int 2t\,\mathbf{i}+3\mathbf{j}\,\mathrm{d}t$

$=t^2\mathbf{i}+3t\mathbf{j}+c$

When $t=0$ s, $v=3\mathbf{i}+13\mathbf{j}$

$3\mathbf{i}+13\mathbf{j}=0\mathbf{i}+0\mathbf{j}+c$

$c=3\mathbf{i}+13\mathbf{j}$

$\mathbf{v}=\left(t^2+3\right)\mathbf{i}+\left(3t+13\right)\mathbf{j}$

17 b When the train is moving NE, the coefficients of the **i** and **j** components are equal and positive.

$$t^2 + 3 = 3t + 13$$
$$t^2 - 3t - 10 = 0$$
$$(t-5)(t+2) = 0$$
$$t = 5, \ -2$$

Ignoring the negative root, as it denotes a time before the train was moving, the train is moving NE at $t = 5$ s (3s.f.).

Challenge

1 a $s(0) = 20$ m

b $$\frac{ds}{dt} = (20 - t^2) \times \frac{1}{2}(t+1)^{-\frac{1}{2}} - 2t(t+1)^{\frac{1}{2}}$$
$$= \frac{(20-t^2) - 4t(t+1)}{2(t+1)^{\frac{1}{2}}}$$
$$= \frac{20 - 4t - 5t^2}{2\sqrt{t+1}}$$

Particle changes direction when $v = \frac{ds}{dt} = 0 \Rightarrow$

$20 - 4t - 5t^2 = 0$

$t = 1.64$ s (ignoring negative root, since $t \geqslant 0$)

So particle changes direction exactly once, when $t = 1.64$ s

c Particle crosses O when $s = 0$

$0 = (20 - t^2)\sqrt{t+1}$

$t = \sqrt{20}$

At $t = \sqrt{20}$ s, $$\frac{ds}{dt} = \frac{20 - 4\sqrt{20} - 5 \times 20}{2\sqrt{\sqrt{20}+1}}$$
$$= \frac{-40 - 2\sqrt{20}}{\sqrt{\sqrt{20}+1}}$$
$$= -2\sqrt{20}\left(\sqrt{20}+1\right)^{\frac{1}{2}}$$

Challenge

2 a $\mathbf{v} = \dot{\mathbf{r}} = (6\omega\cos\omega t)\mathbf{i} - (4\omega\sin\omega t)\mathbf{j}$

$$v^2 = |\mathbf{v}|^2 = 36\omega^2\cos^2\omega t + 16\omega^2\sin^2\omega t$$
$$= 36\omega^2\left(\frac{1}{2}+\frac{1}{2}\cos 2\omega t\right) + 16\omega^2\left(\frac{1}{2}-\frac{1}{2}\cos 2\omega t\right)$$
$$= 18\omega^2 + 18\omega^2\cos 2\omega t + 8\omega^2 - 8\omega^2\cos 2\omega t$$
$$= 26\omega^2 + 10\omega^2\cos 2\omega t$$
$$= 2\omega^2(13 + 5\cos 2\omega t), \text{ as required.}$$

Use the double angle formulae $\cos 2\theta = 2\cos^2\theta - 1$ and $\cos 2\theta = 1 - 2\sin^2\theta$

b As $-1 \leqslant \cos 2\omega t \leqslant 1$

$$2\omega^2(13-5) \leqslant 2\omega^2(13+5\cos 2\omega t) \leqslant 2\omega^2(13+5)$$
$$16\omega^2 \leqslant v^2 \leqslant 36\omega^2$$

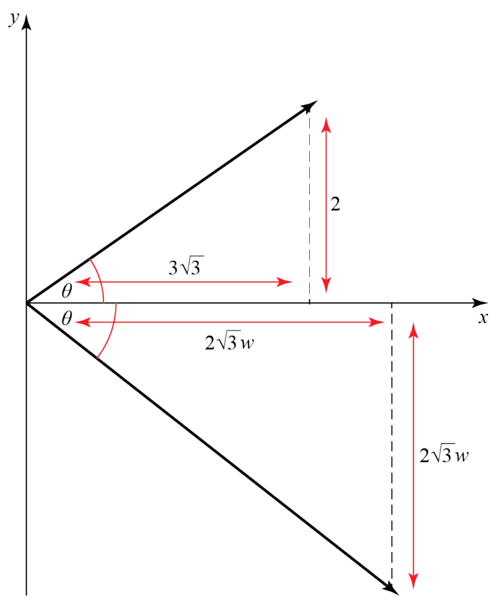
As $v > 0$ and $\omega > 0$, we can take the square root of each term and it will not change the inequality signs:

$4\omega \leqslant v \leqslant 6\omega$, as required.

c When $t = \dfrac{\pi}{3\omega}$

$$\mathbf{r} = \left(6\sin\frac{\pi}{3}\right)\mathbf{i} + \left(4\cos\frac{\pi}{3}\right)\mathbf{j} = 3\sqrt{3}\,\mathbf{i} + 2\,\mathbf{j}$$
$$\dot{\mathbf{r}} = \left(6\omega\cos\frac{\pi}{3}\right)\mathbf{i} - \left(4\omega\sin\frac{\pi}{3}\right)\mathbf{j} = 3\omega\,\mathbf{i} - 2\sqrt{3}\omega\,\mathbf{j}$$

Using $\cos\frac{\pi}{3} = \frac{1}{2}$ and $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$



A diagram is essential here. Once the diagram has been drawn, the problem reduces to basic trigonometry. You find the angles using the inverse tangent button on your calculator.

$$\tan\theta = \frac{2}{3\sqrt{3}} \Rightarrow \theta = 0.3674...^c$$
$$\tan\phi = \frac{2\sqrt{3}\omega}{3\omega} = \frac{2\sqrt{3}}{3} \Rightarrow \phi = 0.8570...^c$$

The angle between $\mathbf{r}$ and $\dot{\mathbf{r}}$ is

$\theta + \phi = 0.3674...^c + 0.8570...^c = 1.224...^c = 70.2°$ (3 s.f.)